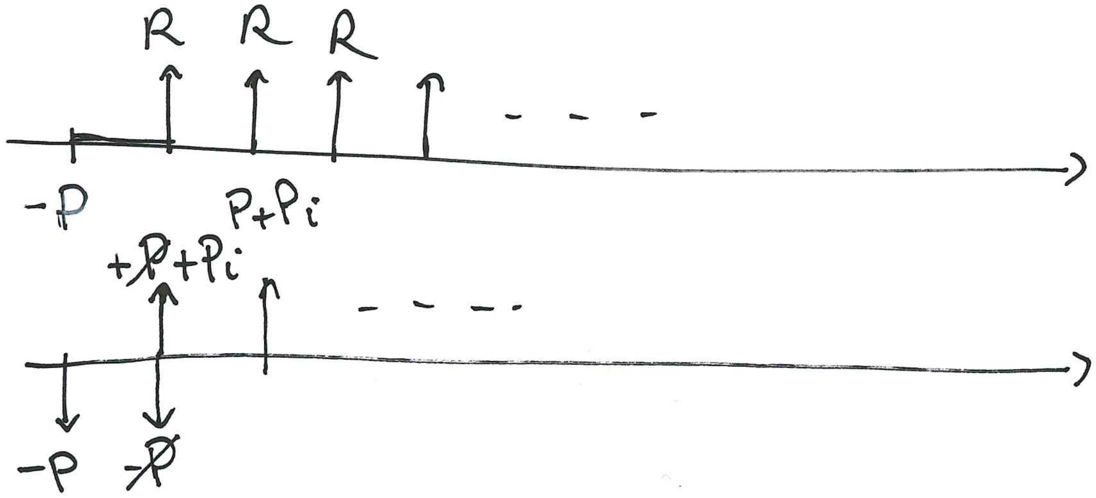
RENDITA PERPETUA

posticip.
immed.



$R = Pi$

Se $R = Pi$

probl. del deposito

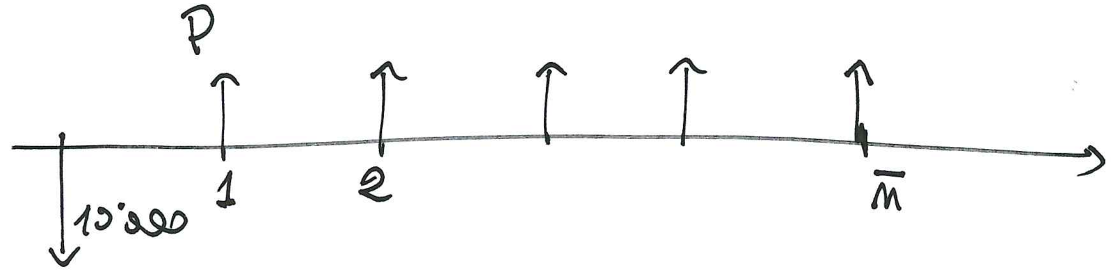
Deposito 10'000 € alla data 0 su un cc al 5% a. cap. comp.

A fine di ogni anni ~~[illegible]~~ vengono prelevati 1'000 €

1) dopo quanto tempo il depos. si esaurisce?

2) qual è il max importo P che si può prelev. affinché il depos. non si esaurisca mai?

1)



$\bar{n}$ = ultimo istante in cui posso prelev.

$n=1$: $\quad 10'000 \cdot (1+i) - P \geq 0$

$n=2$: $0 \leq \left(10'000(1+i) - P\right)(1+i) - P$

$$= 10'000 \cdot (1+i)^2 - P(1+i) - P \quad = M_2$$

$n = 3$:

$$0 \leq M_2(1+i) - P$$

$$= 10'000 \cdot (1+i)^3 - P(1+i)^2 - P(1+i) - P$$

$$M_{\bar{n}+} = 10'000(1+i)^{\bar{n}} - P \cdot s_{\overline{\bar{n}}|i} \geq 0$$

cerco la soluz. $\bar{n}$ più grande possib.

