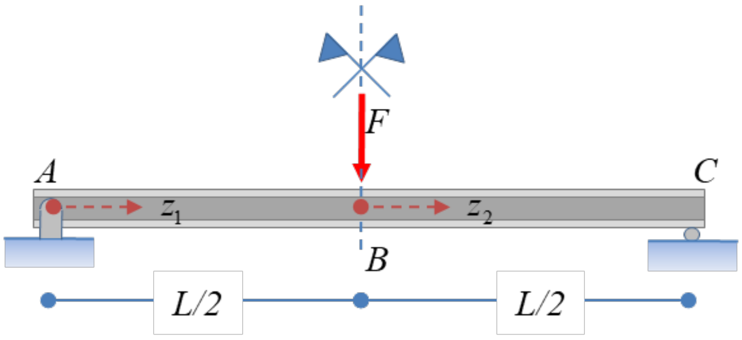
Analisi di una trave appoggiata-appoggiata, sollecitata da una forza trasversale applicata in mezzeria



- definisco alcune variabili simboliche (inizializzazione)

```
syms q F chi_a L El Ix
syms z_1 c1_1 c1_2 c1_3 c1_4
syms z_2 c2_1 c2_2 c2_3 c2_4
```

- carichi

```
chi_a = 0; % assenza di distorsioni termiche
q = 0;
```

- integrazione delle equazioni in gioco

Tratto 1

```
% indefinite di equilibrio
T_1(z_1)    = - int(q,z_1) + c1_1
```

T_1(z_1) = $c_{1,1}$

```
M_1(z_1)    =   int(T_1,z_1) + c1_2
```

M_1(z_1) = $c_{1,2} + c_{1,1}\, z_1$

```
% legame costitutivo
chi_1(z_1) = M_1(z_1)/(El*Ix) + chi_a
```

chi_1(z_1) =

$$\frac{c_{1,2} + c_{1,1}\, z_1}{\mathrm{El}\ \mathrm{Ix}}$$

```
% congruenza
phi_1(z_1) = int(chi_1,z_1) + c1_3
```

phi_1(z_1) =

$$c_{1,3} + \frac{z_1\ (2\, c_{1,2} + c_{1,1}\, z_1)}{2\ \mathrm{El}\ \mathrm{Ix}}$$

```
v0_1(z_1) = -int(phi_1,z_1) + c1_4
```

v0_1(z_1) =

$$c_{1,4} - c_{1,3}\, z_1 - \frac{z_1\ \left(c_{1,1}\, {z_1}^2 + 3\, c_{1,2}\, z_1\right)}{6\ \mathrm{El}\ \mathrm{Ix}}$$

Tratto 2

```
% indefinite di equilibrio
T_2(z_2)   = - int(q,z_2) + c2_1
```

T_2(z_2) = $c_{2,1}$

```
M_2(z_2)   =   int(T_2,z_2) + c2_2
```

M_2(z_2) = $c_{2,2} + c_{2,1}\, z_2$

```
% legame costitutivo
chi_2(z_2) = M_2(z_2)/(El*Ix) + chi_a
```

chi_2(z_2) =

$$\frac{c_{2,2} + c_{2,1}\, z_2}{\mathrm{El}\ \mathrm{Ix}}$$

```
% congruenza
phi_2(z_2) = int(chi_2,z_2) + c2_3
```

phi_2(z_2) =

$$c_{2,3} + \frac{z_2\ (2\, c_{2,2} + c_{2,1}\, z_2)}{2\ \mathrm{El}\ \mathrm{Ix}}$$

```
v0_2(z_2) = -int(phi_2,z_2) + c2_4
```

v0_2(z_2) =

$$c_{2,4} - c_{2,3}\, z_2 - \frac{z_2\, \left(c_{2,1}\, {z_2}^2 + 3\, c_{2,2}\, z_2\right)}{6\, \mathrm{El}\, \mathrm{Ix}}$$

