

Sistemi reticolari piani



UNIVERSITÀ
DEGLI STUDI
FIRENZE

Scuola di Architettura
Corso di Laurea Magistrale quinquennale c.u.



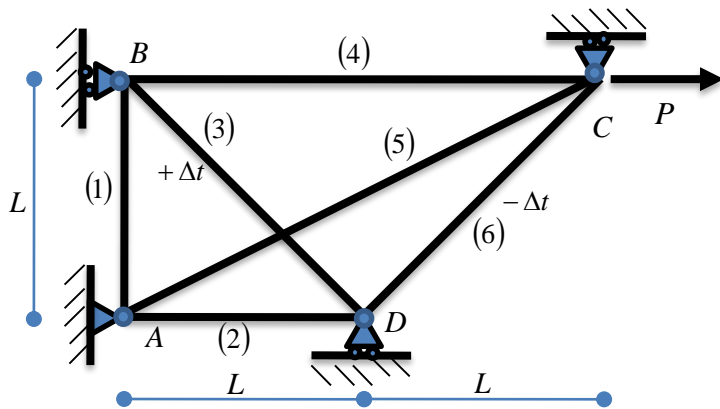
Analisi di sistemi reticolari iperstatici
Metodo degli spostamenti: esempio



Esempio

Si risolva il sistema strutturale schematizzato in figura.

$$P=10,0kN \quad L=1,0m \quad A=100mm^2$$
$$E=210GPa \quad \alpha=11.6 \times 10^{-6} \text{ } ^\circ C^{-1} \quad \Delta t=10^\circ$$

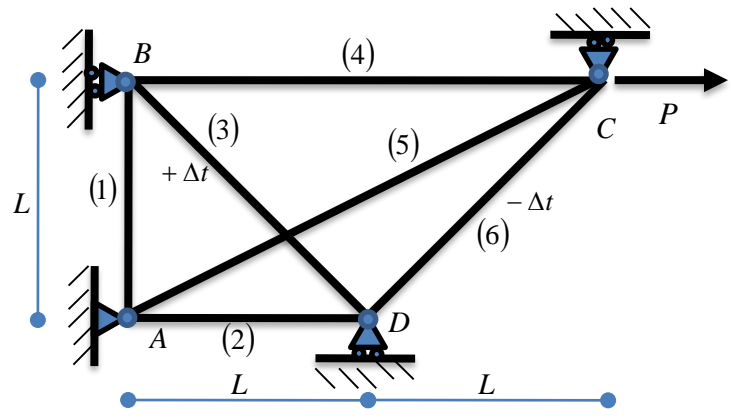




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Il sistema rappresentato in figura chiaramente non ha labilità interne: le bielle sono infatti tali da impedire moti relativi fra i nodi del sistema. Inoltre i vincoli esterni impediscono moti rigidi globali per cui la struttura non ha labilità

$$\ell=0$$

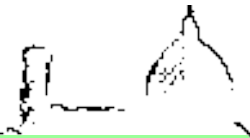
Si ha inoltre

$$n = 4; b = 6; v = 5$$

$$\ell-i = 2n - b - v = 8 - 6 - 5 = -3$$

visto che $\ell=0$ si ha $i=3$. La struttura è allora tre volte iperstatica.

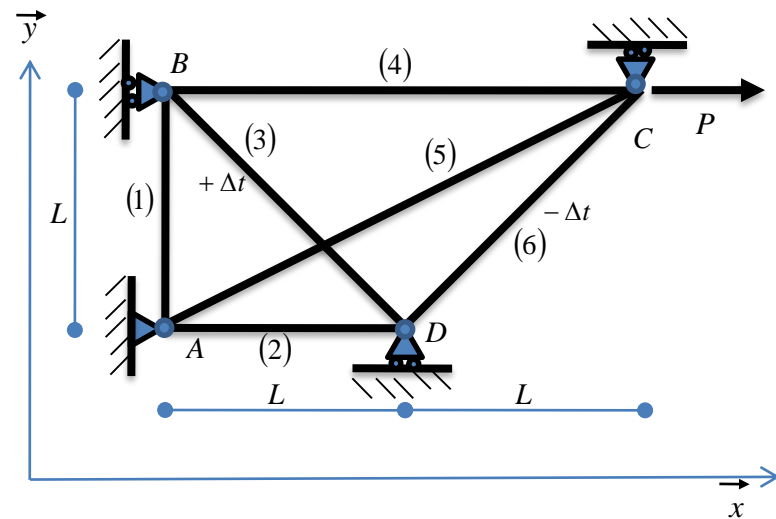
In particolare è esternamente due volte iperstatica ed internamente una volta iperstatica.