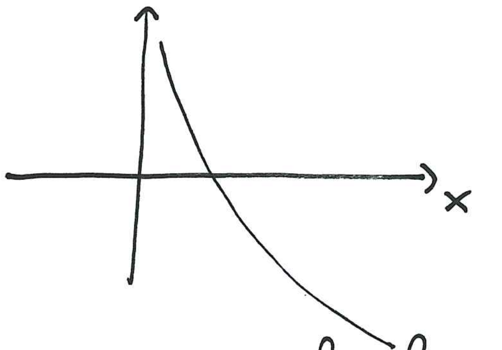
$$10 \cdot (1+i)^{\bar{n}} \geq \frac{-1 + (1+i)^{\bar{n}}}{i}$$

$$10i(1+i)^{\bar{n}} \geq -1 + (1+i)^{\bar{n}}$$

$$(10i - 1)(1+i)^{\bar{n}} \geq -1$$

$$10i - 1 \geq -v^{\bar{n}} \quad , \quad 1 - 10i \leq v^{\bar{n}}$$

$$\log_v(1 - 10i) \geq \log_v(v^{\bar{n}}) = \bar{n}$$



$f(x) = \log_v x$

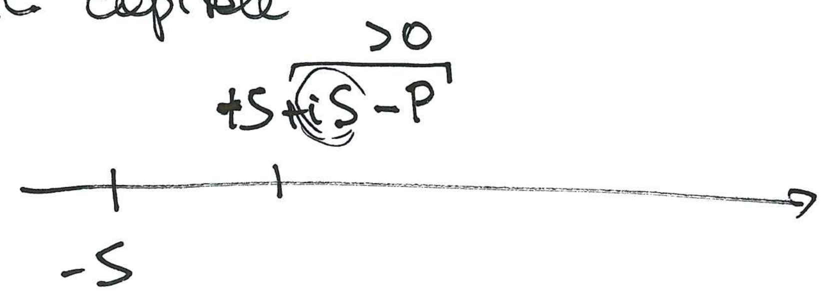
$$x_2 \geq x_1 \rightarrow f(x_2) \leq f(x_1) \quad , f \downarrow$$

$$\bar{m} \leq \log_v(-10i+1) = \frac{\ln(-10i+1)}{\ln v} = 14{,}2$$

$$\Rightarrow \bar{m} = 14$$

2) più alto è P prelevato, prima si esaurisce il depos.

- se $P = iS$ a fine di ogni anno prelevare iS non intacca il capitale che frutta iS, quindi S continua a produrre iS a fine ~~dell~~ di ogni a.
- se $P < iS$ a maggior ragione il prelievo di P non intacca il capitale



- solo se $P \geq iS$ alla fine di 1 anno intacco il capitale S

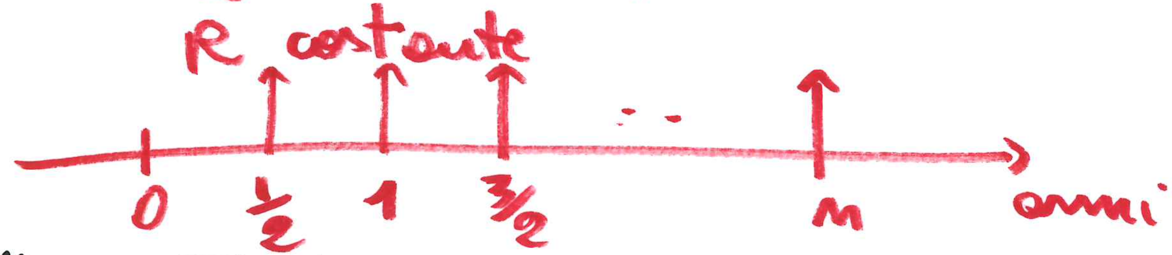
$iS = 10.\cancel{000} \cdot \frac{5}{\cancel{100}} = 500$

$\Rightarrow$ per $P \leq 500$ il deposito non si esaurisce mai

$\rightarrow$ rendita $\{P, P, \ldots \ldots\} / \{1, 2, \ldots\}$ perpetua

Rendita periodica con periodicità $\neq$ anno

tasso di valutaz. i_a annuo :