- calcolo delle costanti di integrazione attraverso le condizioni al contorno

```
[cs1_1, cs1_2, cs1_3, cs1_4,...
 cs2_1, cs2_2, cs2_3, cs2_4] = ...
solve({v0_1(0) == 0;...
       M_1(0) == 0;...
       v0_1(L/2)-v0_2(0) == 0;...
       phi_1(L/2) - phi_2(0) == 0;...
       T_2(0) - T_1(L/2) + F ==0;...
       M_2(0) - M_1(L/2) == 0;...
       v0_2(L/2) == 0;...
       M_2(L/2)==0},...
       {c1_1,c1_2,c1_3,c1_4,...
        c2_1,c2_2,c2_3,c2_4})
```

cs1_1 =

$$\frac{F}{2}$$

cs1_2 = 0

cs1_3 =

$$-\frac{F\, L^2}{16\, \mathrm{El}\, \mathrm{Ix}}$$

cs1_4 = 0

cs2_1 =

$$-\frac{F}{2}$$

cs2_2 =

$$\frac{F\, L}{4}$$

cs2_3 = 0

cs2_4 =

$$\frac{F\, L^3}{48\, \mathrm{El}\, \mathrm{Ix}}$$

- sostituisco le costanti di integrazione determinate nelle funzioni taglio, momento flettente, curvatura, rotazione e spostamento

```
cs = [cs1_1, cs1_2, cs1_3, cs1_4,...
      cs2_1, cs2_2, cs2_3, cs2_4];
cc = [c1_1, c1_2, c1_3, c1_4,...
      c2_1, c2_2, c2_3, c2_4];

T_1(z_1)    = simplify(subs(  T_1(z_1),cc,cs))
```

T_1(z_1) =

$$\frac{F}{2}$$

```
M_1(z_1)   = simplify(subs(  M_1(z_1),cc,cs))
```

M_1(z_1) =

$$\frac{F z_1}{2}$$

```
chi_1(z_1) = simplify(subs(chi_1(z_1),cc,cs))
```

chi_1(z_1) =

$$\frac{F z_1}{2 \,\mathrm{El}\, \mathrm{Ix}}$$

```
phi_1(z_1) = simplify(subs(phi_1(z_1),cc,cs))
```

phi_1(z_1) =

$$-\frac{F \left(L^2 - 4 z_1^2\right)}{16 \,\mathrm{El}\, \mathrm{Ix}}$$

```
v0_1(z_1)  = simplify(subs( v0_1(z_1),cc,cs))
```

v0_1(z_1) =

$$\frac{F z_1 \left(3 L^2 - 4 z_1^2\right)}{48 \,\mathrm{El}\, \mathrm{Ix}}$$

```
T_2(z_2)   = simplify(subs(  T_2(z_2),cc,cs))
```

T_2(z_2) =

$$-\frac{F}{2}$$

```
M_2(z_2)   = simplify(subs(  M_2(z_2),cc,cs))
```

M_2(z_2) =

$$\frac{F \left(L - 2 z_2\right)}{4}$$

```
chi_2(z_2) = simplify(subs(chi_2(z_2),cc,cs))
```

chi_2(z_2) =

$$\frac{F \left(L - 2 z_2\right)}{4 \,\mathrm{El}\, \mathrm{Ix}}$$

```
phi_2(z_2) = simplify(subs(phi_2(z_2),cc,cs))
```

phi_2(z_2) =

$$\frac{F\, z_2\, (L - z_2)}{4\, \text{El}\, \text{Ix}}$$

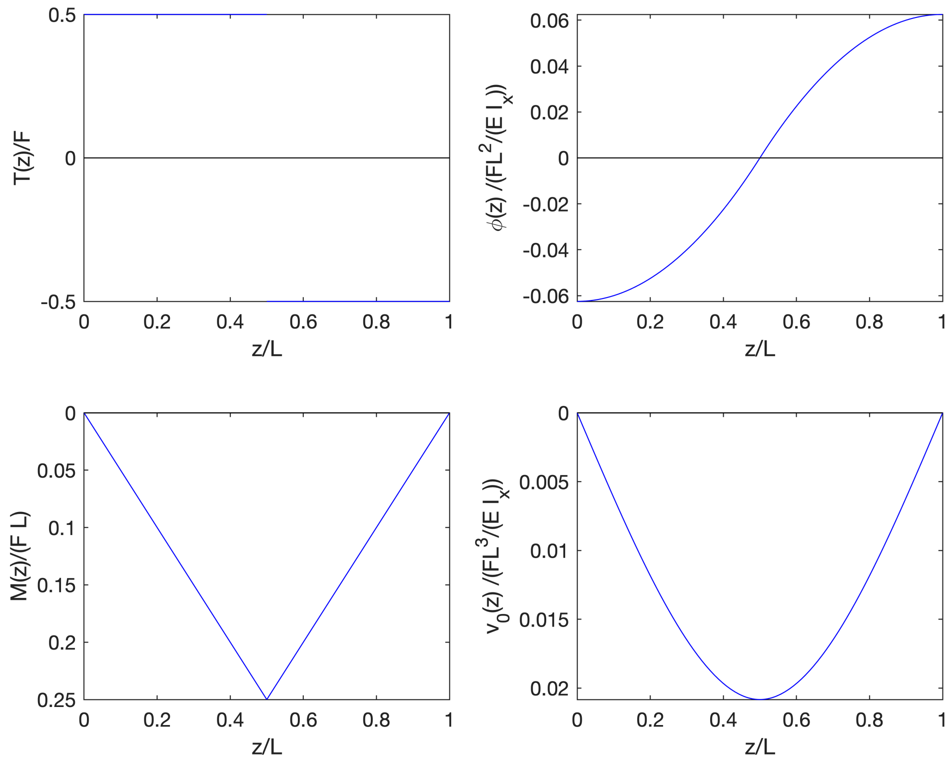
```
v0_2(z_2)  = simplify(subs( v0_2(z_2),cc,cs))
```

