# What is the Power Law?

The power law (also called the scaling law) states that a relative change in one quantity results in a proportional relative change in another. The simplest example of the law in action is a square; if you double the length of a side (say, from 2 to 4 inches) then the area will quadruple (from 4 to 16 inches squared). A power law distribution has the form $Y = k X^{\alpha}$, where:

- X and Y are variables of interest,
- α is the law's exponent,
- k is a constant.

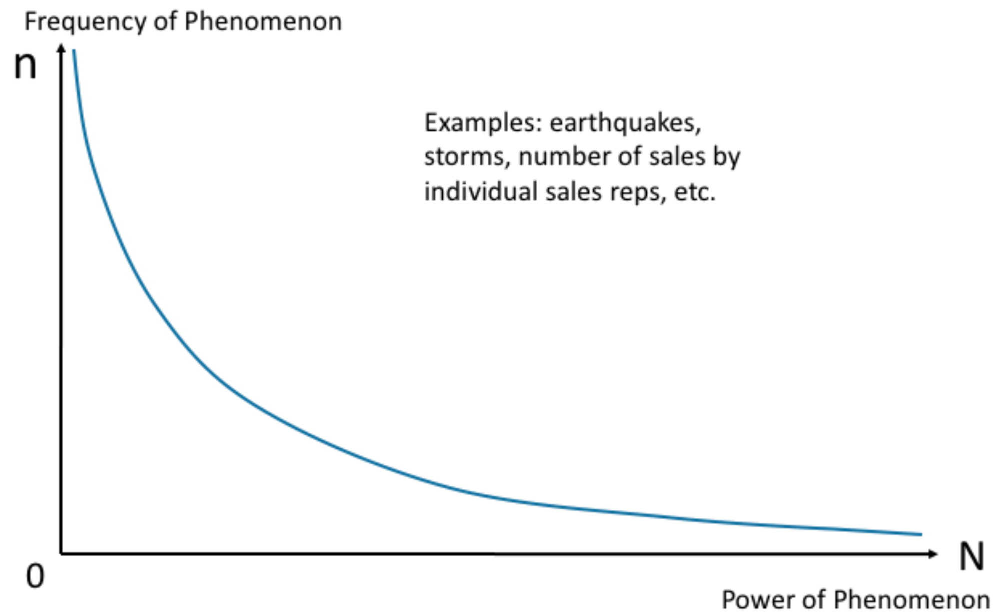
Any inverse relationship like $Y = X^{-1}$ is also a power law, because a change in one quantity results in a negative change in another.

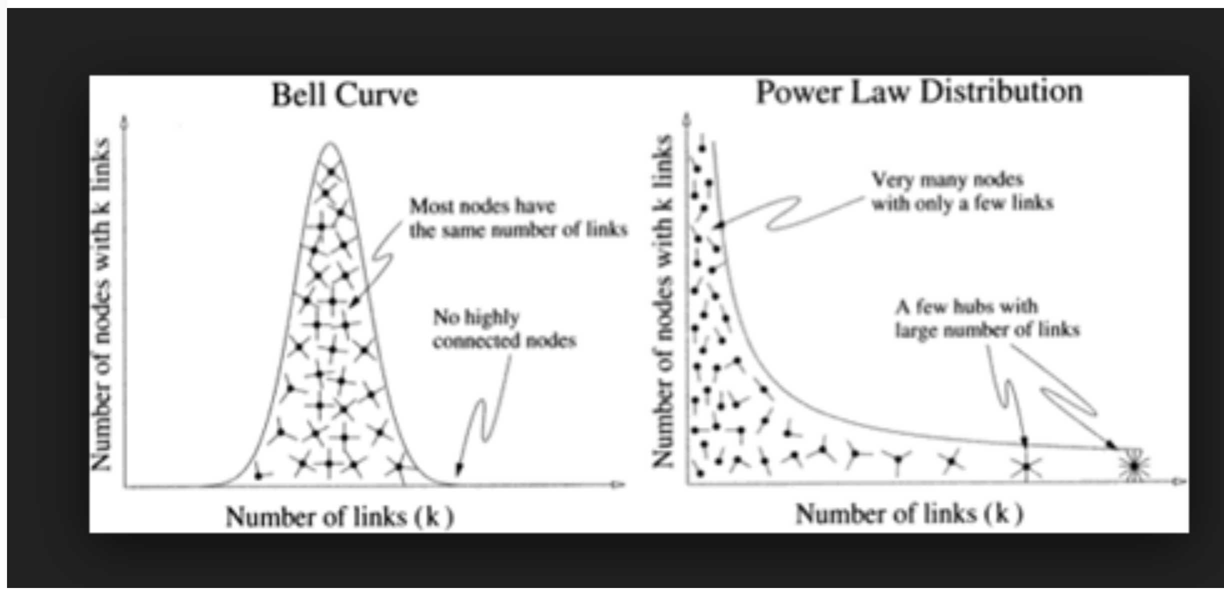
$$Y = kX^{\alpha}, \qquad Y = kX^{-\alpha}$$

