$$A=\begin{pmatrix}3&2&0\\2&2&2\\0&2&1\end{pmatrix} \qquad A=A^t \qquad \det\begin{pmatrix}3-\lambda&2&0\\2&2-\lambda&2\\0&2&1-\lambda\end{pmatrix}=(3-\lambda)(2-\lambda)(1-\lambda)-4(3-\lambda)-4(1-\lambda)$$

$$=(3-\lambda)(\lambda^2-3\lambda+2-4)-4(1-\lambda)=(3-\lambda)(\lambda^2-3\lambda-2)-4+4\lambda=3\lambda^2-9\lambda-6-\lambda^3+3\lambda^2+2\lambda-4+4\lambda=$$
$$-\lambda^3+6\lambda^2-3\lambda-10$$

$$p(\lambda)=(\lambda-2)(-\lambda^2+4\lambda+5)=0$$

$$\lambda^2-4x-5=0 \qquad \lambda=2\pm\sqrt{4+5}=\begin{cases}5\\-1\end{cases}$$

	-1	6	-3	-10
2		-2	8	10
	-1	4	5	0

autovalori : $-1,\ 2,\ 5$

E_{-1} : $\begin{pmatrix}3&2&0\\2&2&2\\0&2&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=-\begin{pmatrix}x\\y\\z\end{pmatrix}$ $\quad\begin{cases}3x+2y=-x\\2x+2y+2z=-y\\2y+z=-z\end{cases}$ $\quad\begin{cases}4x+2y=0\\2x+3y+2z=0\\2y+2z=0\end{cases}$ $\quad \begin{matrix}y=-2x\\y=-z\end{matrix}$ $\quad v_1=(1,-2,2)$

E_{+2} $\begin{cases}3x+2y=2x\\2x+2y+2z=2y\\2y+z=2z\end{cases}$ $\quad\begin{cases}x=-2y\\z=2y\end{cases}$ $\quad v_2=(-2,1,2)$ $\qquad E_5$ $\begin{cases}3x+2y=5x\\2x+2y+2z=5y\\2y+z=5z\end{cases}$ $\quad\begin{cases}x=y\\y=2z\end{cases}$

$v_2=(2,2,1)$

$\{v_1,v_2,v_3\}$ base ortogonale di $\mathbb{R}^3$ costituita da autovettori di A

$u_1=\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$; $u_2=\left(-\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$; $u_3=\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$

$\{u_1,u_2,u_3\}$ base ortonormale

$$D=\begin{pmatrix}\frac13&-\frac23&\frac23\\-\frac23&\frac13&\frac23\\\frac23&\frac23&\frac13\end{pmatrix}$$

D è ortogonale cioè $D^t=D^{-1}$

$$D^tAD=\Lambda \qquad \begin{pmatrix}\frac13&-\frac23&\frac23\\-\frac23&\frac13&\frac23\\\frac23&\frac23&\frac13\end{pmatrix}\begin{pmatrix}3&2&0\\2&2&2\\0&2&1\end{pmatrix}\begin{pmatrix}\frac13&-\frac23&\frac23\\-\frac23&\frac13&\frac23\\\frac23&\frac23&\frac13\end{pmatrix}=\begin{pmatrix}-1&0&0\\0&2&0\\0&0&5\end{pmatrix}$$

Teorema spettrale reale: A simmetrica è diagonalizzabile ed esiste una base ortonormale di $\mathbb{R}^n$ (n = ordine di A) fatta da autovettori di A