v0_2(z_2) =

$$\frac{F\, \left(L^3 - 6\, L\, {z_2}^2 + 4\, {z_2}^3\right)}{48\, \text{El}\, \text{Ix}}$$

- diagrammi

```
figure
subplot(2,2,1)
fplot(subs(T_1(z_1)/F,[L],[1]),[0 0.5],'b')
hold on
fplot(subs(T_2(z_2-0.5)/F,[L],[1]),[0.5 1],'b')
    line(xlim(), [0,0], 'Color', 'k');
    xlabel ('z/L'), ylabel('T(z)/F')
subplot(2,2,3)
fplot(subs(M_1(z_1)/(F*L),[L],[1]),[0 0.5],'b')
hold on
fplot(subs(M_2(z_2-0.5)/(F*L),[L],[1]),[0.5 1],'b')
    set(gca,'Ydir','reverse')
    line(xlim(), [0,0], 'Color', 'k');
    xlabel ('z/L'), ylabel('M(z)/(F L)')
subplot(2,2,2)
fplot(subs(phi_1(z_1)*El*Ix/(F*L^2),[L],[1]),[0 0.5],'b')
hold on
fplot(subs(phi_2(z_2-0.5)*El*Ix/(F*L^2),[L],[1]),[0.5 1],'b')
    line(xlim(), [0,0], 'Color', 'k');
    xlabel ('z/L'), ylabel('\phi(z) /(FL^2/(E I_x))')
subplot(2,2,4)
fplot(subs(v0_1(z_1)*El*Ix/(F*L^3),[L],[1]),[0 0.5],'b')
hold on
fplot(subs(v0_2(z_2-0.5)*El*Ix/(F*L^3),[L],[1]),[0.5 1],'b')
    set(gca,'Ydir','reverse')
    line(xlim(), [0,0], 'Color', 'k');
    xlabel ('z/L'), ylabel('v_0(z) /(FL^3/(E I_x))')
```



- spostamenti e rotazioni notevoli

```
fprintf('spostamento in mezzeria %s',v0_1(L/2))
```

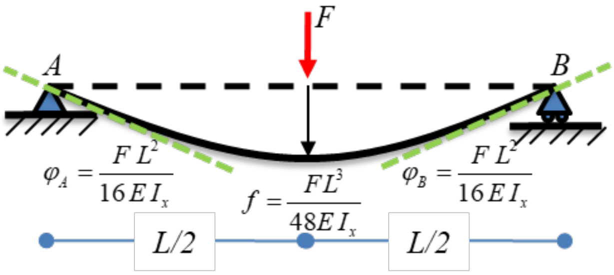
```
spostamento in mezzeria (F*L^3)/(48*El*Ix)
```

```
fprintf('rotazione della sezione iniziale %s',phi_1(0))
```

```
rotazione della sezione iniziale -(F*L^2)/(16*El*Ix)
```

```
fprintf('rotazione della sezione finale %s',phi_2(L/2))
```

```
rotazione della sezione finale (F*L^2)/(16*El*Ix)
```



F
A
B
$\varphi_A = \frac{FL^2}{16EI_x}$
$f = \frac{FL^3}{48EI_x}$
$\varphi_B = \frac{FL^2}{16EI_x}$
L/2
L/2