Irreversible technical progress

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Consider a macroeconomy with a constant level of employment, and technology described at each date s by the linear production function

$$Y_s = A_s K_s$$

Technical progress is partly endogenous, determined by the level of net investment, and partly exogenous due to random and unpredictable shocks that affect productivity either positively or negatively. The equation of technical progress is the following

$$A_{s+1} = A_s + \alpha \cdot \max\{0, K_{s+1} - K_s\} + \varepsilon_{s+1}$$

where $\varepsilon_{s+1} \sim WN$ is the exogenous component, α a parameter in the $(0,1)$ interval, and the expression $\max\{\cdot\}$ means that net investment affects productivity only when it is positive.

There is no external trade, so the budget constraint of the economy is

$$K_{s+1} = (1-\delta) K_s + A_s K_s - C_s$$

Information at date s includes K_s and the sequence of productivity shocks up to ε_s (included). Consider C as the control variable, K as the state variable. For convenience, assume the instantaneous marginal utility of consumption to range between $0 = \lim_{C\to\infty} u'(C)$ and $\infty = \lim_{C\to 0} u'(C)$.

Problem 1 *Characterize the optimum investment-consumption path in terms of Bellman equation for each s.*

Problem 2 *Pass from Bellman to Euler equation [BE CAREFUL: the first derivative of the value function has a discontinuity at the point at which C_s is such that $K_{s+1} = K_s$. Thus, the form of the Euler equation is different depending on whether $K_{s+1} > K_s$ or $K_{s+1} < K_s$. Besides, special attention is required when K_{s+1} lies in the vicinity of K_s, as pointed out in the next problem].*

Problem 3 *Keeping in mind the previously mentioned discontinuity, and availing yourself of graphic analysis and continuity arguments, argue that there is an* *interval (K^-, K^+) such that: (i) $K^- < K_s < K^+$; (ii) given K_s, ε_s, the optimum consumption level C_s corresponds either to $K_{s+1} \leq K^-$ or to $K_{s+1} \geq K^+$; (iii) $K_{s+1} = K^-$ corresponds to an optimum consumption level if and only if also $K_{s+1} = K^+$ does so [thus, in particular cases the optimum consumption level may not be unique].*

Problem 4 *Find the transversality condition.*

Problem 5 *Using the previous results, discuss the impact of productivity shocks (for example, compare the cases of a 'high' and a 'low' ε at a given date s) on the current levels of consumption and investment.*