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SPECO: A systematic and general methodology for calculating efficiencies and costs in thermal systems

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Abstract

A systematic and general methodology for defining and calculating exergetic efficiencies and exergy related costs in thermal systems is proposed. The methodology is based on the Specific Exergy Costing (SPECO) approach, in which (a) the fuel and product of a component are defined by taking a systematic record of all exergy additions to and removals from all the exergy streams of the system, and (b) the costs are calculated by applying basic principles from business administration. Thus, a direct link between the definitions of fuel and product for a component and the corresponding costing equations is established. In particular, the paper shows how to obtain detailed definitions of exergetic efficiencies using separate forms of exergy (thermal, mechanical and chemical) and how, according to these definitions, to conduct an evaluation of costs associated with all the exergy streams entering and exiting a system component. For this case, the cost equations are presented in a general matrix form.

Keywords: Thermal system; Exergy; SPECO

1. Introduction

A comprehensive evaluation of the performance of thermal systems requires a proper definition of the exergetic efficiency and a proper costing approach for each component of the system. The exergetic efficiency of a component is defined as the ratio between product and fuel ($\varepsilon = \dot{E}_P/\dot{E}_F$) [1,2]. The product ($\dot{E}_P$) and the fuel ($\dot{E}_F$) are defined by considering the desired result produced by the component and the resources expended to generate this result.

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The costs associated with each material and energy stream in a system are calculated with the aid of (a) cost balances written for each system component, and (b) auxiliary costing equations. Assuming that the costs of the exergy streams entering a component are known, a cost balance is not sufficient to determine the costs of the exiting exergy streams when the number of exiting streams is larger than one. In this case, auxiliary costing equations must be formulated for the component being considered, the number of these equations being equal to the number of exiting streams minus one.

Different approaches for formulating efficiencies and auxiliary costing equations have been suggested in the literature. These approaches can be divided into two groups: (1) The exergoeconomic accounting methods (e.g. [1–15]) aim at the costing of product streams, the evaluation of components and systems, and the iterative optimization of energy systems; (2) The Lagrangian-based approaches (e.g. [16–24]) have as a goal the optimization of the overall system and the calculation of marginal costs.

Some of the papers in the first group discuss the foundations of exergoeconomic accounting (e.g. [3–6]). Refs [1,2] and [7] present a general and comprehensive way for calculating efficiencies and costs in complex energy systems. Based on these references, a systematic organization of cost equations in conjunction with a clear matrix formulation was given in [8] and [9]. In parallel to the monetary cost associated with an exergy stream, the concept of exergetic cost of an exergy stream was introduced in [8] and [9], representing the external exergy supply needed to obtain the exergy of the stream. In all these papers only total exergy values were used and the auxiliary costing equations were formulated explicitly by using assumptions derived from experience, postulates, or the purpose of the system being analyzed.

A different approach, based on the LIFO (Last In First Out) accounting principle, was presented in [25]. According to this approach, fuels, products and costs are defined by systematically registering exergy and cost additions to and removals from each material and energy stream. In this way, ‘local average costs’ are obtained since the cost per exergy unit of the exergy used in a component is evaluated at the cost at which the removed exergy units were supplied by upstream components. An automatic criterion to generate the auxiliary costing equations based on this principle, the associated computer implementation and an algebraic formulation were presented in [26]. In that paper, the name SPECO was given to this approach because of the need of using specific exergies and costs for registering all additions and removals of exergy and cost.

The basic principles of the SPECO approach were then directly applied to exergy streams instead of material and energy streams in [27–29]. It was demonstrated that these principles are sufficient for systematically defining fuel and product of the components and for formulating the auxiliary costing equations used to calculate either average costs (AVCO approach) or local average costs (LIFO approach).

A thorough discussion of rigorous fuel and product definitions including all the exergy components (mechanical, thermal and chemical) was presented, and two fundamental principles (F and P principles) were formulated to cover most of the existing cases and to easily develop auxiliary equations in the most complex cases when separate components of exergy are considered and when mixing, separation or chemical reactions occur.

Erlach et al. [15] presented a new approach for assigning costs to systems streams. This approach is more flexible than others, allows engineers to actively participate in the cost assigning process and leads to results that are much closer to anticipated values.

