1 The importance of mathematics in economics

Below you can find my very personal viewpoint.

Why should economists study mathematics? Because math helps in dealing with many significant problems in economics. Let's present a list of some of those problems. Many other problems of the same type are listed in Simon and Blume (1994).

A macroeconomic problem.

Let's consider the following version of the so-called IS-LM model you may have studied in Macroeconomic courses.

Let the following exogenous variables (or parameters) be given: $s \in (0,1), g, m \in (0, \infty)$, where s is saving propensity, g is public expenditure, m is supply of money. Let the following functions be given:

$$l : \mathbb{R}^2_{++} \to \mathbb{R}, \quad (y, r) \mapsto l(y, r)$$

and

$$i : \mathbb{R}_{++} \to \mathbb{R}, \quad r \mapsto i(r),$$

where y is GNP, r the interest rate, l the money demand function and i the investment function. Given the following "equilibrium" system

$$\begin{cases} i(r) + g & = & s \cdot y \\ \\ l(y, r) & = & m, \end{cases}$$

what is the effects of changes in the exogenous variable g on the (endogenous) variable y ?

A finance problem

Consider a two period model with $S \in \mathbb{N}$ states in the second period.

$$\begin{array}{ccccccc} & & \text{state } 0 & & & & \text{today} \\ & \swarrow & \downarrow & & \searrow & & \\ 1 & ... & s & & ... & S & \text{tomorrow} \end{array}$$

An asset a is a list of $(S+1)$ real numbers

$$\left[\; q(a), \quad y(a, 1), \quad ..., \quad y(a, s), \quad ..., \quad y(a, S) \; \right],$$

where $q(a)$ is the price of that asset and for any $s \in \{1, ..., S\}$, $y(a, s)$ is the amount of "objects" you get in state s if you buy one unit of that asset.

$z(a)$ is the amount of asset a you decide to buy.

The financial structure of the above economy is then described by the following table, in which each column describes asset $a \in \{1, ..., A\}$ (and A is the total number of available assets):

$$\left[\begin{array}{ccccc} q(1) & ... & q(a) & ... & q(A) \\ y(1,1) & ... & y(a,1) & ... & y(A,1) \\ ... & & ... & & ... \\ y(1,s) & & y(a,s) & & y(A,s) \\ ... & & ... & & ... \\ y(1,S) & & y(a,S) & & y(A,S) \end{array} \right].$$

A portfolio is a list of A real numbers

$$\left[\; z(1), \quad ... \quad z(a), \quad .... \quad z(A) \; \right].$$

How much does it cost to buy a given portfolio?

What is a risk free portfolio?

What do we mean by a no-arbitrage portfolio?

What is insurance?

Which portfolio an investor should choose?

A game theory (or industrial organization) problem

Consider a situation in which two firms interact. Firm 1 and 2 can choose "Advertising or Not advertising". Profits of each firm in all possible situations are described below:

firm 2 $\rightarrow$ / firm 1 $\downarrow$	A	N
A	$(2,2)$	$(4,1)$
N	$(1,4)$	$(3,3)$

Do firms choose to advertise?
What is a Nash equilibrium in this game?

Microeconomic problems.

1. A consumer has a given positive wealth w and she wants to buy some bundle of n goods whose prices are $p_1, p_2, ..., p_n$. What bundle of goods does she choose?
2. What are the choices of a firm which wants to maximize profits?

Economics

A very common definition of Economics is the following one.

"Economics is the study of the use of scarce resources to satisfy unlimited human wants" (Lipsey and others (1990)).

If we take seriously that definition, we have to study the problem of maximizing the satisfaction of human wants for given finite resources. We want to solve a problem of the form

$$\max_x \quad f(x) \qquad s.t. \qquad x \in C,$$

i.e., we want to find $x^* \in C$ such that for any $x \in X$, we have that $f(x^*) \geq f(x)$.

The goal of the course can be seen as to provide mathematical tools to
1. formalize the above problems;
2. give an answer to the related questions.

2 Do economists think mathematics is important for economics?

Most economists do agree with the importance of mathematics, but they believe you can use mathematics in economics without a thorough, deep understanding of the math tools you use. According to them, it is enough to provide **"intuition"** and **"recipes"**.

2.1 Intuition

Intuition is the ability of making conjectures and to "explain" things which are true.[1] Intuition is very important: it guides you in looking for true statements, but our intuition may be wrong in several ways as summarized in the table below.

		it is true		it is false
it seems true		TT		TF
it seems false		FT		FF

[1] Definition of "intuition" found on some dictionaries:
Direct perception of truth, fact, etc., independent of any reasoning process (dictionary.com);
A natural ability or power that makes it possible to know something without any proof or evidence : a feeling that guides a person to act a certain way without fully understanding why (on line Merriam - Webster dictionary).

TT:

1. Two disks in $\mathbb{R}^2$ with nonempty intersection can be separated by a line (Formally: Let X be a topological vector space. If 1. A and B are convex, and non-empty, 2. $A \cap B = \varnothing$, 3. either A or B has an interior point, then A and B can be properly separated by a linear and continuous functional).

2. If a function is continuous, then it has a global maximum on a interval $[a, b]$ - with $a < b$.

...

FT:

1. there exists a one-to-one and onto function $f : \mathbb{R} \to \mathbb{R}^2$;

2. there exists a function $f : \mathbb{R} \to \mathbb{R}$ which is continuous and nowhere differentiable - see https://en.wikipedia.org/wiki/Weierstrass_function

3. there exists a continuous function such that $f(0) = -1$ and $f(1) = 1$ and which is not strictly increasing in some intervals contained in $[0, 1]$

...

TF:

1. The sun rotates around earth;

2. a complete ordering admits a utility function.

...

FF:

Hair and fingernails continue to grow after death;

You will get arthritis from cracking your knuckles.

...

* Things which seem to be true and it took hundred of years to be shown to be true:

(what follows is taken from Wikipedia) In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture) states that no three positive integers a, b, c can satisfy the equation $an + bn = cn$ for any integer value of n greater than 2.

This theorem was first conjectured by Pierre de Fermat in 1637 in the margin of a copy of Arithmetica where he claimed he had a proof that was too large to fit in the margin. The first successful proof was released in 1994 by Andrew Wiles, and formally published in 1995, after 358 years of effort by mathematicians. The theretofore unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most notable theorems in the history of mathematics and prior to its proof it was in the Guinness Book of World Records for "most difficult mathematical problems".

* * Things which seem to be true, but we do not if they are true or false:

Goldback conjecture: any even number strictly bigger than 2 is equal to the sum of two prime numbers.

2.2 Recipes

Recipes are very useful; at the end of your analysis in the attempt to solve a general problem, you should try hard to get one. But without proofs they are just "magic" and one of the goal of science is to fight magic statements.

If you use mathematics in economic analysis at the University level, all statements you make should be either

a. proved, or

b. supported by a precise reference in some book or article.

Let a be the proportion of stated facts whose proofs are provided. The determination of the "appropriate" value of a is quite complicated but it should be the higher the more advanced the University course is. For sure, contemporary (macro)economics has choose a value of a too close to *zero*.