Around 1750 the friendship between d'Alembert and Euler began to deteriorate, probably from several causes. D'Alembert had written his first paper on vibrating strings in 1746, and had discussed his solution with Euler in their correspondence. A few months later Euler gave his own paper on vibrating strings which he published twice and as quickly as possible.[3] Euler's paper was largely a repetition of what d'Alembert had already written and added little of value except for a criticism of the restrictions that d'Alembert had applied to the problem. In his paper d'Alembert had derived and solved the wave equation, the first partial differential equation to be studied in detail. It was an important contribution to mechanics,

[1] Euler did warn d'Alembert that his publisher (unbeknown to Euler) had included a correction by d'Alembert without citing him, and Euler was anxious that d'Alembert should not take offence. Euler to d'Alembert, 28 September 1748; Charles Henry, 'Lettres inédites d'Euler à d'Alembert', *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, XIX (1886), 144–5.

[2] 5 March 1748; quoted from Joseph Bertrand, 'Euler et ses travaux', *Journal des savants* (March 1868), p. 143.

[3] *Nova acta eruditorum* (1749), pp. 512–27 and Akademie der Wissenschaften, Berlin, *Histoire . . . avec les mémoires*, IV (1748), 69–85 (published 1750). There are three studies of the long controversy over the vibrating string. These are: H. Burkhardt, 'Entwicklungen nach oscillirenden Functionen', *Jahresbericht der Deutschen Mathematiker-Vereinigung*, X (1908), 1–1804; J. R. Ravetz, 'Vibrating strings and arbitrary functions', in the *Logic of Personal Knowledge, Essays presented to Michael Polanyi on his Seventieth Birthday, 11th March 1961* (Glencoe, Ill. 1961), pp. 71–88; and Truesdell, 'The Rational Mechanics of Flexible or Elastic Bodies', *Euleri opera omnia*, ser. 2, XI, pt. 2, pp. 237–300. All three are excellent papers. The article by Ravetz is the least technical and emphasizes the important controversy over the nature of a mathematical function.

because it opened the way for the study of oscillations propagated in continuous media.[1]

Even more important for mathematics, however, was the agrument which ensued over the nature of a function. D'Alembert insisted that the calculus could only treat 'continuous' functions or what modern mathematicians would call 'analytic' functions. Although his criteria for allowable functions were never completely specified, his difference with Euler turned on the initial shape of the string before it began to vibrate. D'Alembert said that analysis could only treat curves whose corresponding equation was odd, periodic, and defined everywhere, even outside the interval being considered. In the physical situation, very few initial conditions could satisfy d'Alembert's requirements, and so his ideas severely limited the theory. But it is apparent from d'Alembert's papers that he was not at all interested in the physical problem. In order to solve the wave equation for the given end-conditions, he had to specify restrictions that limited the shape of the string to the familiar sine function, although he tried to demonstrate other periodic functions that would fulfil this requirement. Euler, however, permitted *any* function even a curve 'drawn by hand' defined over the periodic interval. The conditions of periodicity were provided by adding pieces of this curve. Although Euler's solution was not a 'function' as that term was used in the eighteenth century, it did give a powerful way of approaching the physical problem of the vibrating string.

All the major mathematicians of Europe entered this contest, including Daniel Bernoulli, Lagrange, and Laplace. Daniel Bernoulli was interested in solving the problem of the vibrating string, while Euler and d'Alembert soon left the physical problem altogether to argue important points in the theory of functions. It was a great accomplishment of Euler to recognize the generality of these oscillating functions, while d'Alembert stubbornly held to his more traditional view.[2] The vibrating string was a problem that d'Alembert was apparently willing to argue until eternity. Lagrange marvelled at his tenacity. Even when the rest of the mathematical profession had gone over to Euler's side, d'Alembert would continue to fire off an occasional broadside. The controversy continued until his death; only at the very end did he admit defeat and even then it was in a manuscript that never reached the publishers.[3]

The year 1749 was, as we have seen, a crucial year for the *philosophes*

[1] D'Alembert had already given the first solution for motion in a continuous medium in his analysis of the motion of a heavy hanging cord, *Traité de dynamique* (1758), pp. 168–9; and Truesdell, *Euleri opera omnia*, ser. 2, XI, pt. 2, pp. 191–2.

[2] Lagrange's paper 'Recherches sur la nature et la propagation du son' (1759) was a turning point in the whole controversy since he adopted an entirely new approach—that of a string loaded with mass points taken to a limit. He supported Euler's position with powerful new arguments. In the subsequent months Euler made extensive use of Lagrange's discovery which essentially closed the dispute over the vibrating string.

[3] Unpublished ninth volume of the *Opuscules mathématiques*, Bibl. Inst. MS. 1790, fol. 271.