In the second group of exergoeconomic approaches, a Lagrangian method was used to solve an optimization problem and calculate marginal costs as Lagrange multipliers associated with exergy streams. The pioneer work in [16] was followed by [17–24] where a significant effort was devoted to

the development of a functional structure representing the interactions among components in terms of fuel and product. The fact that marginal costs could be calculated in the optimum as Lagrange multipliers suggested to calculate these costs starting directly from their definition. Accordingly, the cost formation process was described through exergy derivatives in the 'Structural theory' [30] to calculate exergetic or monetary costs associated with different exergy flows. This principle was extended to include investment costs in [31]. In these papers, it is shown how to directly obtain the component cost balances and the auxiliary costing equations using derivatives, once the fuel and product have been defined. Further developments of the Structural Theory, proposed as a standard for thermoeconomics, are presented in [32].

Exergoeconomic accounting methods use principles from business administration. Cost balances are explicitly formulated and resources used in the production process are valued at the costs at which they were purchased or generated; this information is used in the formulation of the auxiliary costing equations.

Lagrangian-based approaches, on the other side, employ mathematical techniques to arrive at costs. It can be easily demonstrated that the same cost balances and auxiliary equations used in accounting methods can be obtained through partial derivatives in the Lagrangian-based approaches.

As concluded in [33], the differences between various exergoeconomic approaches are mainly limited to the way of representing the productive structure, in case such a structure is used. As long as the fuel and product definitions are the same, the costs calculated by various approaches are the same.

The present paper summarizes the results of all previous papers associated with the SPECO approach ([25–29]) and enlarges the application of the SPECO procedure to components that were not treated in detail by this procedure in the past (see e.g. dissipative components). Fuel, product and cost equations are given as a 'recipe', leaving to the mentioned references the details needed to develop these equations. The general matrix formulation of the costing equations proposed in [8] and further developed in [34,35] is applied here to SPECO costing equations in the more complex case in which exergy is separated into its thermal, mechanical and chemical components. This is a very practical tool for applications and demonstrates the possibility of easily transforming into an algebraic form a criterion based on the recording of additions and removals of exergy along the exergy streams. From the application point of view, the distance between the suggested approach and other well-formalized approaches (see e.g. [8,17,18]) is therefore narrowed. The main objective of this paper is to supply a systematic and unambiguous procedure for evaluating fuels and products for system components, and exergetic and monetary costs associated with all material and energy streams of the overall energy conversion system being considered. This provides a close insight of the process development in terms of exergy and money, which can be used conveniently both at system design level (in the optimization of the configuration and the design variables) and at operation level (particularly for the diagnosis of malfunctions). Unlike other methodologies, the fuel and product definitions and the auxiliary cost equations are developed here at the component level also in the most complex case in which the separate components of exergy are considered. Therefore, these definitions are in the SPECO method independent of the overall system configuration.

2. The SPECO method

The general method consists of the following three steps.

2.1. Step 1: identification of exergy streams

Initially, a decision must be made with respect to whether the analysis of the components should be conducted using total exergy or separate forms of the total exergy of a material stream (e.g. thermal, mechanical and chemical exergies). Considering separate exergy forms improves the accuracy of the results. However, this improvement is often marginal and not necessary for extracting the main conclusions from the exergoeconomic evaluation. Since the increase in the computational efforts is significant when separate exergy forms are considered, the decision to be made in this step should be based on the purpose of the study and on the system being evaluated. Components in which the evaluation of the separate forms of exergy may significantly improve the accuracy of the results are, for example, those in which an exothermal chemical reaction occurs, such as gasification reactors (see Section 5.3). Also, a distinction between thermal and mechanical exergy in components with significant pressure losses may provide more accurate results. After a decision with respect to the exergy forms to be used is made, all exergy streams associated with the entering and exiting material and energy streams are identified and the exergy values are calculated.

2.2. Step 2: definition of fuel and product

The product is defined to be equal to the sum of

- all the exergy values to be considered at the outlet (including the exergy of energy streams generated in the component) plus
- all the exergy increases between inlet and outlet (i.e. the exergy additions to the respective material streams) that are in accord with the purpose of the component.

Similarly, the fuel is defined to be equal to

- all the exergy values to be considered at the inlet (including the exergy of energy streams supplied to the component) plus
- all the exergy decreases between inlet and outlet (i.e. the exergy removals from the respective material streams) minus
- all the exergy increases (between inlet and outlet) that are not in accord with the purpose of the component.