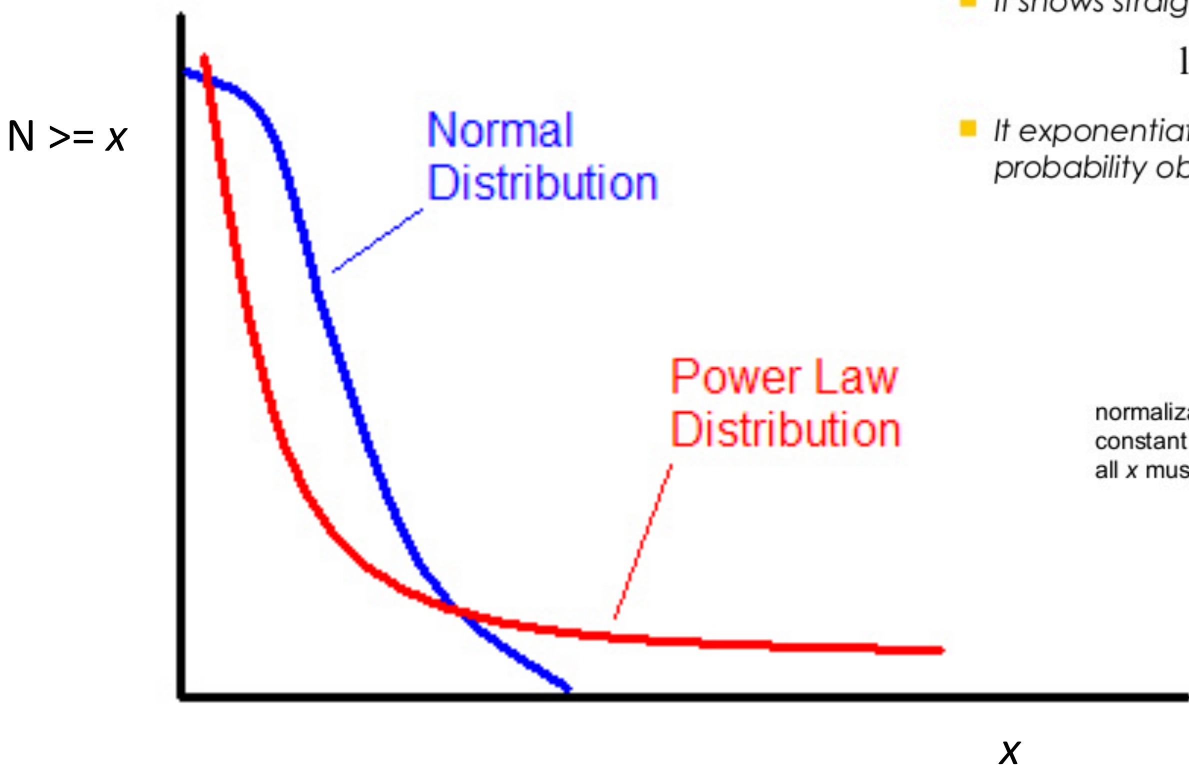
**Vilfredo Pareto**

| | |
|---|---|
| **Born** | 15 July 1848<br>Paris, France |
| **Died** | 19 August 1923 (aged 75)<br>Céligny, Switzerland |
| **Nationality** | Italian |
| **Institutions** | University of Lausanne |

# Basic Power Law






- *It shows straight line when a log-log plotted, as*

$$\ln(p(x)) = c - \alpha \ln(x)$$

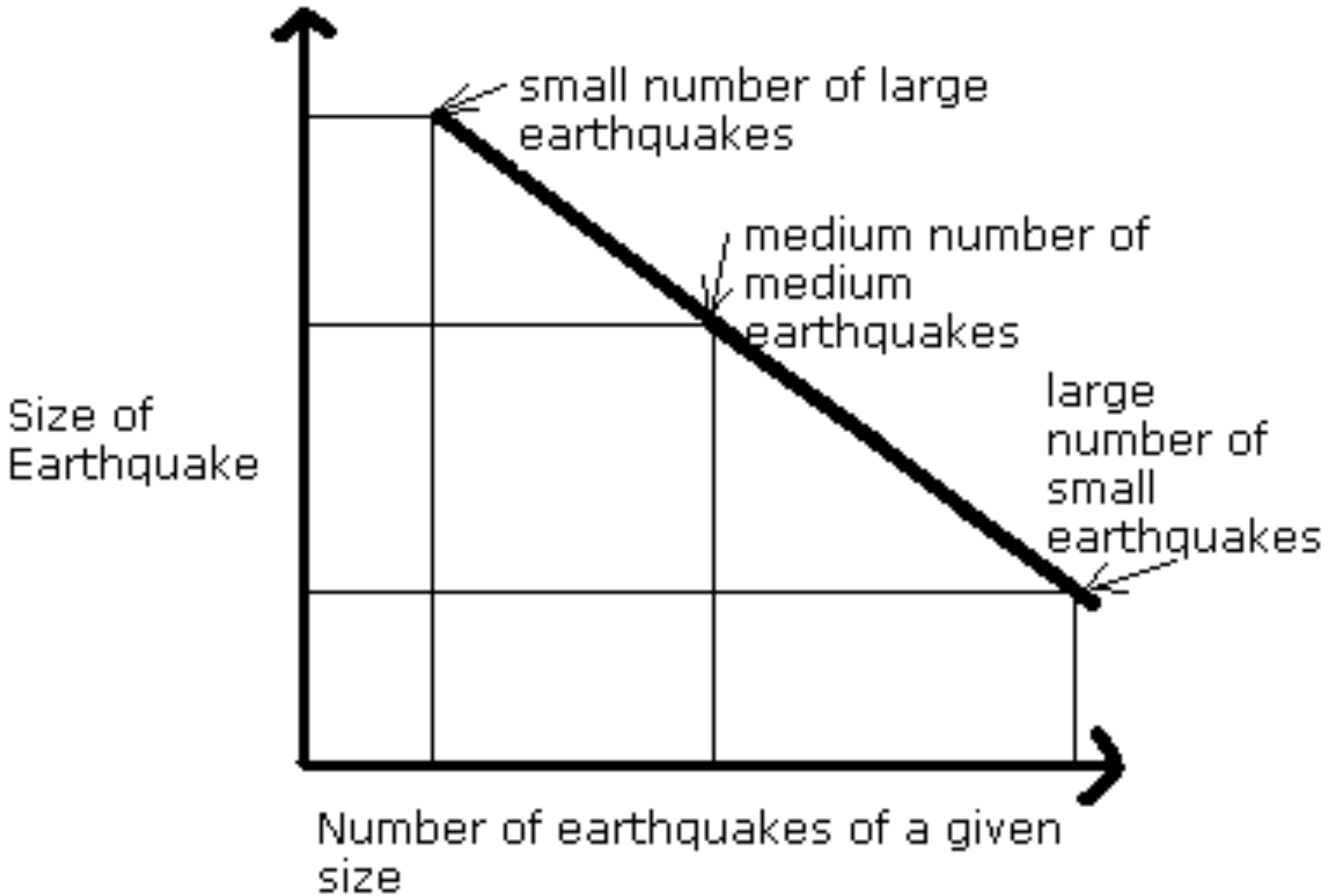
- *It exponentiate on both sides such that p(x), that is the probability observed item having size 'x', given by*