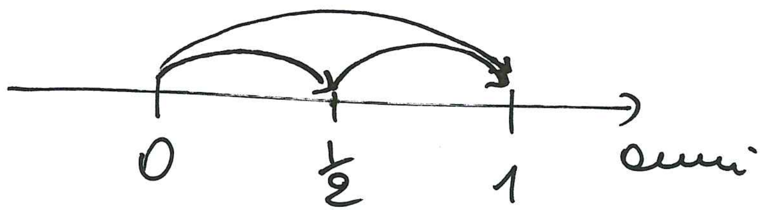
$$V_0(\vec{r}) = R \cdot \sum_{k=1}^{2n} (1+i_a)^{-\frac{k}{2}}$$

$$= R \cdot \underbrace{\sum_{k=1}^{2n} v_2^k}$$

$$= R \cdot a_{\overline{2n}|i_2}$$

$$(1+i_a)^{+\frac{1}{2}} = (1+i_2)^{+1}$$

$$1+i_a = (1+i_2)^2$$

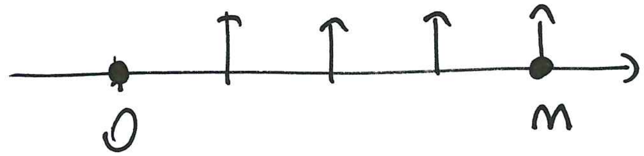


se periodicità della $\vec{r}$ è $\frac{1}{m}$ di anno, i_a, n anni:

$$V_0(\vec{r}) = R \cdot \sum_{k=1}^{n \cdot m} (1+i_a)^{-\frac{k}{m}}$$

$$= R \cdot \sum_{k=1}^{nm} v_m^k$$

$$= R \, a_{\overline{n \cdot m}|\, i_m}$$

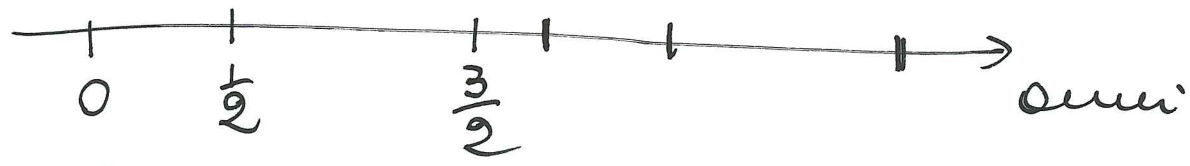


$$V_n(\vec{r}) = R \, a_{\overline{n \cdot m}|\, i_m} \cdot (1+i_a)^n$$

$$= R \, a_{\overline{nm}|\, i_m} \cdot (1+i_m)^{m \cdot n}$$

$$= R \cdot \frac{1 - v_m^{mn}}{i_m} \cdot u_m^{mn} = R \cdot s_{\overline{nm}|\, i_m}$$

se rendita aperiodica



devo applicare la formula generale

$$V_0(\vec{r}) = \sum_{k=1}^{n.\,rate} R_k \,(1+i_a)^{-t_k}$$

<u>EX</u> rendita italiana 5% :

paga semestralm. $R = 2.5\% \cdot N$,

N = nominale

$\vec{r}$ perpetua , $i_a = 3\%$

$N = 10'000$

$V_0(\vec{r})$?

