

4. CONCLUSION

Between 1972 and 1977, there was a substantial decline in both primary and secondary energy intensity in the service sector. Had 1977 energy technology been used in 1972, the service sector would have used 23% less primary energy and 12% less secondary energy. The major exceptions to this trend were in the medical service and education sectors. In spite of the overall trend toward increased energy efficiency, the service sector actually became more electricity-intensive. Had 1977 energy technology been used in 1972, the service sector would have used about 9% more electricity. Although electricity is a more efficient end-use fuel its generation requires a substantial amount of primary energy. The service sector currently consumes about 25% of U.S. electricity output and meeting an increasing demand for electricity in this sector may pose problems in the future.

REFERENCES

1. W. Walker, *Energy Policy* 13, 458 (1985).
2. D. Jorgensen, *Challenge* 23, 16 (1980).
3. C. Bullard and R. Herendeen, *Energy Policy* 1, 268 (1975); B. Hannon and T. Blazek, *Energy Syst. Policy* 8, 85 (1984).
4. U.S. Energy Information Administration, State Energy Data Report, Consumption Estimates 1960-82. Washington, D.C. (May, 1984).
5. U.S. Energy Information Administration, *Annual Energy Review* 1984. Washington, D.C. (April 1985).
6. R. Miller and P. Blair, *Input-Output Analysis*. Prentice-Hall, Englewood Cliffs, N.J. (1985).
7. D. Kowalczyk, *Energy Policy* 13, 169 (1985).
8. B. Barker, *EPRI J.* December 1983; M. Wayne, *EPRI J.* June 1982.
9. P. LeBel, *Energy Economics and Policy*, p. 179. Johns Hopkins Press, Baltimore (1982).

THERMO-ECONOMIC FUNCTIONAL ANALYSIS AND OPTIMIZATION

CHRISTOS A. FRANGOPOULOS

Department of Naval Architecture and Marine Engineering, National Technical University of Athens, Athens 106 82, Greece

(Received 18 August 1986)

Abstract—Thermo-economic functional analysis is a method for optimal design or improvement of complex thermal systems. Thermodynamic concepts are combined with economic considerations in a systems approach. Units are the basic elements of the system; each unit has a particular quantified function (purpose or product). The distribution of functions establishes inter-relations between units or between the system and the environment and leads to a functional diagram of the system. The optimization minimizes the total cost of owning and operating the system, subject to constraints revealed by the functional diagram and analysis. The general formulation and a numerical example are presented.

NOMENCLATURE

b_{ri}	cost coefficients (constants)	<i>Greek letters</i>	
C	annual fixed charge rate	η_{II}	second-law efficiency
c_e	price of electricity (\$/kJ)	η_M	$= W/(M\Delta h)$ for the turbine-generator $= M\Delta h/W$ for the pump-motor
c_f	price of fuel (\$/kJ)	λ	vector of all the Lagrange multipliers
c_{pw}	specific heat of water	σ	number of units
e_f^h	chemical essergy of fuel	τ	number of units and junctions
f_{am}	pump material factor (constant)	v	specific volume
h	enthalpy per unit mass	ϕ	maintenance factor
ℓ_r	number of outputs from unit r	ω	number of units, junctions and branching points
M	mass	<i>Subscripts</i>	
m_r	number of inputs to unit r	i	the i th decision variable
N	hours of operation per year	o	reference state
n_3	number of transfer units of condenser	r	the r th unit ($r = 0$ is for the environment)
n_r	number of decision variables for unit r	w	water
P	pressure	<i>Overmarks</i>	
p	parameter	\cdot	per unit time
R_3	overall heat transfer resistance of condenser	$-$	reference quantity (constant)
S	entropy	\cdot	fixed value of an otherwise variable quantity
S	negentropy		
s	entropy per unit mass		
T	temperature		
v	velocity		
\dot{W}	power		
x	vector of decision variables		
Y	constraint function		
y	vector of all the inputs and outputs		
$y_{r,1}$	the product of unit r		
$y_{r,k}$	the k th output from r		
$y_{r,j}$	the j th input to r		