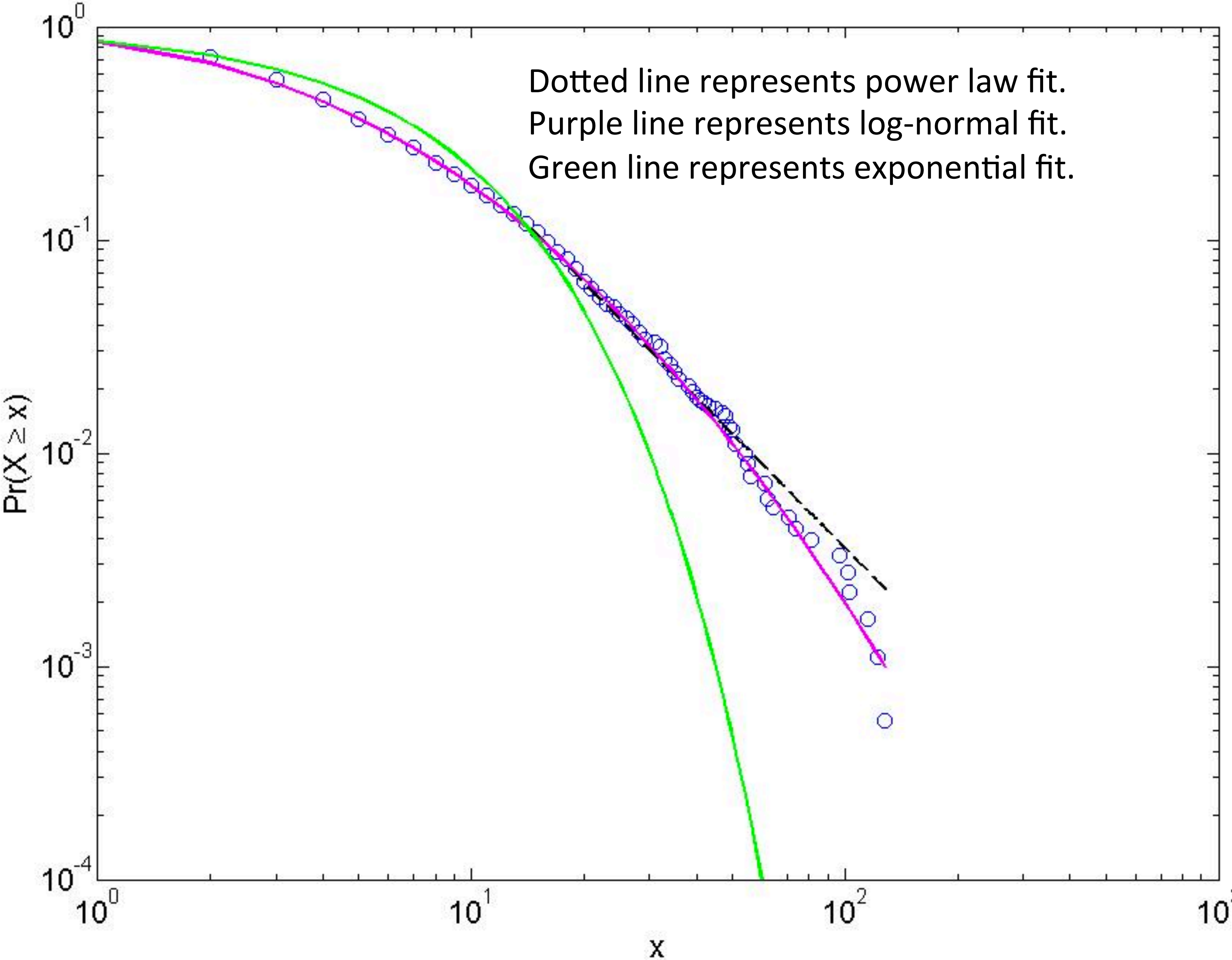
$$p(x) = Cx^{-\alpha}$$

normalization constant (probabilities over all $x$ must sum to 1)

power law exponent $\alpha$

- Rank the dat in creasing order, $x_1$, $x_2$,…$x_n$
- Generate a vector of values N, N-1, N-2,…,N-N
- Plots on a binary diagram the couples ($x_1$,N), ($x_2$, N-1),…($x_n$,0)
- Transform the axes in log-log → the red curve will be a straight line→ presence of a power law

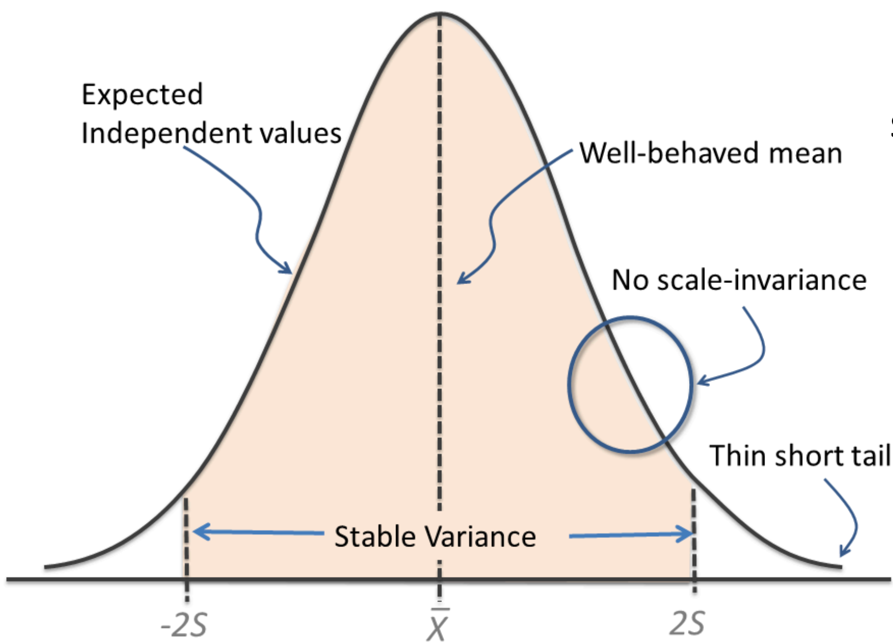

small number of large earthquakes
medium number of medium earthquakes
large number of small earthquakes
Size of Earthquake
Number of earthquakes of a given size



Dotted line represents power law fit.
Purple line represents log-normal fit.
Green line represents exponential fit.
Pr(X ≥ x)
x
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^0$
$10^1$
$10^2$
$10^3$

Complex systems often display **scale invariance**, i.e. similar patterns appear over and over in many scales of observation, in a fractal manner. Scale invariance is often associated with a power-law frequency distribution.