8

$$R = 10'\cancel{000} \cdot \frac{2.5}{\cancel{100}} = 250 \text{ €}$$

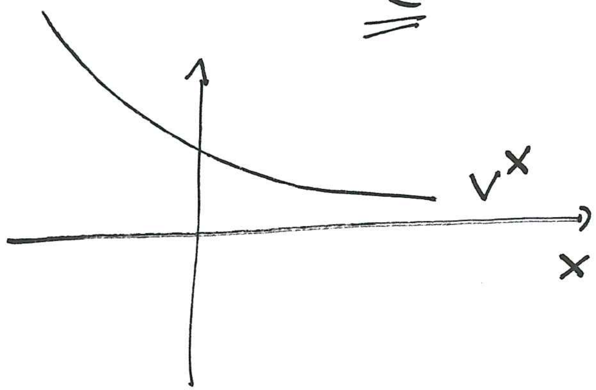
$$i_2 = (1.03)^{\frac{1}{2}} - 1$$

$$V_0 = \frac{R}{i_2} = 16'790,74 \text{ €}$$

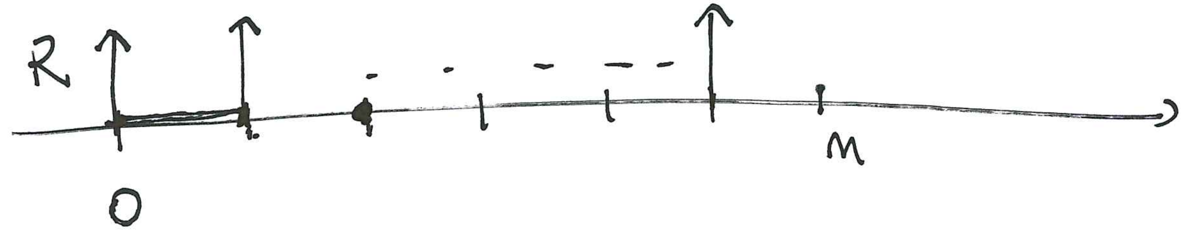
$$V_0(\vec{n}) = R \cdot a_{\overline{2n}|i_2} = R \cdot \frac{1 - v_2^{2n}}{\underline{\underline{i_2}}}$$

$$v_2^{2n} \xrightarrow[n \to +\infty]{} 0$$

$$\frac{1}{1+i_2} < 1 \quad \text{se } i_2 > 0$$



Rendita rata cost. ANTICIPATA, immed. posticip. finita



$$V_0(\vec{\pi}) = R\sum_{k=0}^{n-1} \cancel{R}\cdot v^k$$

$$\sum_{k=0}^{m} v^k = \frac{1-v^{m+1}}{1-v}$$

$$v \neq 1$$

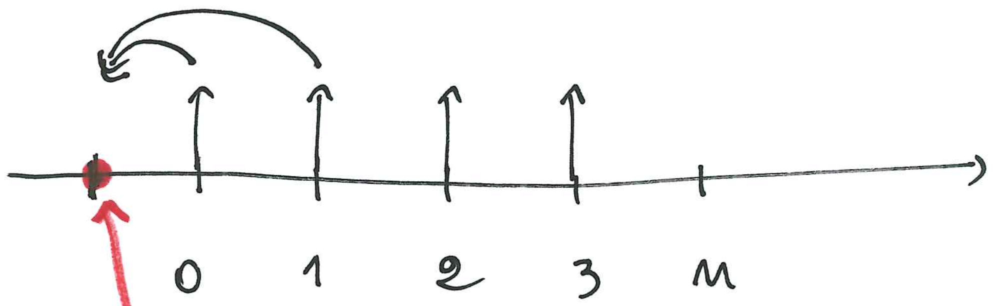
$$= R\cdot \frac{1-v^n}{1-v}$$

$$1-v = d = i\cdot v = \frac{i}{1+i}$$

$$= R\cdot \frac{1-v^n}{\frac{i}{1+i}}$$

$$= R\,\frac{1-v^n}{i}\cdot(1+i)$$

$$= R\cdot \underbrace{a_{\overline{n}|i}\cdot(1+i)}\; = R\cdot \ddot{a}_{\overline{n}|i}$$



$a_{\overline{n}|i}$ = valore di una rendita unitaria di n rate 1 periodo prima dell'ist. del primo pagam.

$$= V_{-1}(\vec{r})$$

$$= R \cdot (v^1 + v^2 + v^3 + v^4)$$

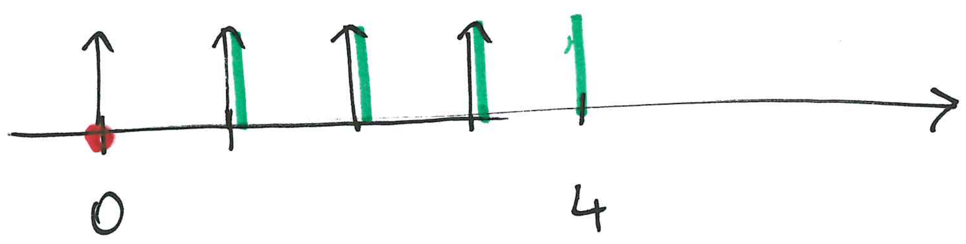
$$= R \cdot \sum_1^4 v^k = R \cdot a_{\overline{4}|i}$$