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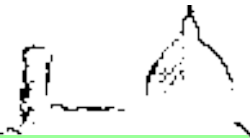


Si definiscono i seguenti vettori delle *incognite del problema*

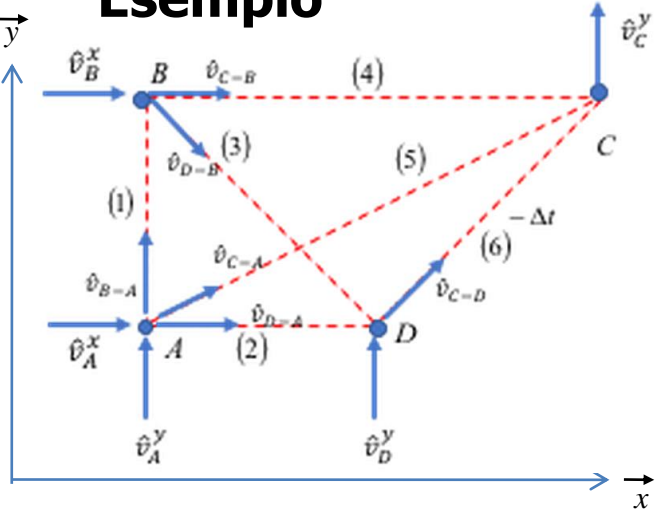
$$\underline{u}_f = \begin{bmatrix} u_y^B \\ u_x^C \\ u_x^D \end{bmatrix}; \quad \underline{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}; \quad \underline{N} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}; \quad \underline{R} = \begin{bmatrix} H_A \\ V_A \\ H_B \\ V_C \\ V_D \end{bmatrix}$$

ed i seguenti vettori dei carichi

$$\underline{F} = \begin{bmatrix} F_y^B \\ F_x^C \\ F_x^D \end{bmatrix} = \begin{bmatrix} 0 \\ P \\ 0 \end{bmatrix}; \quad \underline{\delta}_a = \alpha \Delta t L \sqrt{2} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$



Esempio

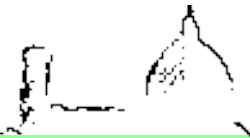


Determinazione delle matrici di congruenza

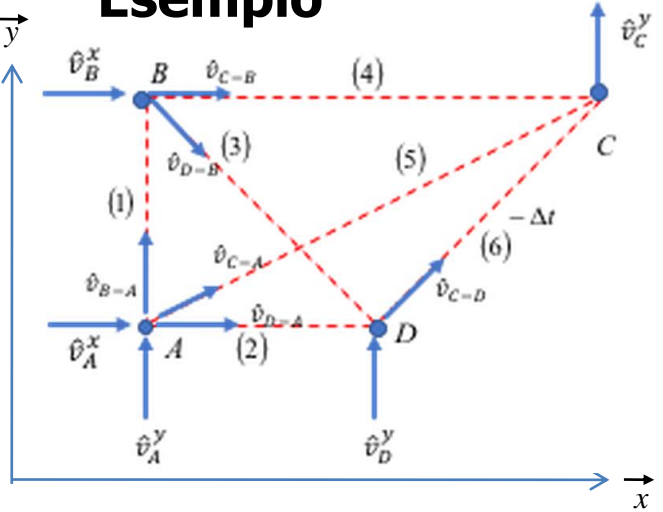
$$\underline{u}_f = \begin{bmatrix} u_y^B \\ u_x^C \\ u_x^D \end{bmatrix};$$

$$\underline{\delta} = [C_{tot}]\underline{u} = [C_f]\underline{u}_f + [C_v]\underline{u}_v$$