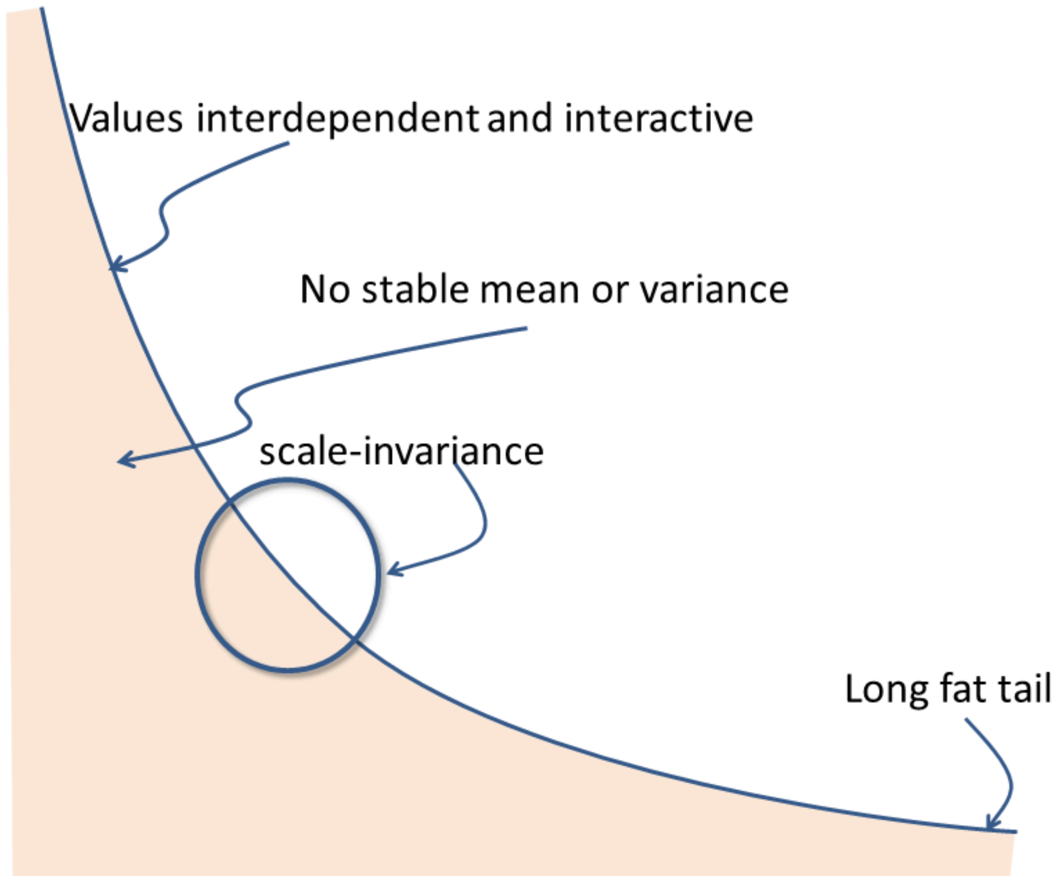
For example, the sizes of fire events typically display this type of distribution. This means that, for each fire of size s, the amount of fires whose size is, say, s/2, does not depend on s (unless s is extremely large or small).




Main Characteristics of Normal Distribution

a fragment of a normal curve doesn't show similarity to the rest of the curve – i.e. lacks scale-invariance. Smaller sample size doesn't represent the population.

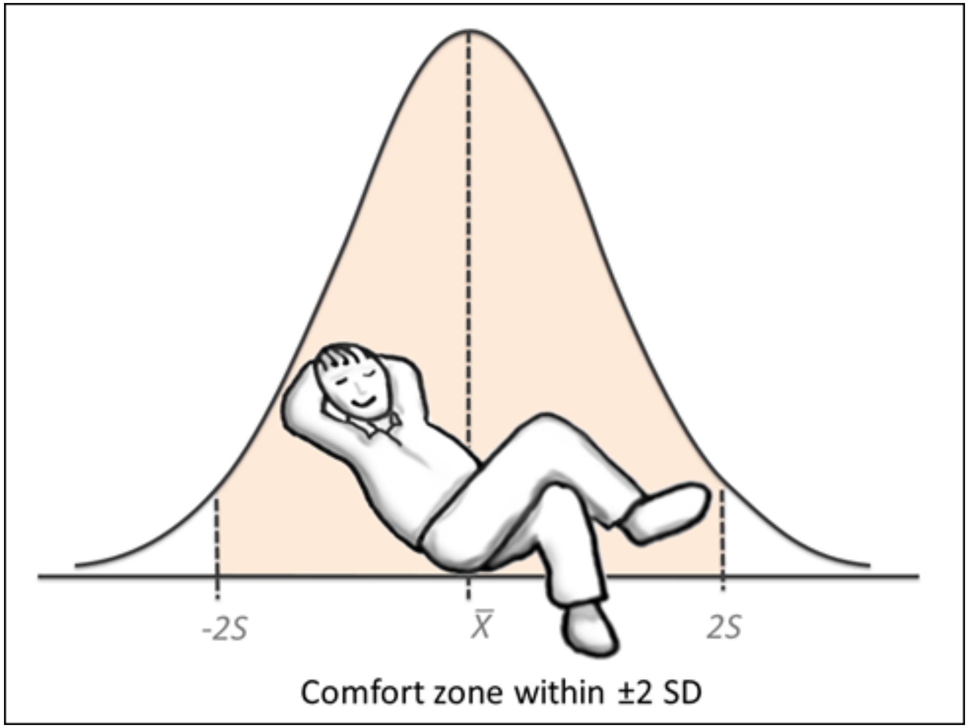
Pareto distribution, defined by Vilfredo Pareto, is a power law distribution and nothing like the normal distribution. It is not symmetric. It doesn't have a stable or well-behaved mean or variance. So these values become meaningless. It has a long thick tail, allowing the presence of more extremes. It assumes that samples are interdependent. Furthermore, like a fractal, for most parts it has **scale-invariance**, parts of curve show similarity to the whole (self-similarity and self-affinity).




