MACRO I: EXAMINATION PAPER #1

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The paper is based on the two-generation OLG model explained in class and presented in Wickens chapter 6.3.

Assume instantaneous utility to be represented by the function

$$u(c) = a + b \ln c$$

with a, b positive parameters. Total utility over the life-cycle for an individual born at s is therefore

$$U = (1 + \beta) a + b (\ln c_{1S} + \beta \ln c_{2S})$$

with $\beta = \frac{1}{1+\theta}$ as usual, and c_{1s} and c_{2s} representing consumption of an individual born at s when (respectively) young and old .

Population at s is formed by young individuals (born at s) in number N_{1s} and old individuals (born at $s - 1$) in number $N_{2s} = N_{1s-1}$. The rate of growth of population n is constant over time,

$$\frac{N_{1s}}{N_{1s-1}} = 1 + n$$

Only the young are employed in production and are paid wages in return.

Production technology is represented by an aggregate Cobb-Douglas production function

$$Y = K^{\alpha} N^{1-\alpha}$$

with $\alpha \in (0, 1)$ and $N = N_1$ at all dates. In per-labourer terms (small type)

$$y = k^{\alpha}$$

Assume that production is managed efficiently so that, given the wage rate w_s and the interest rate r_s at each s, factors are employed up to maximum profitability characterized by

$$\begin{aligned} w_s &= (1 - \alpha) k_s^{\alpha} \\ r_s &= \alpha k_s^{\alpha-1} - \delta \end{aligned}$$

FF: *An entirely voluntary, fully-funded pension system*

Without need to assume compulsory contributions as in Wickens sect. 6.3.5.1, suppose that the public pension system works as a private investment fund. Young individuals born at s decide freely to distribute their labour income between consumption and savings; savings are invested in the capital stock employed in production. As this is the only asset available in the economy, returns on capital at next period's rate r_{s+1} constitute pensions. No intergenerational bequests are allowed.

Calling $R_{s+1} = 1 + r_{s+1}$, the budget constraint for an individual born at s is

$$(w_s - c_{1s}) R_{s+1} - c_{2s} = 0$$

Problem 1 *Find c_{1s} and c_{2s} as explicit functions of w_s, R_{s+1} in the optimal consumption-investment plan.*

Problem 2 *By using the total resource constraint of the economy and the equality between savings and invested capital, [i] demonstrate that the dynamics of the capital-labour ratio is ruled by the following first-order non-linear difference equation*

$$(1+n)(1+\beta) k_{s+1} = \beta (1-\alpha) k_s^{\alpha} \qquad (1)$$

[ii] Find the two values of k that correspond to steady states of this dynamics. [iii] Using a graphical argument, argue that one of the two steady states is unstable and the other one is stable.

PP: *A mixed Pay-As-You-Go system*

Differently from Wickens sect. 6.3.5.2, now assume that each worker pays the Government (G) a compulsory contribution τ fixed independently of income. The total amount of the contributions collected by G is immediately redistributed to individuals in old age, who receive a fixed tax-exempt pension p per-head. The system is planned to work on a balanced-budget basis: fiscal receipts τN_{1s} cover the pension payments $pN_{2s} = pN_{1s-1} = p\frac{N_{1s}}{1+n}$, so as not to contribute to the creation of public debt. The balanced budget constraint is

$$\frac{p}{1+n} - \tau = 0$$

Beyond participating in the compulsory public pension scheme, workers are free to save from their post-tax income and to invest their savings in physical capital, as in the fully-funded system. The individual budget constraint over the life-cycle is therefore

$$(w_s - \tau - c_{1s}) R_{s+1} + p - c_{2s} = 0$$

Equilibrium in the economy requires the usual equality between total savings and total assets

$$\begin{aligned} K_s &= (w_{s-1} - \tau - c_{1s-1}) N_{1s-1} \\ &= \frac{c_{2s-1} - p}{R_s (1+n)} N_{1s} \end{aligned}$$

In per-labourer terms,

$$k_s = \frac{c_{2s-1} - p}{R_s (1+n)}$$

Problem 3 *The same as problem 1 with reference to this mixed PAYG system.*

Problem 4 *Find the dynamic equation that links k_{s+1} to k_s. Argue that [i] for p (or, equivalently, τ) near enough to zero, the dynamics has two steady states, one of which is stable. [ii] The two steady states get closer to each other as p increases, until a value $\overline{p}$ is reached such that the steady states collapse into a semi-stable one. [iii] For $p > \overline{p}$ no steady state exists and k converges inevitably to zero. [Warning: don't try to follow an entirely algebraic course. A simple but reasonably rigorous argument can be developed on the basis of a mix of algebra and graphic analysis.]*

Problem 5 *Compare the steady state levels of consumption per worker over the entire life-cycle ($c_1 + c_2$) under FF and under PAYG with p positive and less than $\overline{p}$. [Warning: since an explicit formula for the steady-state value of k under PAYG is not easily found, base your answer on the qualitative results found in answering problem 4]*