

Detailed program of the course.

Matematica per le applicazioni Economiche 2  
e  
Laboratorio.

Tutti i paragrafi cerchiati nell'indice  
degli appunti in rete, come riportato  
di seguito.

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