1. INTRODUCTION

Earlier work by Tribus, Evans and El-Sayed on thermo-economics^{1,2} led to the concept of thermo-economic isolation (TI) introduced by Evans.³ In order to approach thermo-economic isolation, Tribus¹ and Evans *et al.*⁴ made use of the functions or purposes of thermal system components, but the conditions for approaching TI remained to be derived. The attempt to derive formally the complete set of conditions for TI has led us to the development of a new and generally applicable method for optimal design or improvement of complex thermal systems, which we call thermo-economic functional analysis (TFA).⁵

2. FUNCTIONAL ANALYSIS

2.1 Concepts and definitions

A thermal system (thermal power plant, refrigeration plant, chemical plant, etc.) consists of a set of inter-related units.⁶ Each unit has one particular function (purpose or product).

Functional analysis is "the formal, documented determination of the function of the system"⁷ as a whole and of each unit individually.⁵

2.2 The functional diagram of a system

The picture of a system will be composed of small geometrical figures representing the units and a network of lines representing the distribution of the unit functions. This is the functional diagram of the system. Junctions (where the functions of two or more units merge) and branching points (where the function of a unit is distributed to more than one unit) are considered to be fictitious units, unless they correspond to real components of the plant.

3. THERMO-ECONOMIC FUNCTIONAL OPTIMIZATION

The optimization objective is the minimization of the total cost of owning and operating the system, with income included as a negative cost.

3.1 Total optimization

The system is considered to be operating at the steady state with products of a specific type, but not necessarily of predetermined quantity. The objective function is

$$\min_{\mathbf{x}} \dot{F} = \sum_{r=1}^{\sigma} \dot{Z}_r + \sum_{k=1}^{\ell_0+m_0} \dot{\Gamma}_{0,k} \quad (1)$$

where \dot{F} = cost rate of owning and operating the whole system, \dot{Z}_r = amortized capital cost rate of unit r including fixed charges and maintenance, $\dot{\Gamma}_{0,k}$ = cost rate associated with $y_{0,k}$. For $1 \leq k \leq \ell_0$, $\dot{\Gamma}_{0,k}$ is positive and represents expenditure corresponding to services purchased by the system from the environment; penalties for environmental pollution or other social hazards can be quantified and included. For $\ell_0 + 1 \leq k \leq \ell_0 + m_0$, $\dot{\Gamma}_{0,k}$ is negative and represents profit from selling products.

Costs are functions of decision variables and quantities purchased or sold as follows:

$$\dot{Z}_r = Z_r(x_r, y_{r,1}) \equiv Z_r, \quad (2a)$$

$$\dot{\Gamma}_{0,k} = \Gamma_{0,k}(y_{0,k}) \equiv \Gamma_{0,k}, \quad (2b)$$

$$\dot{F} = F(\mathbf{x}, \mathbf{y}) \equiv F. \quad (2c)$$

Both sides of Eqs. (2) are expressions for cost rate, but the left hand side (e.g. \dot{Z}_r) represents the cost rate quantity itself, while the right hand side (e.g. Z_r) represents the mathematical functional operation which generates its numerical value.

In view of Eqs. (2), the objective function takes the form

$$\min_{\mathbf{x}, \mathbf{y}} F = \sum_{r=1}^{\sigma} Z_r(x_r, y_{r,1}) + \sum_{k=1}^{\ell_0+m_0} \Gamma_{0,k}(y_{0,k}). \quad (3)$$

The equality constraints are

$$Y_{r,j} = Y_{r,j}(x_r, Y_{r,1}) \equiv Y_{r,j} \quad (r = 0, 1, \dots, \omega, \quad j = 1, 2, \dots, m_r), \quad (4)$$

$$y_{r',k} = y_{r,j} \quad (r' = 0, 1, \dots, \omega, \quad k = 1, 2, \dots, \ell_r). \quad (5)$$

The constraints (4) give the inputs to the unit r as functions of the product $y_{r,1}$ and the decision variables x_r of the unit. The constraints (5) represent interconnections between

units or between a unit and the environment, which are revealed by the functional diagram of the system.

Inequality constraints would not alter the basic principles of the method, but they will not be considered.