In order to implement the foregoing definitions of product and fuel in practice, decisions need to be made. In evaluating the performance of a component it is, in general, meaningful and appropriate to operate with exergy differences associated with each material stream between inlet and outlet of the component. Exergy differences (exergy additions to, or removals from a material stream) should be calculated for all exergy streams associated with a change of physical exergy (or thermal and mechanical exergy) and in most cases for exergy streams associated with the conversion of chemical exergy.

However, in some cases involving the conversion of chemical exergy (e.g. conversion of the chemical exergy of a solid fuel into chemical and thermal exergy through a gasification process), the purpose of owning and operating the component dictates that the chemical exergy at the outlet is considered on the product side and the chemical exergy at the inlet on the fuel side. In general, when the analysis is

conducted using only total exergies, a difference in total exergy values between inlet and outlet of the considered material stream should be used unless the chemical exergy represents the main exergy form of this material stream and the purpose of the component dictates that the chemical exergies at the inlet and outlet should be considered separately at the fuel and product sides, respectively. As an example, when only total exergies are used in the definition of the exergetic efficiency of a gasifier, the exergy of the 'fuel' for the component equals the value of the exergy of the entering fuel stream.

Thus, in the definition of fuel and product of a component, a decision must be initially made for each chemical (reactive [14]) exergy stream (and consequently also for the corresponding total exergy stream) in the system with respect to whether the exergy difference between inlet and outlet or just the exergy values at the inlet and outlet should be considered separately on the fuel and product side, respectively. This decision is based on the purpose of owing and operating the component. We ask, for example, the question whether the purpose of the component is to supply chemical exergy to a stream, to consume a part of the chemical exergy of a stream, or to provide at the outlet a different type of chemical exergy than is available at the inlet. Only in the last case, no differences of chemical or total exergies are used in the definition of exergetic efficiency.

In any case, regardless of what decisions are made in order to define the product and fuel for each component, it should be emphasized that, once defined, they lead to unequivocally determined auxiliary equations, as discussed in the next step.

2.3. Step 3: cost equations

Exergoeconomics rests on the notion that exergy is the only rational basis for assigning costs to the interactions a thermal system experiences with its surroundings and to the sources of inefficiencies within it [14]. Thus, for entering and exiting streams of matter with associated rates of exergy transfer (exergy streams) $\dot{E}_i$ and $\dot{E}_e$, power $\dot{W}$, and the exergy transfer rate associated with heat transfer $\dot{E}_q$ we write, respectively

$$\dot{C}_i = c_i \dot{E}_i = c_i \dot{m}_i e_i \tag{1}$$

$$\dot{C}_e = c_e \dot{E}_e = c_e \dot{m}_e e_e \tag{2}$$

$$\dot{C}_w = c_w \dot{W} \tag{3}$$

$$\dot{C}_q = c_q \dot{E}_q \tag{4}$$

Here c_i, c_e, c_w and c_q denote average costs per unit of exergy, $\dot{C}_i$, $\dot{C}_e$, $\dot{C}_w$, and $\dot{C}_q$ are cost streams associated with the corresponding exergy streams, whereas e_i and e_e denote mass-related specific exergies. We refer to the approach associated with Eqs. (1)–(4) as *exergy-based monetary costing* or, for short, *exergy costing* (not to be confused with exergetic costs).

Exergy costing involves cost balances formulated for each system component separately. A cost balance applied to the kth component shows that the sum of cost rates associated with all exiting exergy streams equals the sum of cost rates of all entering exergy streams plus the appropriate charges (cost rate) due to capital investment and operating and maintenance expenses. The sum of the last two terms is denoted by $\dot{Z}$. Accordingly, for a kth component receiving a heat transfer and generating power, for

example, we write

$$\sum_e (c_e \dot{E}_e)_k + c_{w,k}\dot{W}_k = c_{q,k}\dot{E}_{q,k} + \sum_i (c_i \dot{E}_i)_k + \dot{Z}_k \tag{5}$$

In general, if there are N_e exergy streams exiting the component being considered, we have N_e unknowns and only one equation, the cost balance. Therefore, we need to formulate $N_e - 1$ auxiliary equations. This is accomplished with the aid of the F and P principles presented next.