| | A | B | C | D | |
|--------------------------------------|----------------------|----------------------|---------------------|---------------------|---------|
| $\hat{v}_{B-A} \rightarrow \delta_1$ | $-(\hat{v}_{B-A})_x$ | $(\hat{v}_{B-A})_x$ | 0 | 0 | u_x^A |
| $\hat{v}_{D-A} \rightarrow \delta_2$ | $-(\hat{v}_{D-A})_x$ | 0 | 0 | 0 | u_y^A |
| $\hat{v}_{D-B} \rightarrow \delta_3$ | 0 | $-(\hat{v}_{D-B})_x$ | 0 | $(\hat{v}_{D-B})_x$ | u_x^B |
| $\hat{v}_{C-B} \rightarrow \delta_4$ | 0 | $-(\hat{v}_{C-B})_x$ | $(\hat{v}_{C-B})_x$ | $(\hat{v}_{C-B})_y$ | u_y^B |
| $\hat{v}_{C-A} \rightarrow \delta_5$ | $-(\hat{v}_{C-A})_x$ | 0 | $(\hat{v}_{C-A})_x$ | $(\hat{v}_{C-A})_y$ | u_x^C |
| $\hat{v}_{C-D} \rightarrow \delta_6$ | 0 | 0 | $(\hat{v}_{C-D})_x$ | $(\hat{v}_{C-D})_y$ | u_y^C |
| | | | | | u_x^D |
| | | | | | u_y^D |



Esempio

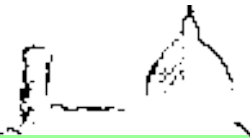


Determinazione delle matrici di congruenza

$$\underline{u}_f = \begin{bmatrix} u_y^B \\ u_x^C \\ u_x^D \end{bmatrix};$$

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| | A | | B | | C | | D | | |
|--------------------------------------|----------------------|----------------------|----------------------|----------------------|---------------------|---------------------|----------------------|----------------------|---------|
| $\hat{v}_{B-A} \rightarrow \delta_1$ | $-(\hat{v}_{B-A})_x$ | $-(\hat{v}_{B-A})_y$ | $(\hat{v}_{B-A})_x$ | $(\hat{v}_{B-A})_y$ | 0 | 0 | 0 | 0 | u_x^A |
| $\hat{v}_{D-A} \rightarrow \delta_2$ | $-(\hat{v}_{D-A})_x$ | $-(\hat{v}_{D-A})_y$ | 0 | 0 | 0 | 0 | $(\hat{v}_{D-A})_x$ | $(\hat{v}_{D-A})_y$ | u_y^A |
| $\hat{v}_{D-B} \rightarrow \delta_3$ | 0 | 0 | $-(\hat{v}_{D-B})_x$ | $(\hat{v}_{D-B})_y$ | 0 | 0 | $(\hat{v}_{D-B})_x$ | $(\hat{v}_{D-B})_y$ | u_x^B |
| $\hat{v}_{C-B} \rightarrow \delta_4$ | 0 | 0 | $-(\hat{v}_{C-B})_x$ | $-(\hat{v}_{C-B})_y$ | $(\hat{v}_{C-B})_x$ | $(\hat{v}_{C-B})_y$ | 0 | 0 | u_y^B |
| $\hat{v}_{C-A} \rightarrow \delta_5$ | $-(\hat{v}_{C-A})_x$ | $-(\hat{v}_{C-A})_y$ | 0 | 0 | $(\hat{v}_{C-A})_x$ | $(\hat{v}_{C-A})_y$ | 0 | 0 | u_x^C |
| $\hat{v}_{C-D} \rightarrow \delta_6$ | 0 | 0 | 0 | 0 | $(\hat{v}_{C-D})_x$ | $(\hat{v}_{C-D})_y$ | $-(\hat{v}_{C-D})_x$ | $-(\hat{v}_{C-D})_y$ | u_y^C |
| | | | | | | | | | u_x^D |
| | | | | | | | | | u_y^D |