Main Characteristics of Power Distribution

Systems that show a Paretian Distribution have interdependent, interactive, and/or self-organizing elements that disallow linearity. The causal impacts on the system may not produce effects that are proportional: butterfly effect on weather systems for example (proportionality of cause and effect). Likewise we cannot calculate the impact of multiple causes acting on the system by summing them up (superposition).

Lacking stable or well-behaved mean and independent elements means that these systems do not show a trend toward equilibrium, as the interdependent and self-organizing behavior of the elements do not allow this to happen.




disregard the interdependency and interactivity of elements in a system.

Wilderness of Pareto

Actual universe is complex or chaotic in behavior. Extremes happen all the time (by universe time, not our blink of existence). The second law of thermodynamics tells us that the universe doesn't tend towards equilibrium or balance, actually it prefers chaos.

Linear causality and superposition are not valid for most natural systems. Nature and society, and related systems are all complex, messy, and unpredictable, where extremes can happen, individual elements can eventually make big impact, and the systems may not settle in a preset equilibrium.

To survive in Pareto world you pay attention to extremes, especially **low-probability**, high impact extremes.

Einstein was a low-probability/high impact human being, so was Hitler, they were outliers in the Gaussian world.

To survive the unpredictable world, where individual extremes can have high impact you know that you cannot build a perfect Richter 14 earthquake proof building. You don't build for robustness, you build for **resilience**: shorter buildings, better evacuation, post-earthquake survival, less dense cities, and etc..

***In the complex system of nature and society elements constantly interact with and modify each other and the system itself. Causality is not linear.***

# Fractal structures optimize entropy production in complex dissipative systems

**TIME, STRUCTURE AND FLUCTUATIONS**

Nobel Lecture, 8 December, 1977

by

ILYA PRIGOGINE

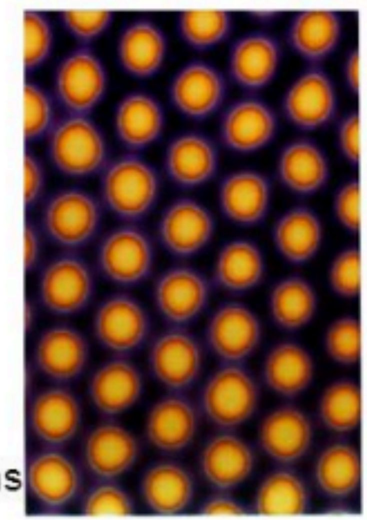
Université Libre de Bruxelles, Brussels, Belgium
and the University of Texas at Austin, Austin, Texas, USA