For the solution of the optimization problem, the method of Lagrange multipliers has been chosen, which leads readily to the special cases of decomposition and thermo-economic isolation.⁵ The Lagrangian is

$$L = \sum_{r=1}^{\sigma} Z_r + \sum_{k=1}^{\ell_0+m_0} \Gamma_{0,k} + \sum_{r=0}^{\omega} \sum_{j=1}^{m_r} \lambda_{r,j} (Y_{r,j} - y_{r,j}) + \sum_{r'=0}^{\omega} \sum_{k=1}^{\ell_r} \lambda_{r',k} (y_{r,j} - y_{r',k}). \quad (6)$$

The necessary conditions for an extremum are

$$\nabla_{\mathbf{x}} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = 0, \quad (7a)$$

$$\nabla_{\mathbf{y}} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = 0, \quad (7b)$$

$$\nabla_{\boldsymbol{\lambda}} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = 0. \quad (7c)$$

It is proved in Ref. 5 that the Lagrangian takes the convenient form

$$L = \sum_{r=1}^{\sigma} (\Gamma_r - \sum_{k=1}^{\ell_r} \lambda_{r,k} y_{r,k}) + \sum_{k=1}^{\ell_0+m_0} (\Gamma_{0,k} - \lambda_{0,k} y_{0,k}), \quad (8)$$

where

$$\Gamma_r \equiv Z_r + \sum_{j=1}^{m_r} \lambda_{r,j} Y_{r,j}. \quad (9)$$

The conditions (7c) lead to a restatement of the constraints given in Eqs. (4) and (5). The conditions (7a, b) lead to

$$\partial \left(\sum_{r=1}^{\sigma} \Gamma_r \right) / \partial x_{ri} = 0 \quad (r = 1, 2, \dots, \sigma, \quad i = 1, 2, \dots, n_r), \quad (10)$$

$$\lambda_{0,k} = \partial \Gamma_{0,k} / \partial y_{0,k} \quad (k = 1, 2, \dots, \ell_0 + m_0), \quad (11)$$

$$\lambda_{r,k} = \partial \Gamma_r / \partial y_{r,k} \quad (r = 1, 2, \dots, \omega, \quad k = 1, 2, \dots, \ell_r). \quad (12)$$

The concept of internal economy introduced by El-Sayed and Evans² and the corresponding interpretation of Lagrange multipliers as economic indicators of prices are maintained here. According to Eqs. (11), (12), each Lagrange multiplier can be viewed as the marginal price of the corresponding function (product) y .

The optimization problem is solved by using the following procedure: (a) select a set of values for the decision variables \mathbf{x} ; (b) solve Eqs. (4), (5) for the dependent variables \mathbf{y} ; (c) determine the Lagrange multipliers from Eqs. (11), (12). We stop when Eq. (10) is satisfied to an acceptable approximation. Otherwise, we select a new set for \mathbf{x} and repeat the procedure.

3.2 Fixed-product optimization

This case arises when the income functions $\Gamma_{0,k}(y_{0,k})$, $\ell_0 + 1 \leq k \leq \ell_0 + m_0$, are not known, but the products are fixed and known quantities:

$$y_{0,k} = \hat{y}_{0,j} \quad (k = \ell_0 + j, \quad j = 1, 2, \dots, m_0), \quad (13)$$

The objective function in such a situation is

$$\min_{x,y} F = \sum_{r=1}^{\sigma} Z_r(x_r, y_{r,1}) + \sum_{k=1}^{\ell_0} \Gamma_{0,k}(y_{0,k}). \quad (14)$$

Equation (6) is valid if the upper limit $\ell_0 + m_0$ of the summation over $\Gamma_{0,k}$ is replaced by ℓ_0 , while Eq. (8) takes the form

$$L = \sum_{r=1}^{\sigma} \left(\Gamma_r - \sum_{k=1}^{\ell_r} \lambda_{r,k} y_{r,k} \right) + \sum_{k=1}^{\ell_0} (\Gamma_{0,k} - \lambda_{0,k} y_{0,k}) + \sum_{j=1}^{m_0} \lambda_{0,j} \hat{y}_{0,j} \quad (15)$$

The procedure to solve the optimization problem is similar to the one described in Sec. 3.1.