2.4. The F and P principles

The F principle refers to the removal of exergy from an exergy stream within the component being considered when for this stream the exergy difference between inlet and outlet is considered in the definition of the fuel. The F principle states that the specific cost (cost per exergy unit) associated with this removal of exergy from a fuel stream must be equal to the average specific cost at which the removed exergy was supplied to the same stream in upstream components. In this way, we obtain one auxiliary equation per each removal of exergy so that the number of auxiliary equations provided by the F principle is always equal to the number $N_{e,F}$ of exiting exergy streams that are associated with the definition of the fuel for the component. Note that no auxiliary costing equation is required for an entering exergy stream for which no difference between inlet and outlet is considered in the fuel definition.

The P principle refers to the supply of exergy to an exergy stream within the component being considered. The P principle states that each exergy unit is supplied to any stream associated with the product at the same average cost, which is denoted with c_P. Since each stream to which exergy is supplied corresponds to an exiting stream, the number of auxiliary equations provided by the P principle is always equal to $N_{e,P} - 1$ where $N_{e,P}$ is the number of exiting exergy streams that are included in the product definition.

Since each exiting exergy stream is associated either with the fuel or with the product, the total number of exiting streams (N_e) is equal to the sum ($N_{e,F} + N_{e,P}$). Thus, the F and P principles together provide the required $N_e - 1$ auxiliary equations. The F principle is a general accounting principle and can be applied in conjunction with average or non-average costs. As shown in Eq. (12), the average cost per exergy unit c_P can be calculated from the cost balance and the equations obtained by applying the F principle.

2.5. On the principles used in the SPECO approach

The foundation of the SPECO method can be found in the approach presented in [25]. According to this approach, fuels and products are defined by systematically registering exergy additions to and removals from each material and energy stream. The associated record of cost additions to and removals from the same streams in conjunction with the application of the LIFO principle are then used to calculate ‘local average costs’ [26]. This is done by evaluating the cost per exergy unit of the exergy removed from a stream (that is used as ‘fuel’ in a component) at the cost at which the removed exergy units (only!) were supplied by upstream components. This is the basic idea included in the formulation of an F principle to define the auxiliary costing equations associated with the exergy streams involved in

the fuel definition. The SPECO approach maintains the criterion of registering systematically the exergy additions and removals from the exergy streams, which is demonstrated in [27–29] to be less ambiguous for defining the fuel and product of system components. The extension of the LIFO concept leads then to a general formulation of the F principle for the average costing method. The cost per exergy unit of the exergy removed from a stream is equal to the average cost at which all the exergy units added to this stream were supplied by upstream components.

Thus, in both cases a simple and absolutely general accounting principle is used. Compared to other existing criteria for cost accounting, arbitrariness is reduced by the fact that the criterion considers the exergy stream like a continuous 'thread' throughout the components that use the associated exergy. This can be considered true for all existing components appearing in energy systems but those in which a chemical transformation of a fuel resource into another fuel resource occurs (see e.g. a gasification reactor). In the latter it is therefore 'natural' to interrupt this 'thread' and start a new thread for the downstream process.

Thinking about an exergy stream as being 'continuous' leads on the product side to the straightforward definition of the P principle which assigns the same unit cost for the added exergy to every exergy stream belonging to the product. However, this principle has more the characteristics of a 'postulate', and is therefore more arbitrary, the level of arbitrariness being reduced when all product exergy streams belong to the same material stream. When, conversely, these product streams belong to different material streams, the level of arbitrariness is higher and can be reduced only by modifying the aggregation level within the component being considered (i.e. by splitting the component into different zones or parts). This is possible only when in each part it is possible to identify separate fuel exergy streams to be associated with the product exergy streams belonging to each material stream.

3. Applications of the SPECO method

The application of the SPECO method is demonstrated here with the aid of two general cases, shown in Figs. 1a and 2a. These two cases cover all situations that might be encountered. Fig. 1a covers all cases except those in which the chemical composition of a stream changes and exergy differences between inlet and outlet should be considered for this stream. The latter is covered in Fig. 2a. The reader should note that in real components not all of the exergy-stream situations shown in the general case of Fig. 1 exist simultaneously. Figs. 1b and 2b show the graphical representations (productive structures) of the fuel and product terms for the two general cases. Although not necessary, these productive structures can be helpful in understanding the fuel and product definitions, as discussed in Section 4.