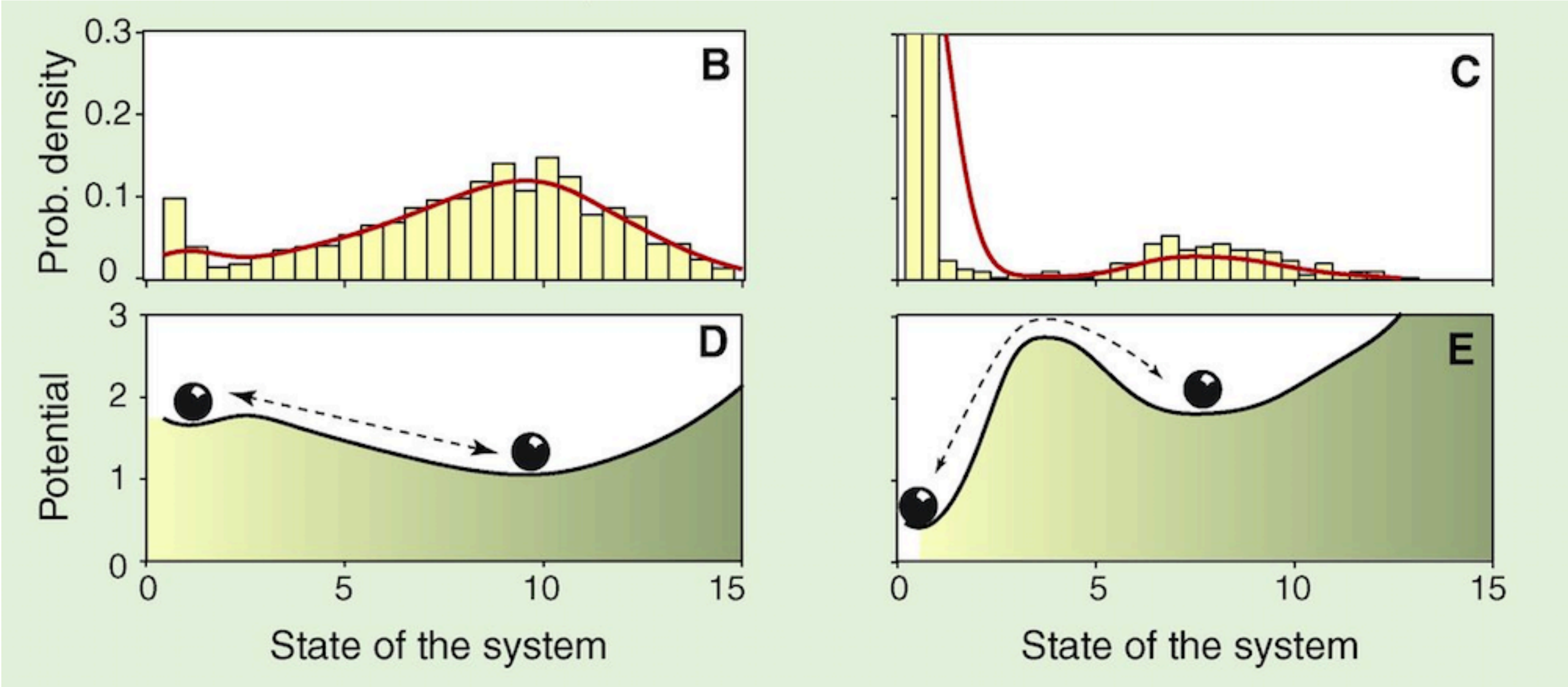
**Ilya Prigogine**

| | |
|---|---|
| **Born** | Ilya Romanovich Prigogine<br>25 January 1917<br>Moscow, Russian Empire |
| **Died** | 28 May 2003 (aged 86)<br>Brussels, Belgium |
| **Nationality** | Belgian |

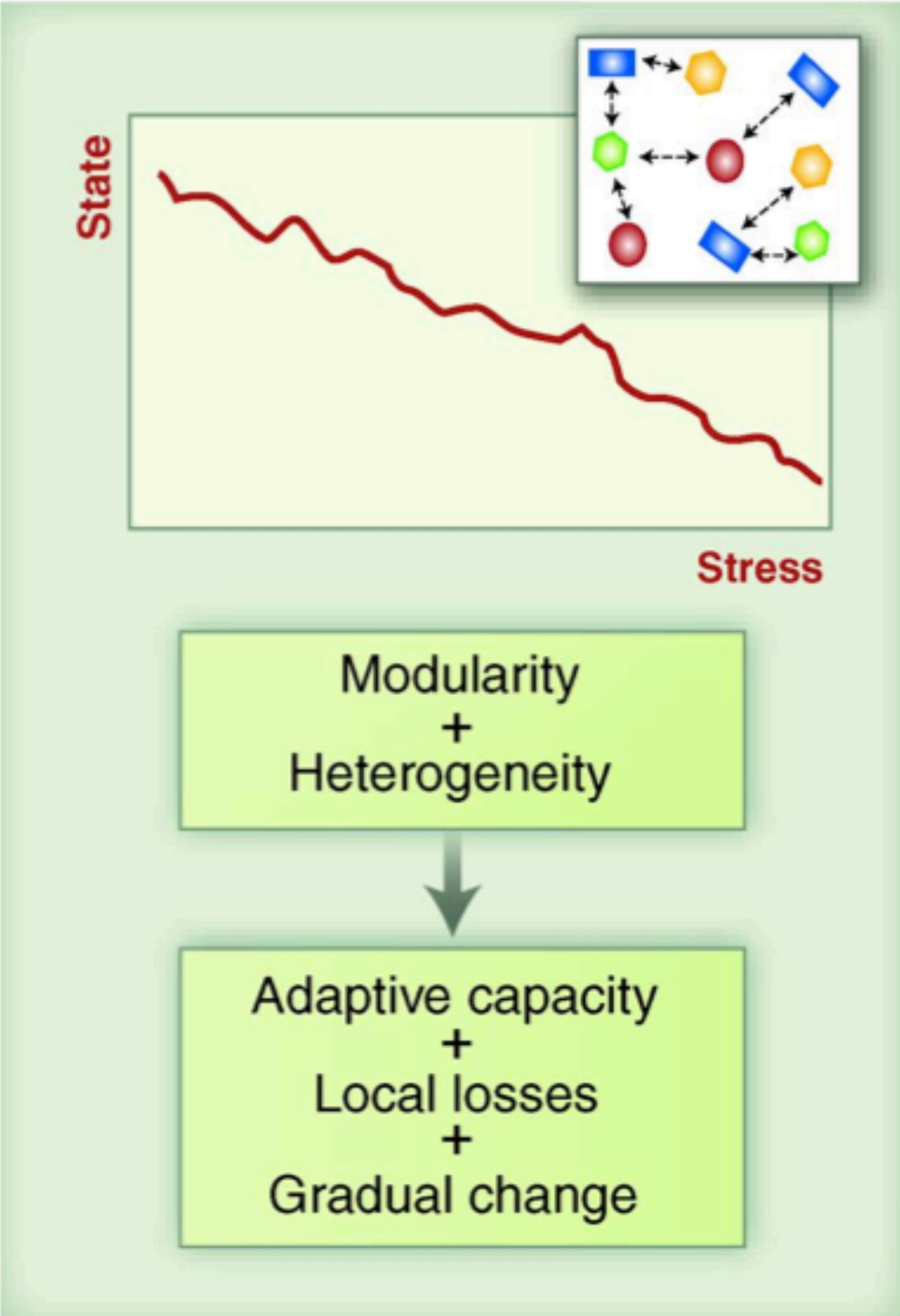
- Dissipative structures are organized arrangement in non-equilibrium systems that are dissipating energy and thereby generate entropy