3.3 Main actions for TFA and O

In summary, the thermo-economic functional analysis and optimization of a thermal system consists of the following actions: (1) identifying the functions of the system as a whole and of each individual unit; (2) constructing the functional diagram of the system; (3) formulating the optimization problem, i.e. (a) selecting the decision variables, (b) deriving the cost functions $Z_r(x_r, y_{r,1})$ and $\Gamma_{0,k}(y_{0,k})$, (c) deriving the constraint functions $Y_{r,j}(x_r, y_{r,1})$, (d) deriving the explicit form of the optimization equations; (4) solving the system of optimization equations. Parametric study and sensitivity analysis may follow.

4. APPLICATION OF TFA AND O TO A FOUR-UNIT THERMAL POWER PLANT

4.1. Description of the plant and main assumptions

The method will now be applied to a simplified thermal power plant. The system is considered to be made up of four units: (1) boiler, (2) turbine-generator, (3) condenser and condensing supply works (e.g. cooling towers, cooling-water-circulating pumps etc.), and (4) boiler feed pump (see Fig. 1).

A distinction must be drawn between a unit and a component. The cooling tower or cooling-water-circulating pump are components. However, in this example, they are not considered to be separate units but are instead combined with the condenser in a unit having one function. The number of units in a system is not unique and depends on the available information and desired results. The designer may select high (many units) or low resolution (a few units), depending on the objectives.

It is assumed that the plant produces only electrical power at a specified rate \dot{W} . The power required by the pump is supplied from outside the system. All of the components are well insulated and losses through the pipes connecting the components are negligibly small.

4.2. The functional diagram of the system

The function of each unit will be measured by means of essergy (as defined by Evans⁹) and entropy. To identify the functions and their distribution, the following procedure is used: a closed loop of an essergy-carrying stream is considered, Fig. 2. The units take from this stream or from outside the system the amount of essergy required for their operation,

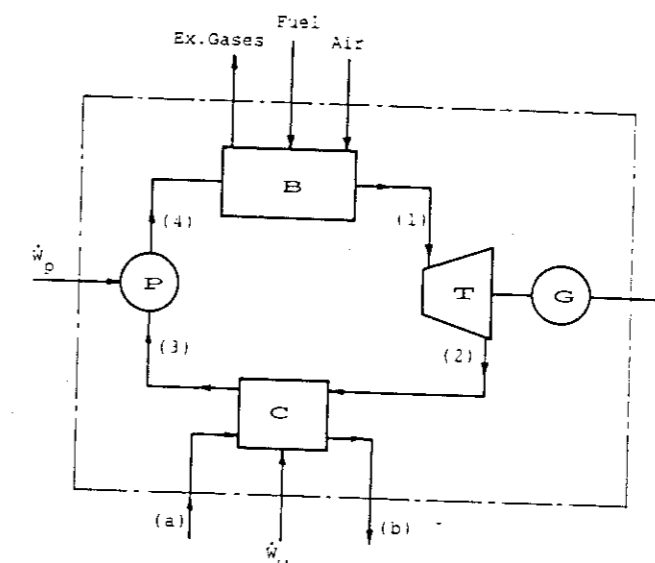


Fig. 1. A four-unit thermal power plant (B = boiler, T = turbine, G = generator, C = condenser, P = pump).

and give their product either back to the stream or to the environment. The operation of the units creates entropy, which is rejected to the environment through the condenser. In other words, the condenser is supplying the system with the negative of entropy, (negentropy, as introduced by Brillouin⁹ and used by Smith¹⁰ to quantify the function of the condenser). A negentropy-carrying stream runs parallel to the essergy stream but in the opposite direction (see Fig. 2).