Esempio

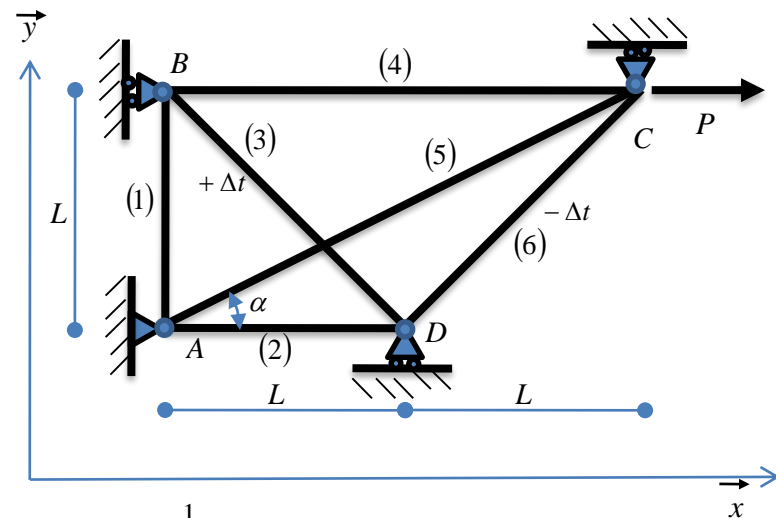
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$$[C_f] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ 0 & 2\sqrt{5}/5 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}$$

$$[C_v] = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 0 & -1 & 0 & 0 \\ -2\sqrt{5}/5 & -\sqrt{5}/5 & 0 & \sqrt{5}/5 & 0 \\ 0 & 0 & 0 & \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}$$



$$\tan \alpha = \frac{1}{2}$$

$$\sin \alpha = \frac{\sqrt{5}}{5}$$

$$\cos \alpha = \frac{2\sqrt{5}}{5}$$

1.2 matrice delle rigidità assiali

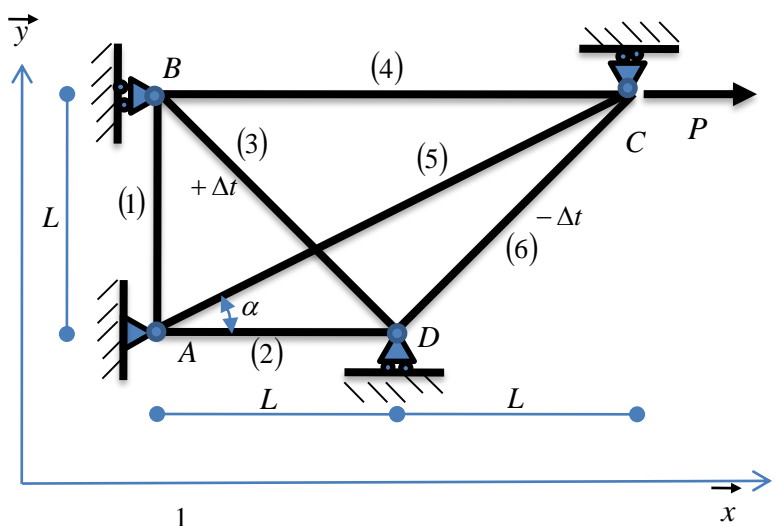
$$[k] = \frac{EA}{L} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$



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1.3 forze nodali compressive

$$\underline{\hat{F}} = \underline{F} + [C_f]^T [k] \underline{\delta}_\alpha = \begin{pmatrix} \frac{EA\alpha\Delta t}{\sqrt{2}} \\ P - \frac{EA\alpha\Delta t}{\sqrt{2}} \\ 2 \frac{EA\alpha\Delta t}{\sqrt{2}} \end{pmatrix}$$