Convection patterns



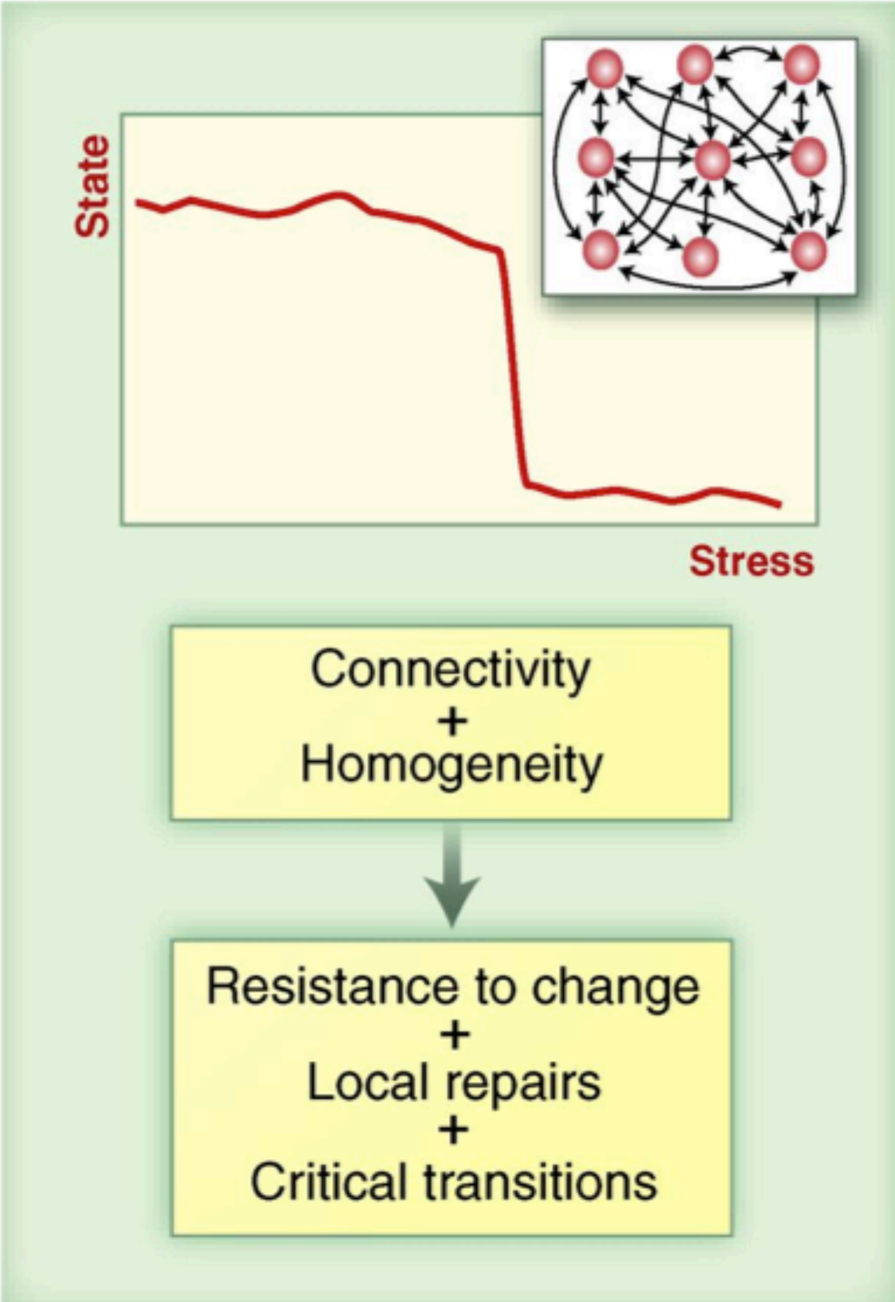



(A) Flickering to an alternative state as a warning signal in highly stochastic systems. In such situations, the frequency distribution of states (B and C) can be used to approximate the shape of the basins of attraction of the alternative states (D and E).