The purpose of owning and operating the component shown in Fig. 1a [29] allows us to identify that the exergy streams 1, 2, 5, 6, and 9 are associated with the fuel whereas the streams 3, 4, 7, 8, and 10 are associated with the product of the component. Thus, the fuel ($\dot{E}_F$) and product ($\dot{E}_P$) of the component are, respectively:

$$\dot{E}_F = \dot{E}_1 + \dot{E}_2 + (\dot{E}_{5i} - \dot{E}_{5e}) + (\dot{E}_{6i} - \dot{E}_{6e}) + \dot{E}_9 \tag{6}$$

$$\dot{E}_P = \dot{E}_3 + \dot{E}_4 + (\dot{E}_{7e} - \dot{E}_{7i}) + (\dot{E}_{8e} - \dot{E}_{8i}) + \dot{E}_{10} \tag{7}$$

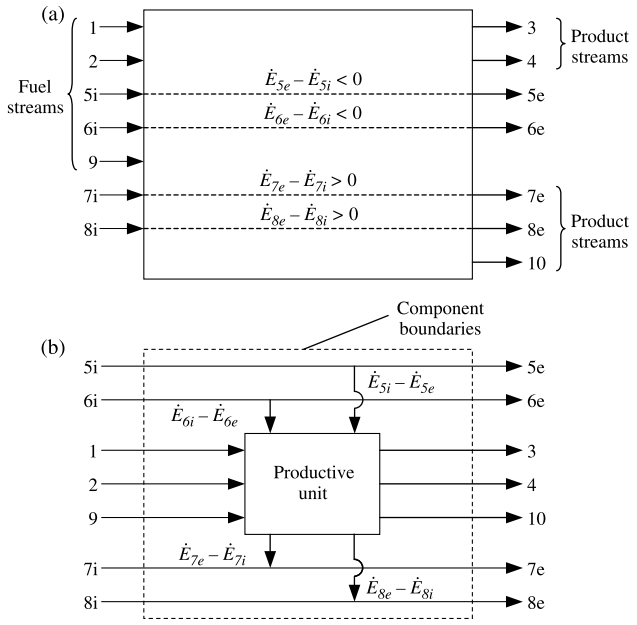

Fig. 1. (a) Schematic of a component in a thermal system to define fuel, product, and auxiliary costing equations. All streams shown in the figure are exergy streams. (b) The productive structure for the component shown in (a).

From the F principle we obtain

$$c_{5e} = c_{5i} \tag{8}$$

and

$$c_{6e} = c_{6i} \tag{9}$$

The P principle leads to the following equations:

$$c_3 = c_4 = \frac{\dot{C}_{7e} - \dot{C}_{7i}}{\dot{E}_{7e} - \dot{E}_{7i}} = \frac{\dot{C}_{8e} - \dot{C}_{8i}}{\dot{E}_{8e} - \dot{E}_{8i}} = c_{10} \tag{10}$$

The cost balance for the component shown in Fig. 1a is

$$\dot{C}_3 + \dot{C}_4 + \dot{C}_{5e} + \dot{C}_{6e} + \dot{C}_{7e} + \dot{C}_{8e} + \dot{C}_{10} = \dot{C}_1 + \dot{C}_2 + \dot{C}_{5i} + \dot{C}_{6i} + \dot{C}_{7i} + \dot{C}_{8i} + \dot{C}_9 + \dot{Z} \tag{11}$$

Since we assume that the cost rate $\dot{Z}$ and all costs associated with all entering streams are known, we can calculate the seven unknowns associated with the exiting streams $\dot{C}_3, \dot{C}_4, \dot{C}_{5e}, \dot{C}_{6e}, \dot{C}_{7e}, \dot{C}_{8e}$ and $\dot{C}_{10}$ by solving the system of Eqs. (8)–(10) (four equations) and (14). Note that Eq. (10) implicitly define a unit cost of product, c_P, made explicit as follows

$$c_P = c_3 = c_4 = \frac{\dot{C}_{7e} - \dot{C}_{7i}}{\dot{E}_{7e} - \dot{E}_{7i}} = \frac{\dot{C}_{8e} - \dot{C}_{8i}}{\dot{E}_{8e} - \dot{E}_{8i}} = c_{10} \tag{10a}$$