2. matrice di rigidezza del sistema

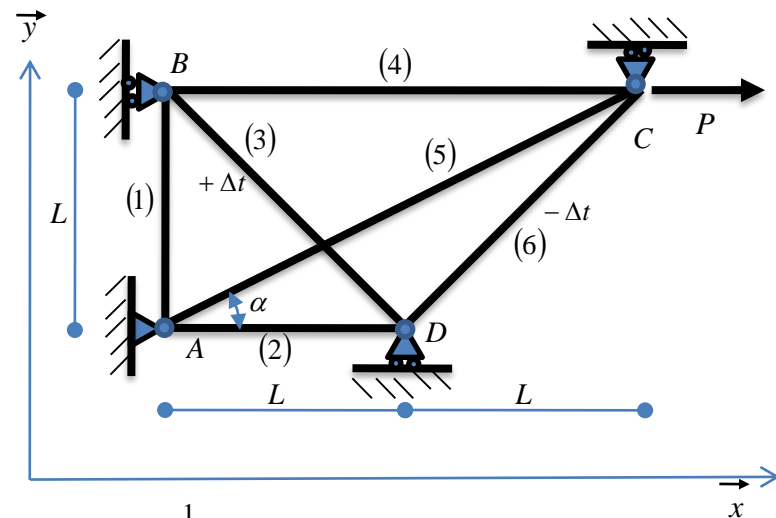
$$[K] = [C_f]^T [k] [C_f] = \frac{EA}{L} \begin{bmatrix} \frac{4 + \sqrt{2}}{4} & 0 & \frac{1}{2\sqrt{2}} \\ 0 & \frac{4}{5\sqrt{5}} + \frac{2 + \sqrt{2}}{4} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 1 + \frac{1}{\sqrt{2}} \end{bmatrix}$$



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3.1 calcolo degli spostamenti nodali

$$\underline{u}_f = [K]^{-1} \underline{\hat{F}} \cong \begin{bmatrix} 0.016 \\ 0.375 \\ 0.170 \end{bmatrix} mm$$

3.2 calcolo degli allungamenti assiali

$$\underline{\delta} = [C_f] \underline{u}_f \cong \begin{bmatrix} 0.016 \\ 0.170 \\ 0.132 \\ 0.375 \\ 0.336 \\ 0.145 \end{bmatrix} mm$$

3.3 calcolo degli sforzi normali

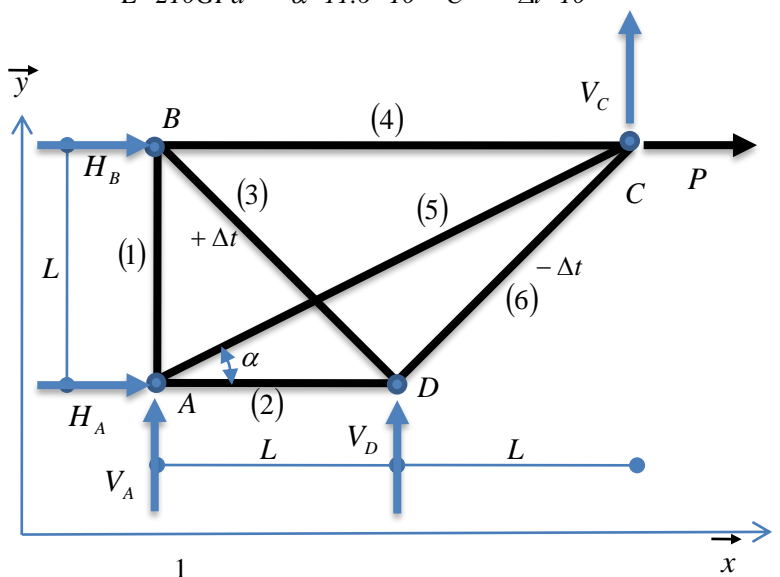
$$\underline{N} = [k] (\underline{\delta} - \underline{\delta}_a) = \begin{bmatrix} 0.338 \\ 3.580 \\ -0.477 \\ 3.939 \\ 3.151 \\ 4.585 \end{bmatrix} kN$$



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3.4 calcolo delle reazioni vincolari
 Per il calcolo delle reazioni vincolari si scrivono le equazioni di equilibrio nodali nelle direzioni vincolare esternamente

$$\begin{bmatrix} H_A \\ V_A \\ H_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 & -2\sqrt{5}/5 & 0 \\ -1 & 0 & 0 & 0 & -\sqrt{5}/5 & 0 \\ 0 & 0 & -\sqrt{2}/2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{5}/5 & \sqrt{2}/2 \\ 0 & 0 & -\sqrt{2}/2 & 0 & 0 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}$$