$$\Rightarrow V_0(\vec{r}) = V_{-1}(\vec{r}) \cdot (1+i) = R\, a_{\overline{4}|i} \cdot (1+i)$$

$$\ddot{a}_{\overline{n}|i} = \frac{1-v^n}{i \cdot v} = \frac{1-v^n}{d}$$

$$a_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i}}{1+i} \underset{i>0}{<} \ddot{a}_{\overline{n}|i}$$

in 0 la rendita antic. VALE di PIÙ della analoga postic.

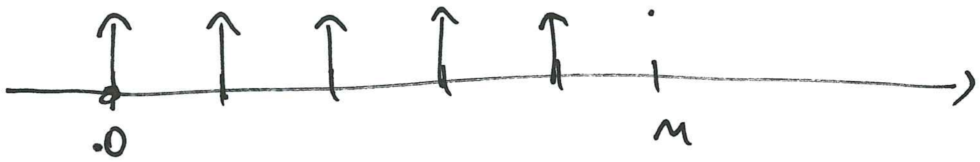


se $i>0$:

(R, 1) in 0 vale $R\nu < R$

(R, 0) in 0 vale R

Rendite anticipate



$$V_0(\vec{r}) = R\,\ddot{a}_{\overline{n}|i}$$

$$V_n(\vec{r}) = V_0(\vec{r})\cdot(1+i)^n = R\cdot\frac{1-v^n}{i}\cdot(1+i)\cdot u^n =$$

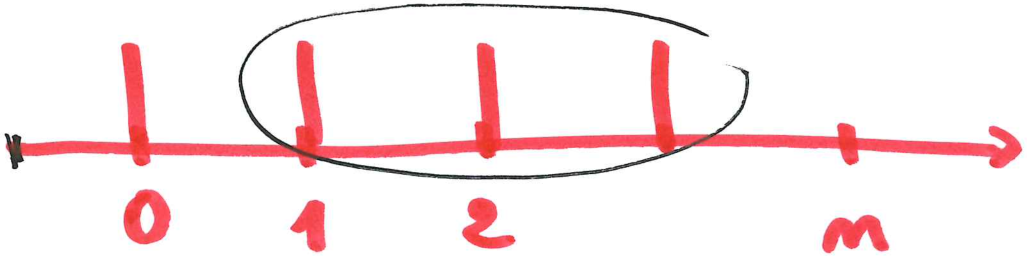
$$= R\,\frac{1-v^n}{d}\,u^n = R\cdot\frac{u^n-1}{d}$$

<u>es.</u> 3% a. , $R = 12'000$ annui , $n = 10$ a.

$$V_0(\vec{r}) = 12'000\cdot\frac{1-1.03^{-10}}{0.03} = 105'433{,}31\ €$$

<u>cfr</u> posticipata : 102'362,43 €

Proprietà



1

$$\ddot{a}_{\overline{n}|i} = a_{\overline{n-1}|i} + 1 \qquad R = 1$$

Rendita antícip. <u>perpetua</u> :

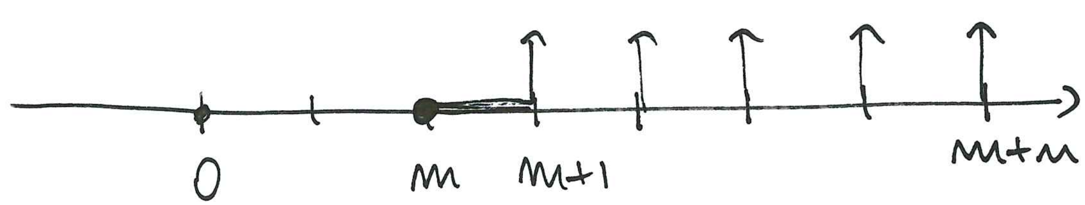
$$V_0(\vec{r}) = \frac{R}{i} + R = R \cdot \left(\frac{1}{i} + 1\right) = R\left(\frac{1+i}{i}\right) =$$