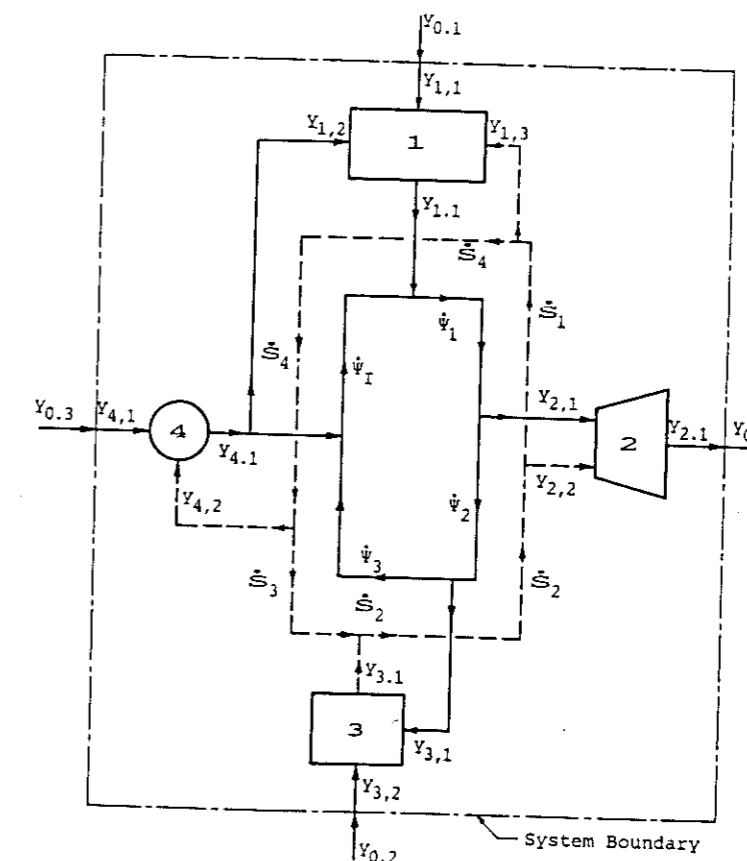


Fig. 2. The power plant with essergy and negentropy streams.

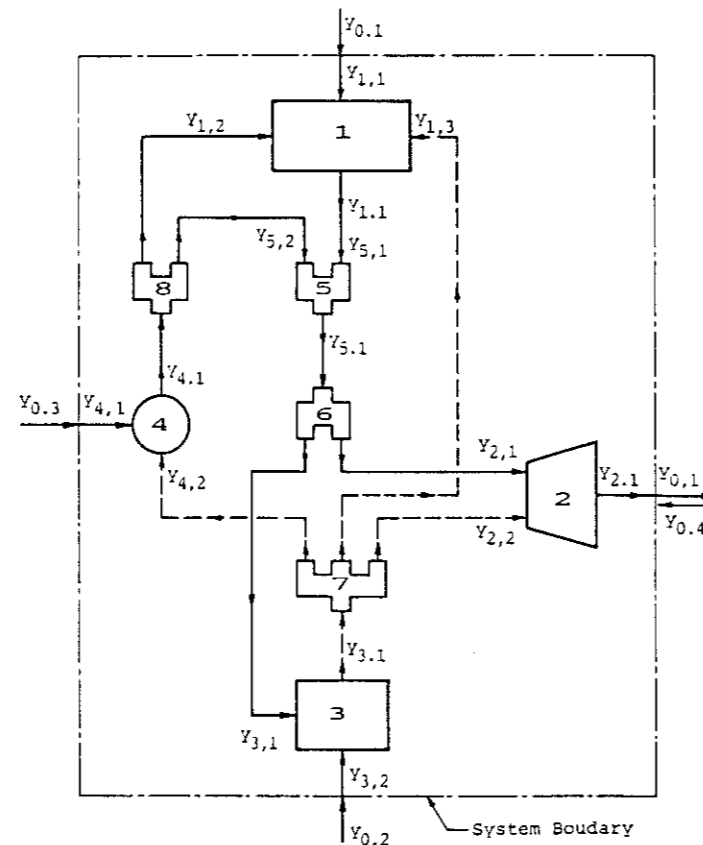


Fig. 3. Functional diagram of a four-unit thermal power plant.

Essergy and negentropy balances along each stream lead to expressions for the $y_{r,j}$, which are given in Appendix A. The concepts introduced in Sec. 2.2 and the procedure described here lead to the functional diagram of the system (see Fig. 3).

An arrow pointing toward a unit does not necessarily represent a stream (of mass, energy, essergy, etc.) entering the unit. For example, exhaust gases of a boiler form a stream exiting the boiler; but, the service of getting rid of exhaust gases is provided to the boiler by an other unit (not shown in Fig. 3, because of the resolution level selected). Similarly, if the boiler is to be penalized for environmental pollution, then the corresponding expenditure will depend on an appropriate measure of pollution, to be represented by an arrow, say $y_{1,4}$ pointing toward the unit.