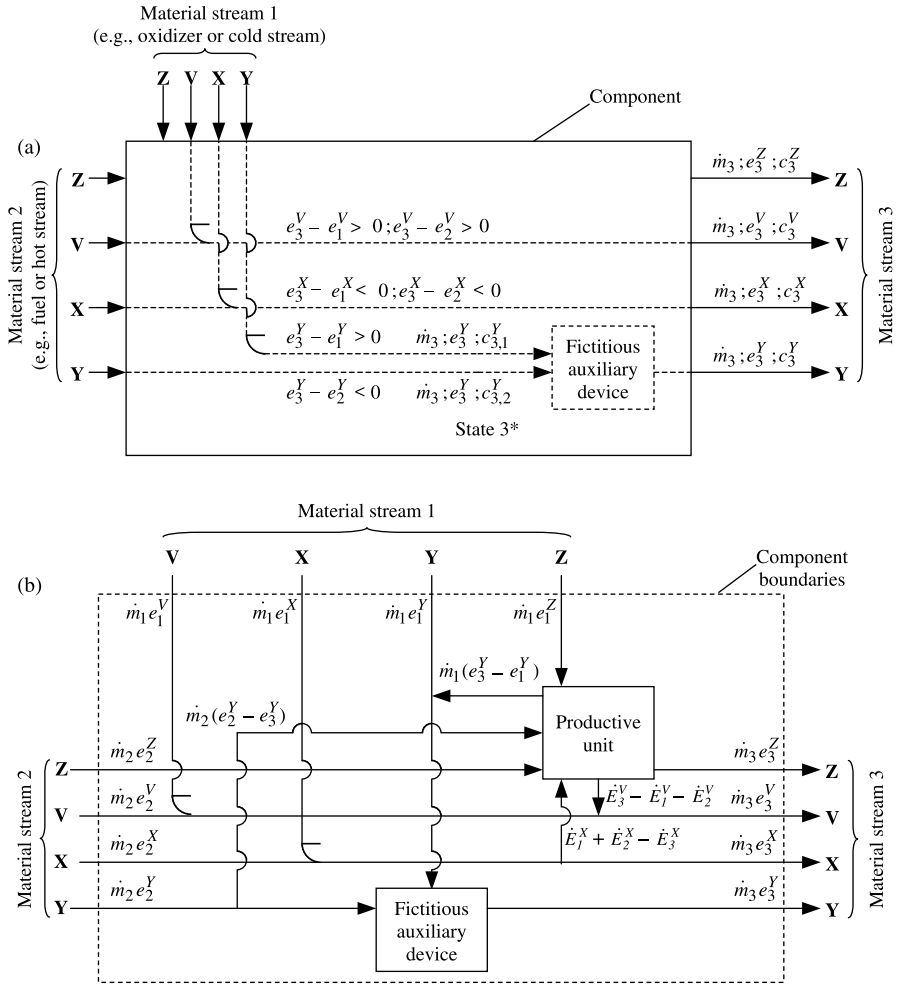

Fig. 2. (a) Schematic of a component in which the chemical exergy of material streams changes between inlet and outlet. (b) Represents the productive structure for the component shown in (a).

Then, the cost balance can be written as

$$c_P \dot{E}_P = \dot{C}_1 + \dot{C}_2 + (\dot{C}_{5i} - \dot{C}_{5e}) + (\dot{C}_{6i} - \dot{C}_{6e}) + \dot{C}_9 + \dot{Z} \tag{12}$$

where $\dot{E}_P$ is given in Eq. (7). Eq. (12) demonstrates that the average cost (c_P) at which each exergy unit is supplied to all streams associated with the product can be calculated by just combining the cost balance (Eq. (12)) with the equations obtained from the F principle (Eqs. (8) and (9)) alone. After calculating the value of c_P, Eqs. (10a) can be used to obtain the cost associated with each exiting stream that is used in the definition of the product.

Let us now turn to the case represented in Fig. 2a, when

(a) the composition of a stream changes because of mixing, separation or chemical reaction, and
(b) differences in the exergy streams need to be considered in accord with the purpose of the component (see step 2 of Section 2).

If one of these two conditions is not fulfilled, the case being considered is covered by the example shown in Fig. 1a.