$$= \frac{R}{d}$$

$$= \frac{R}{i} \cdot (1+i)$$

$$= \lim_{n \to +\infty} R \cdot \frac{1-v^n}{d} = \frac{R}{d}$$

Rendita differita di m periodi

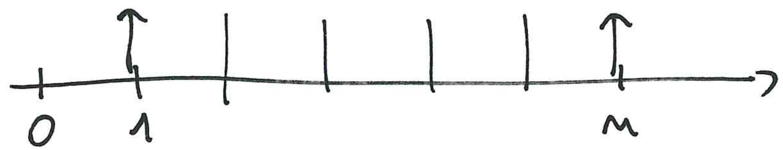


posticip.

$$V_0(\vec{r}) = R \cdot a_{\overline{n}|i} \cdot v^m$$

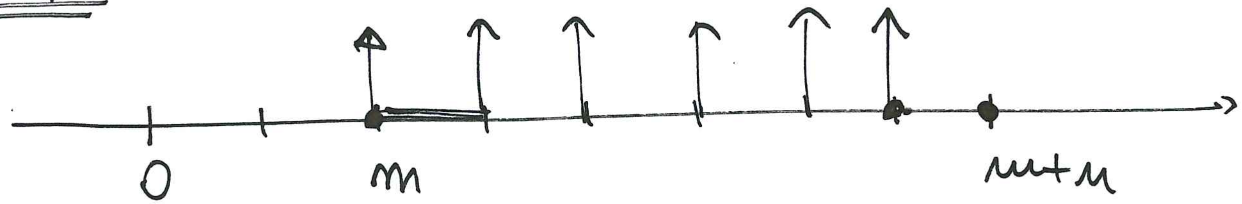
$$V_{m+n}(\vec{r}) = R \cdot a_{\overline{n}|i} \cdot \cancel{v^m} \cdot u^{n+\cancel{m}} = \underbrace{R\, a_{\overline{n}|i}\, u^n} = R\, s_{\overline{n}|i}$$

NON dipende da m



$s_{\overline{n}|i}$ = valore alla data dell'ultimo pagam., di una rend. postic., rata R, n rate

caso anticipato



$$V_0(\vec{r}) = \underbrace{R \cdot \ddot{a}_{\overline{n}|i}}_{\text{val. in } m} \cdot v^m$$

$$V_{m+n} = R \cdot \ddot{a}_{\overline{n}|i} \cdot \cancel{v^m} \cdot u^{\cancel{m}+n} = \underset{\substack{\uparrow \\ \text{non dipende da } m}}{R\, \ddot{s}_{\overline{n}|i}}$$

$\ddot{s}_{\overline{n}|i}$ = valore all' ist. ~~di pagam~~ di fine contratto di un reddito antic. di n rate R

= valore ad un certo istante di un flusso di n rate R che termina 1 periodo prima

<u>ES.</u> $R = 12'000$ € annui, $n = 10$ a., posticip., $i = 3\%$ a.
DIFFERITA di 3 a.

$$V_0(\vec{r}) = R \cdot a_{\overline{10}|i} \cdot v^3 = 93'676{,}12 \text{ €}$$

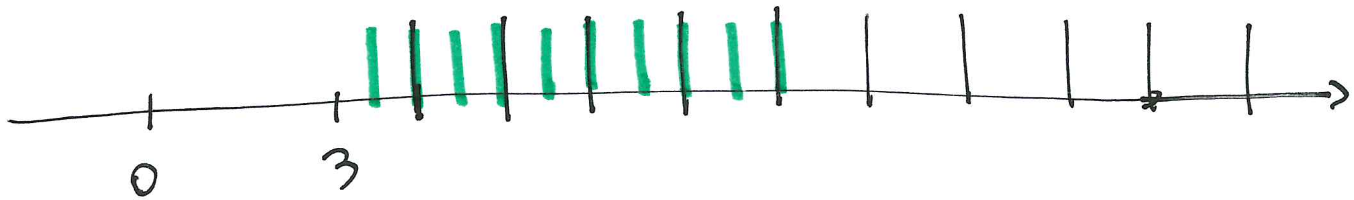
caso immediato $= 102'362{,}43$

Valore della rendita IMMEDIATA è superiore a quello della DIFFERITA perché si comincia a incassare R prima