4.3 Optimization of the system

Among the many quantities involved, the decision variables are selected to be

$$\mathbf{x} = (T_1, \eta_{H1}, T_2, T_b, v_w, \eta_{H4}). \quad (16)$$

The remaining quantities are either dependent variables or parameters. The objective function is given by Eq. (14) with $\sigma = 4$ and $\ell_0 = 3$.

The procedure described at the end of Sec. 3.1 has been used to solve the optimization problem for $\dot{W} = 20$ MW. Figures 4 and 5 show the influence of fuel and electricity prices on optimum values of decision variables. The effect of parameter uncertainties on the optimum solution may be obtained by using sensitivity analysis. The change (uncertainty) in the objective function due to a change (uncertainty) in a particular parameter may be estimated from

$$\Delta F = (\partial F / \partial p_j) \Delta p_j. \quad (17)$$

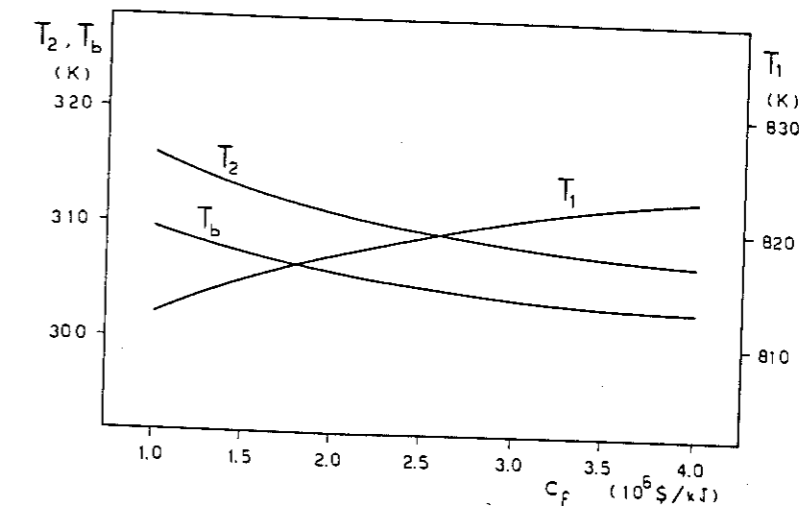


Fig. 4. Effect of fuel price on optimum values of decision variables.

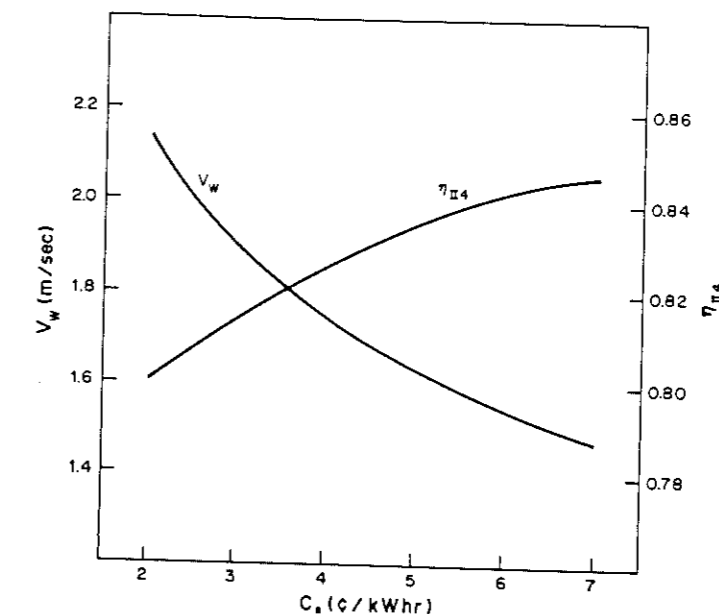


Fig. 5. Effect of electricity price on optimum values of decision variables.

The maximum possible uncertainty in the objective function caused by uncertainties in a set of parameters is

$$\Delta F_{\max} = \sum_j \left| \partial F / \partial p_j \right| \Delta p_j, \quad (18)$$

while the most probable uncertainty is

$$\Delta F_{\text{prob}} = \sqrt{\sum_j [(\partial F / \partial p_j) \Delta p_j]^2}. \quad (19)$$

As an example, sensitivity analysis results are presented in Table 1. Primarily economic parameters have been selected, because of the inherent uncertainty in deriving or predicting values of these parameters. Cost data found in the literature are only accurate to ± 25 –30%.