$[c_v]^{-1}$

$$\rightarrow \begin{bmatrix} H_A \\ V_A \\ H_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -6.398 \\ -1.747 \\ -3.602 \\ 4.652 \\ -2.904 \end{bmatrix} kN$$

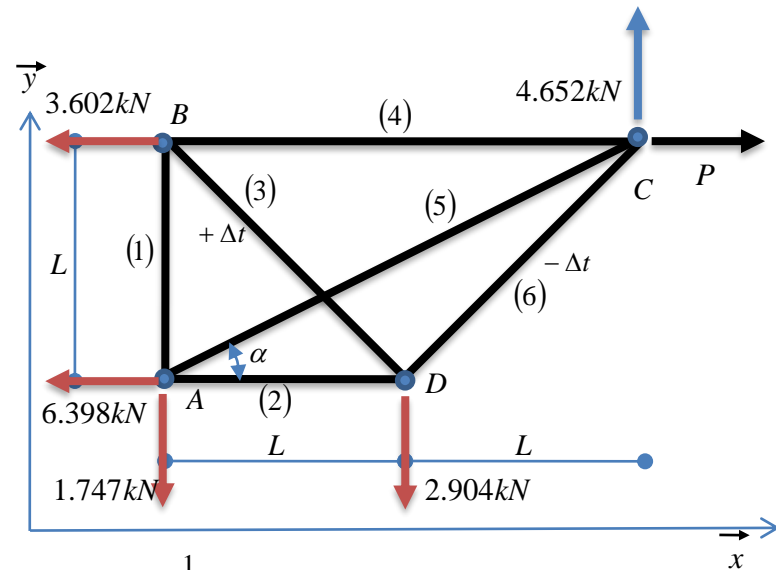


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3.4 calcolo delle reazioni vincolari
Nel diagramma di corpo libero schematizzato a fianco si riportano le reazioni vincolari ora determinate



$$\tan \alpha = \frac{1}{2}$$

$$\sin \alpha = \frac{\sqrt{5}}{5}$$

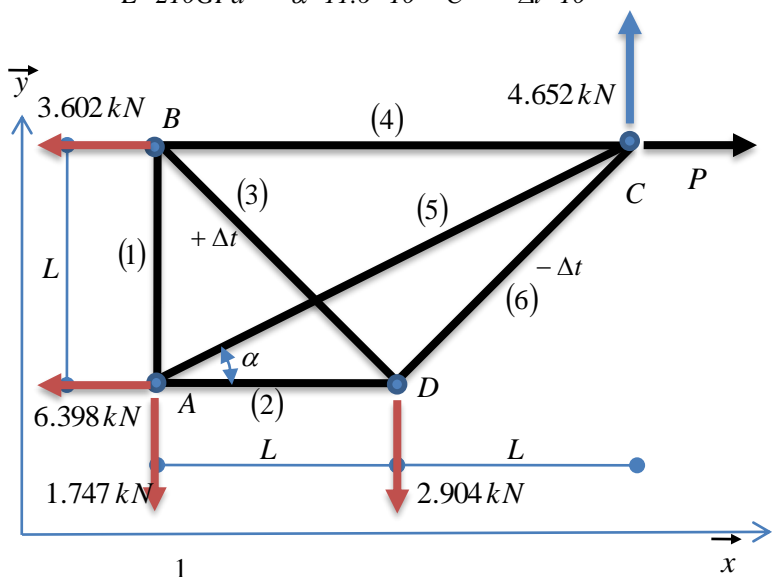
$$\cos \alpha = \frac{2\sqrt{5}}{5}$$



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4. verifica

Al fine di verificare i valori determinati, si impongono le equazioni di equilibrio globale. Esse devono essere identicamente soddisfatte.

$$\begin{cases} \sum F_x = 0 \rightarrow -6.398 - 3.602 + 10 = 0 \rightarrow 0 = 0 \\ \sum F_y = 0 \rightarrow -1.747 - 2.904 + 4.652 = 0 \rightarrow 0 = 0 \\ \sum M_{(A)} = 0 \rightarrow 3.602L + 4.652 \times 2L - 10L - 2.904L = 0 \rightarrow 0 = 0 \end{cases}$$