<u>ES.</u> $R = 12'000$ annui, $n = 5$ anni, postic ip., $i = 3\%$ a. differita di 3 anni, rate semestr.

$$V_0(\vec{r}) = R \cdot a_{\overline{10}|i_2} \cdot (1+i_2)^{-6} = 101'334{,}76$$
$$> 93'676{,}12$$

$$\sum_{k=1}^{10} R_k \cdot (1+i_2)^{-\frac{k}{2}}$$

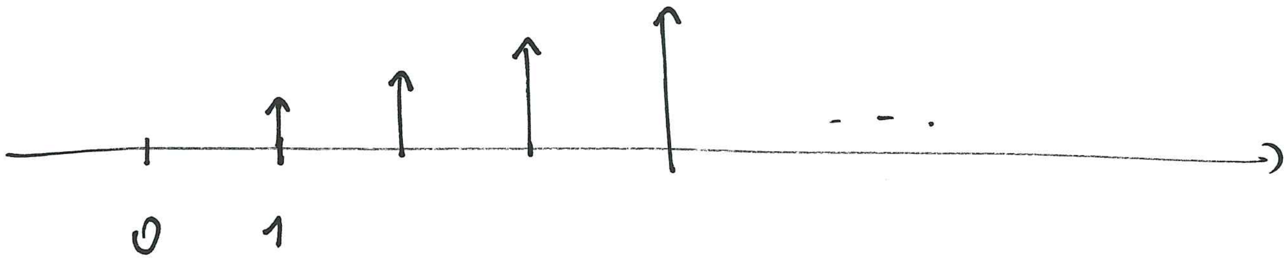


Rendita con rate in PROGRESS. GEOMETRICA

posticipata, periodica, durata n

R_1, $R_k = R_{k-1} \cdot q$, $k \geq 2$, q = ragione $\neq 0$

se $q > 1$



$$V_0(\vec{r}) = \sum_{k=1}^{m} R_k \cdot v^k$$

$$R_k = R_{k-1} \cdot q = R_{k-2} \cdot q^2 = \dots = R_1 q^{k-1}$$

$k-(k-1)$

$$V_0(\vec{r}) = \sum_{k=1}^{m} R_1 q^{k-1} \cdot v^k$$

$$= \frac{R_1}{q} \cdot \sum_{k=1}^{m} (qv)^k$$

$$\sum_{k=1}^{m} v^k = \frac{1-v^m}{i} =$$

$$= \frac{1-v^m}{\frac{1}{v}-1} \quad \forall v \neq 1$$

$$i = \frac{1}{v} - 1$$

$$\sum_{k=1}^{m} r^k = \frac{1-r^m}{\frac{1}{r}-1} \quad \forall\, r \neq 1$$

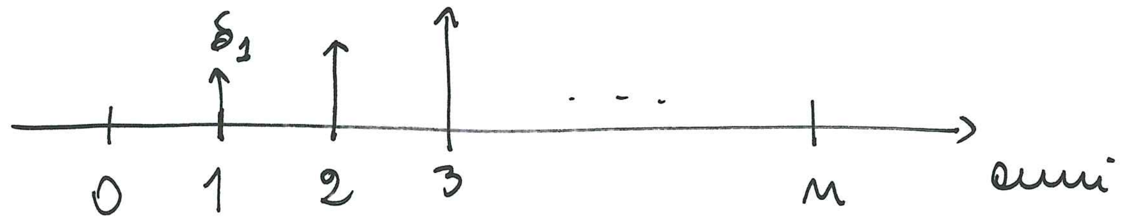
$$r = qv \implies \sum_{k=1}^{m} (qv)^k = \frac{1-(qv)^m}{\frac{1}{qv}-1} \quad \forall\, qv \neq 1$$

1) $qv \neq 1$: $V_0(\vec{r}) = \frac{R_1}{q} \cdot \frac{1-(qv)^m}{\frac{1}{qv}-1} = R_1 \cdot \frac{1-(qv)^m}{\frac{1}{v}-q} = \boxed{R_1 \cdot v \, \frac{1-(qv)^m}{1-qv}}$

2) $qv = 1$: $V_0(\vec{r}) = \boxed{\frac{R_1}{q} \cdot m}$

EX Azienda paga dividendi agli azionisti per 10 anni, annualmente, in modo posticip., ogni anno il 10% in più dell precedente.
Un azionista, via via che incassa i divid. li mette sul suo cc al 3% e.