Table 1. An example of sensitivity analysis

j	Parameter (p_j)			$\frac{\partial F}{\partial p_j}$	$\Delta p_j^{(*)}$	ΔF_j
	Symbol	Nominal Value	Units			
1	c_f	2.0×10^{-5}	\$/kJ	7.7378×10^{-4}	2.0×10^{-7}	1.5476×10^{-2}
2	c_e	8.33×10^{-6}	\$/kJ	2.3541×10^{-2}	8.33×10^{-7}	1.9610×10^{-4}
3	b_{11}	740.	\$/kW ^{0.8}	3.7912×10^{-5}	74.	2.8055×10^{-3}
4	b_{12}	0.8	-	2.8941×10^{-1}	0.08	2.3153×10^{-2}
5	b_{21}	3000.	\$/kW ^{0.7}	7.2855×10^{-6}	300.	2.1857×10^{-3}
6	b_{22}	0.7	-	2.1645×10^{-1}	0.07	1.5152×10^{-2}
7	b_{31}	217.	\$/m ²	1.2885×10^{-5}	21.7	2.7961×10^{-4}
8	b_{32}	577.	\$/kg/s	5.7447×10^{-6}	57.7	3.3147×10^{-4}
9	b_{41}	378.	\$/kW ^{0.71}	6.4086×10^{-7}	37.8	2.4225×10^{-5}
10	b_{42}	0.71	-	1.1835×10^{-1}	0.071	8.4029×10^{-5}
11	f_m	1.06	-	5.3079×10^{-2}	0.106	5.6264×10^{-3}
12	c	18.2	%	3.0914×10^{-3}	1.82	5.6264×10^{-3}
$\Delta F_{\max} = 7.0940 \times 10^{-2} \quad \Delta F_{\text{prob}} = 1.0813 \times 10^{-3}$						
(*) for $\Delta p_j/p_j = 0.10$						

Acknowledgements—The author expresses his sincere gratitude to R. B. Evans for advice and encouragement. Support provided by the Georgia Institute of Technology and the National Technical University of Athens is gratefully acknowledged.

REFERENCES

1. M. Tribus and R. B. Evans, A Contribution to the Theory of Thermoeconomics, UCLA Report No. 62-36, University of California at Los Angeles, Los Angeles, Calif. (1962).
2. Y. M. El-Sayed and R. B. Evans, *J. Engng Power* 92, 27 (1970).
3. R. B. Evans, *Energy* 5, 805 (1980).
4. R. B. Evans, W. A. Hendrix and P. V. Kadaba, Essergetic functional analysis for process design and synthesis, in *Efficiency and Costing: Second Law Analysis of Processes* (Edited by R. A. Gaggioli), p. 239. ACS Symposium Series No. 235, Washington, D.C. (1983).
5. C. A. Frangopoulos, Thermoeconomic Functional Analysis: A Method for Optimal Design or Improvement of Complex Thermal Systems. Ph.D. Thesis, Georgia Institute of Technology, Atlanta, Ga (1983).
6. N. J. T. A. Kramer and J. Smit, *Systems Thinking: Concepts and Notions*. Martinus Nijhoff Social Sciences Division, Leiden, The Netherlands (1977).
7. S. L. Dickerson and J. E. Robertshaw, *Planning and Design*. Lexington Books, Lexington, Mass. (1975).
8. R. B. Evans, A Proof that Essergy is the only Consistent Measure of Potential Work. Ph.D. Thesis, Dartmouth College, Hanover, N.H. (1969).
9. L. Brillouin, *Science and Information Theory*, 2nd ed. Academic Press, New York (1962).
10. M. S. Smith, Effect of Condenser Design upon Boiler Feedwater Essergy Costs in Power Plants. M.S. Thesis, Georgia Institute of Technology, Atlanta, Ga (1981).
11. R. M. Garceau, Thermoeconomic Optimization of a Rankine Cycle Cogeneration System. M.S. Thesis, Georgia Institute of Technology, Atlanta, Ga (1982).
12. J. G. Singer ed., *Combustion: Fossil Power Systems*, 3rd ed. Combustion Engineering Inc., Windsor, Conn. (1981).
13. H. Popper, *Modern Cost Engineering Techniques*. McGraw-Hill, New York (1970).
14. R. S. Hall, J. Matley and K. J. McNaughton. *Chem. Engng* 89, 80 (1982).