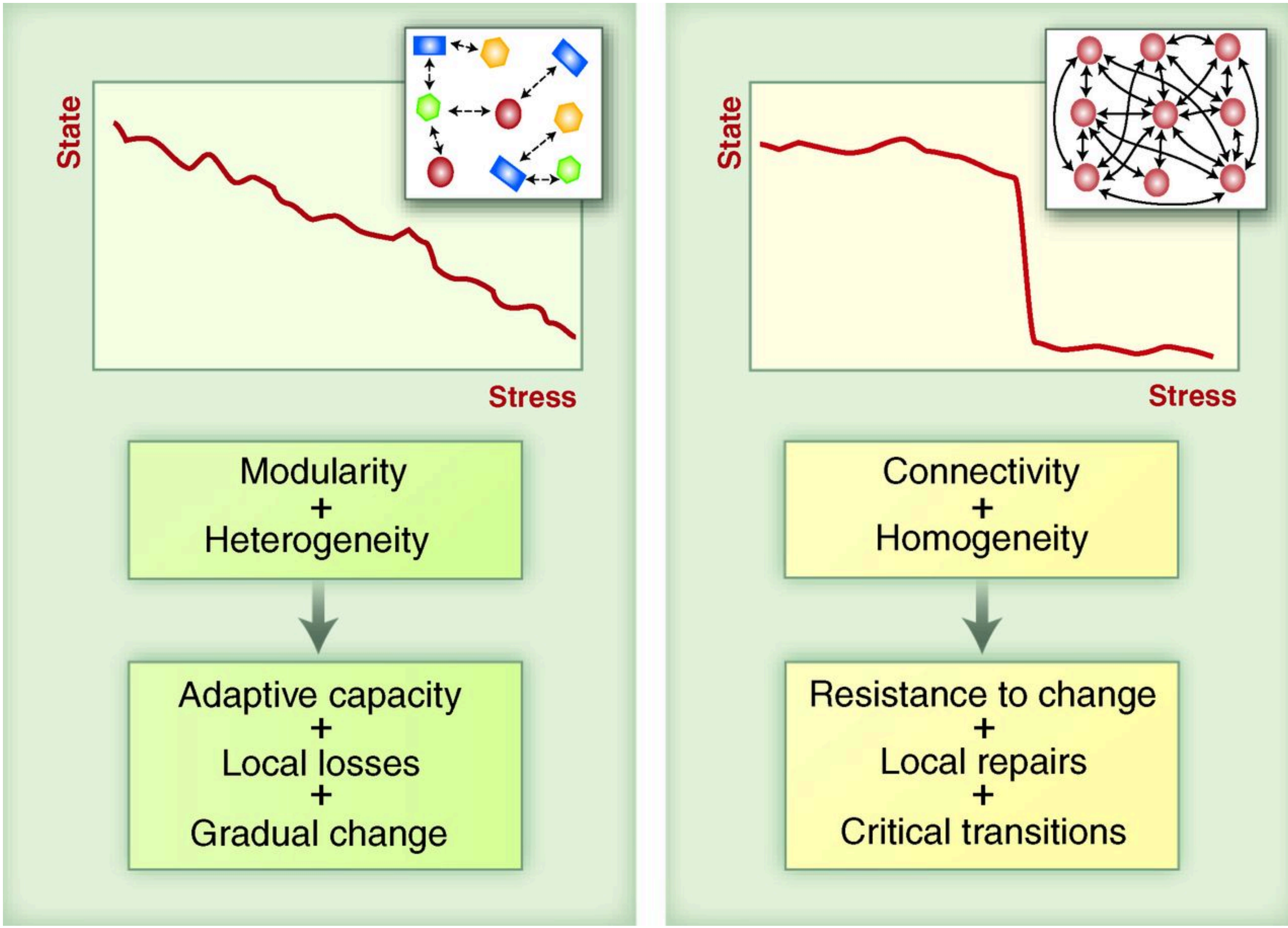


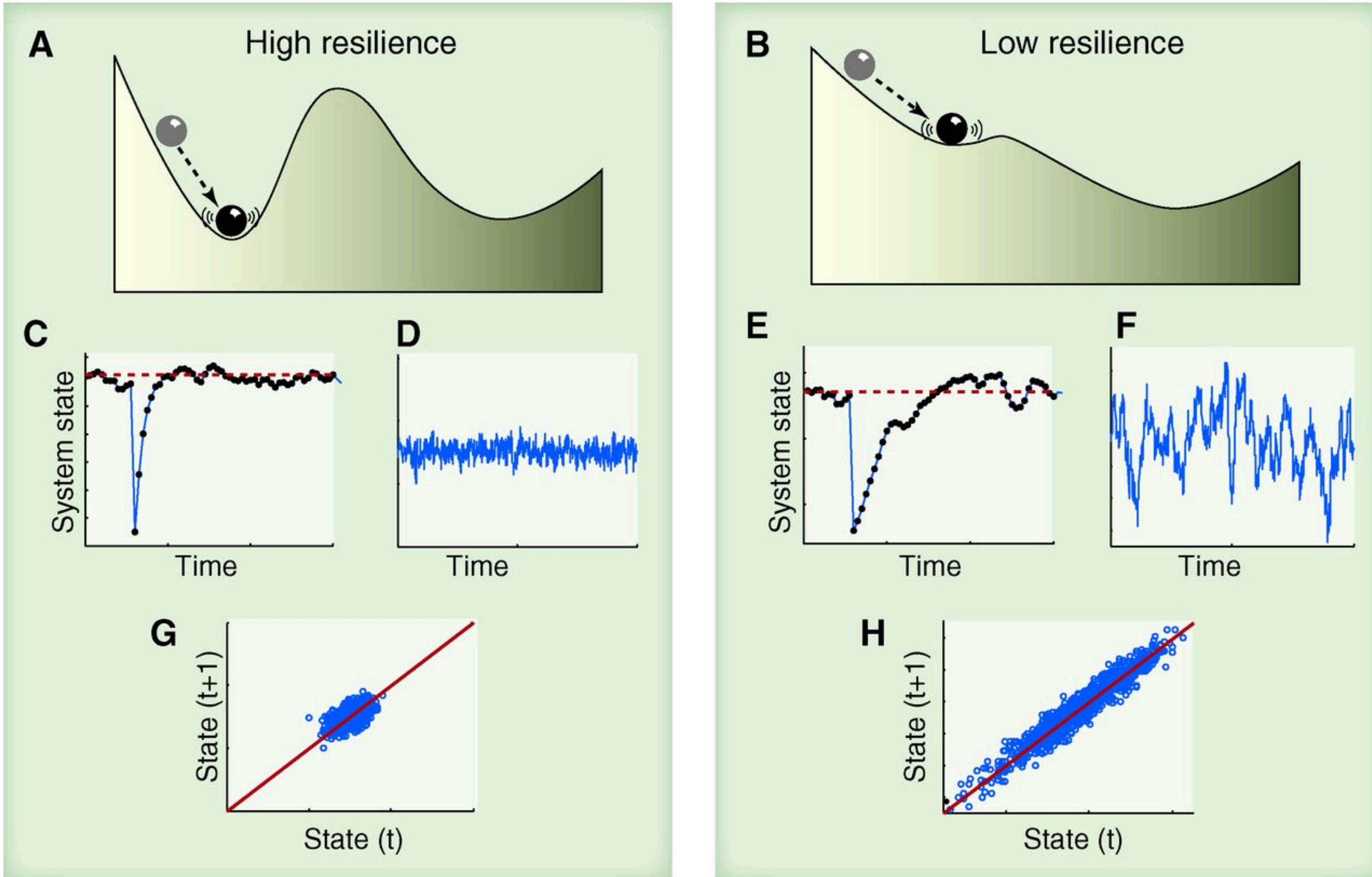

The connectivity and homogeneity of the units affect the way in which distributed systems with local alternative states respond to changing conditions.

Networks in which the components differ (are heterogeneous) and where incomplete connectivity causes **modularity** tend to have adaptive capacity in that ***they adjust gradually to change***.




By contrast, **in highly connected networks**, local losses tend to be “repaired” by subsidiary inputs from linked units until at a critical stress level the system collapses. The particular structure of connections also has important consequences for the robustness of networks, depending on the kind of interactions between the nodes of the network.


State
Stress
Modularity
+
Heterogeneity
Adaptive capacity
+
Local losses
+
Gradual change
State
Stress
Connectivity
+
Homogeneity
Resistance to change
+
Local repairs
+
Critical transitions




Critical slowing down as an indicator that the system has lost resilience and may therefore be tipped more easily into an alternative state. Recovery rates upon small perturbations (C and E) are slower if the basin of attraction is small (B) than when the attraction basin is larger (A). The effect of this slowing down may be measured in **stochastically induced fluctuations** in the state of the system (D and F) as increased variance and "memory" as reflected by lag-1 autocorrelation (G and H).

Different classes of generic observations that can be used to indicate the potential for critical transitions in a complex system.