Valore dei dividendi nel cc alla data $n = 10$ a. ?



$$\delta_1 , \quad \delta_k = \delta_{k-1} + \delta_{k-1} \frac{10}{100} = \delta_{k-1} \, 1.1 \quad , \; q = 1.1$$

$$V_n(\vec{r}) = V_0(\vec{r}) \cdot u^n$$

$$V_0(\vec{r}) = \delta_1 \cdot v \cdot \frac{1-(qv)^n}{1-qv} \qquad qv = \frac{1.1}{1.03} > 1$$

$$V_n(\vec{r}) = \underbrace{\delta_1 \cdot v \cdot \frac{1-(qv)^n}{1-qv} \cdot 1.03^n} = \delta_1 \cdot 17,85$$

$\delta_1 = 0,5$ € per ogni azione

possiedo 100 azioni

$$\Rightarrow \quad V_n(\vec{r}) = 100 \cdot 0,50 \cdot 17,85 = 892,73 \text{ €}$$

Se via via che incasso i dividendi invece di deposit. sul cc li metto nel cassetto

il valore in n di quanto ricevuto è

$$V_n = \sum_{1}^{n} \underbrace{\delta_k}_{\delta_1 \cdot q^{k-1}} = \sum_{k=1}^{10} \delta_1 q^{k-1} = \frac{\delta_1}{q} \cdot \underbrace{\sum_{k=1}^{10} q^k}_{\frac{1-q^{10}}{\frac{1}{q}-1}}$$

$$\sum_{k=1}^{n} \delta_k = \frac{\delta_1}{q} \cdot \frac{1-q^{10}}{\frac{1}{q}-1} = \delta_1 \frac{1-q^{10}}{1-q} = \delta_1 \cdot 15{,}9$$

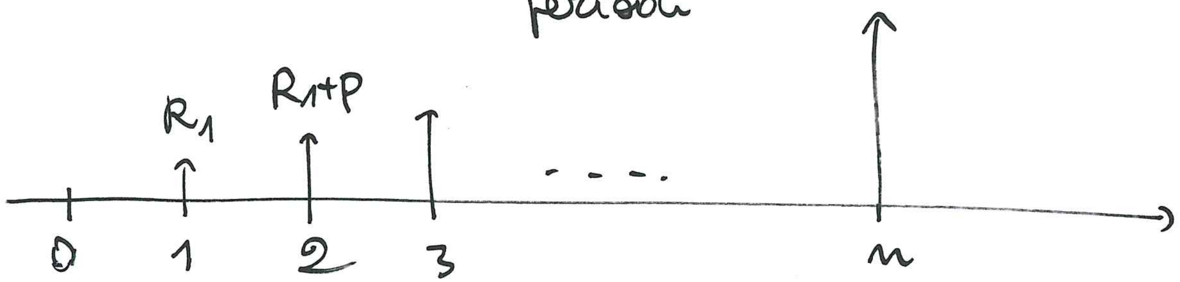
$\delta_1 = 0{,}5$ € , 100 azioni

$$V_n = 0{,}50 \cdot 15{,}9 \cdot 100 = 796{,}90 \text{ €}$$

Rendite con rate in progress. ARITM.

R_1 , $k \geq 2$, $R_k = R_{k-1} + p$

caso posticipato, periodico, durata n periodi, tasso periodale i



caso $p > 0$

$$V_0(\vec{r}) = \sum_{k=1}^{n} R_k \cdot v^k$$

$$R_k = R_{k-1} + p = R_{k-2} + 2p = \dots = R_1 + (k-1)p$$

$$V_0(\vec{r}) = \sum_1^n \left[R_1 + (k-1)p \right] v^k$$

$$= \underbrace{\sum_1^n R_1 \cdot v^k}_{R_1 \cdot a_{\overline{n}|i}} + \sum_1^n (k-1) p \, v^k$$