APPENDIX A

Expressions for the Functions of Fig. 2 and 3

The essergy flows related to the environment are:

$$y_{1,1} = \dot{e}_f^h, \quad y_{3,2} = \dot{M}_w v_w (P_a - P_b) / \eta_{Mw}, \\ y_{4,1} = \dot{W}_4 = \dot{M}(h_4 - h_3) / \eta_{M4}, \quad y_{2,1} = \dot{W} = \eta_{M2} \dot{M}(h_1 - h_2).$$

Other essergy flows are:

$$y_{1,2} = \dot{M} v_4 (P_4 - P_1), \quad y_{2,1} = \dot{M}(\psi_1 - \psi_2), \quad y_{3,1} = \dot{M}(\psi_2 - \psi_3), \\ y_{4,1} = \dot{M}(\psi_4 - \psi_3), \quad \psi_1 = \psi_3 + y_{4,1} - y_{1,2} = \dot{M}[\psi_4 - v_4(P_4 - P_1)], \\ y_{1,1} = \psi_1 - \psi_1 = \dot{M}[\psi_1 - \psi_4] + v_4(P_4 - P_1),$$

where ψ is the flow essergy per unit mass:

$$\psi = h - h_0 - T_0(s - s_0) \quad \text{and} \quad \dot{\psi} = \dot{M}\psi.$$

The negentropy flows are:

$$y_{3,1} = \dot{S}_2 - \dot{S}_3 \rightarrow y_{3,1} = \dot{M}T_0(s_2 - s_3), \\ y_{1,3} = \dot{S}_1 - \dot{S}_4 \rightarrow y_{1,3} = \dot{M}T_0(s_1 - s_4), \\ y_{2,2} = \dot{S}_2 - \dot{S}_1 \rightarrow y_{2,2} = \dot{M}T_0(s_2 - s_1), \\ y_{4,2} = \dot{S}_4 - \dot{S}_3 \rightarrow y_{4,2} = \dot{M}T_0(s_4 - s_3),$$

where:

$$\dot{S}_i = -\dot{M} s_i (i = 1, 2, 3, 4).$$

The last forms of $y_{3,1}$, $y_{1,3}$, $y_{2,2}$, and $y_{4,2}$ are obtained by multiplying negentropy flows by the reference temperature T_0 and changing the sign. This convenient procedure is used in order to obtain all functions as positive quantities in the same units.

APPENDIX B

Cost Functions

The following equations have been derived in Ref. 5 by using information from several sources (e.g. Refs. 11-14):

$$Z_1 = \zeta_{11} y_{1,1}^{\alpha_1} g_{1p} g_{1r} g_{1T}, \quad Z_2 = \zeta_{21} y_{2,1}^{\alpha_2} g_{2p} g_{2T}, \\ Z_3 = T_2 (\zeta_{31} n_3 R_3 + \zeta_{32} / c_{pw}) y_{3,1} / [T_0 (T_0 - T_2)], \\ Z_4 = \zeta_{41} y_{4,1}^{\alpha_4} f_{4m} g_{4r}, \quad \Gamma_{0,k} = c_k y_{0,k} \quad (k = 1, 2, 3),$$

where:

$$\zeta_{ii} = (C/3.6 \times 10^5 N) \phi_i b_{ri}, \quad g_p = \exp[(P_i - \bar{P}_1)/b_{13}], \\ g_{1r} = 1 + [(0.45 - \bar{\eta}_{11})/(0.45 - \eta_{11})]^{\alpha_1}, \\ g_{r,r} = 1 + b_{rs} \exp[(T_1 - \bar{T}_1)/b_{r6}] (r = 1, 2), \\ g_{rr} = 1 + [(1 - \bar{\eta}_{1r})/(1 - \eta_{1r})]^{\alpha_r} (r = 2, 4).$$

Z_1 - Boiler

Z_2 - ST

Z_3 - Condenser

Z_4 - pump