In the SPECO method we follow the exergy changes along a material stream. Fig. 2a is used to facilitate the discussion when two or more material streams are mixed before they exit or when a material stream is separated from another stream. In this figure two material streams 1 and 2 enter the component and exit as stream 3 at state 3 ($\dot{m}_3 = \dot{m}_1 + \dot{m}_2$) after mixing with or without chemical reaction. The total exergy associated with each material stream is separated into the exergy forms V, X, Y and Z (for example, thermal, mechanical, nonreactive and reactive exergy [14]).

In dealing with the system component shown in Fig. 2a we could proceed in the general way suggested by the SPECO method by considering separately eight exergy streams exiting the component. These streams would be obtained by assuming that the mass flow rates $\dot{m}_1$ and $\dot{m}_2$ at the outlet are separated for each of the four exergy forms V, X, Y and Z. For a given exergy form at the outlet, the specific exergy is the same but the cost per exergy unit is different for each mass flow rate, which results in eight unknowns at state 3. However, in some situations this treatment of the outlet streams leads to considerations at the product or fuel side that contradict the purpose of the component (see e.g. [29]). Moreover, it increases significantly and very often unnecessarily the number of cost variables to be calculated and thus the computational efforts.

Ref. [29] demonstrates how these contradictions can be eliminated and under what conditions we can use only one instead of two cost variables per each exergy form at the outlet. In the following we correct, simplify and summarize the cost equations necessary to calculate all costs at the component outlet, a detailed discussion of which is provided in [29]. To cover all possibilities we study the following four general cases. The first three cases refer to exergy differences between inlet and outlet, whereas in the fourth one the exergy at the inlet belongs to the fuel and the exergy at the outlet to the product.

3.1. Case 1: the specific exergy increases for both material streams

The specific exergy of the exergy form V in Fig. 2a increases for both material streams 1 and 2 between inlet and outlet ($e_3^V > e_1^V$ and $e_3^V > e_2^V$). The sum of both increases of the exergy form V, $\dot{m}_1(e_3^V - e_1^V) + \dot{m}_2(e_3^V - e_2^V) = \dot{E}_3^V - \dot{E}_1^V - \dot{E}_2^V$, evidently must be part of the product and the following equation is derived with the aid of the P principle:

$$c_P = \frac{\dot{C}_3^V - \dot{C}_1^V - \dot{C}_2^V}{\dot{E}_3^V - \dot{E}_1^V - \dot{E}_2^V} \tag{13}$$

Eq. (13) shows that, when the specific exergy of an exergy form (V) increases between inlet and outlet for both material streams that are mixed within a component, it is not necessary to work with the separate increases for each material stream but it is sufficient to only consider the overall increase in the exergy form V for the sum of streams 1 and 2. Then, no auxiliary costing equation is needed. Eq. (13) does not represent an auxiliary costing equation since c_P is an unknown variable in addition to the ones associated with the exiting streams. Examples of this case include the thermal (or physical) exergy of fuel and oxidator in a combustion chamber or a gasification reactor. A conclusion similar to that expressed by Eq. (13) for a mixing process, when the specific exergy of the exiting stream exceeds that of both entering streams, applies to a separation process too.

3.2. Case 2: the specific exergy decreases for both material streams

For the exergy form X in Fig. 2a, the specific exergy for both material streams 1 and 2 decreases between inlet and outlet. The total exergy decrease $\dot{m}_1(e_1^X - e_3^X) + \dot{m}_2(e_2^X - e_3^X) = \dot{E}_3^V - \dot{E}_1^V - \dot{E}_2^V$, evidently must be part of the fuel of the component being considered.

With the aid of the F principle we obtain

$$c_3^X = \frac{\dot{m}_1 c_1^X + \dot{m}_2 c_2^X}{\dot{m}_1 + \dot{m}_2} \tag{14}$$

The auxiliary Eq. (14) shows that when the specific exergy of an exergy form (X) decreases between inlet and outlet for both material streams that are mixed within a component, it is not necessary to use the separate decreases for each material stream but it is sufficient to only consider the overall decrease in the exergy form X for the composite of streams 1 and 2. Then, we need to apply only the auxiliary costing Eq. (14). Examples of this case include the mechanical exergy of fuel and oxidator in a combustion chamber or a gasification reactor or a hot-water/cold-water mixing valve. This conclusion for a mixing process, when the specific exergy for both material streams between inlet and outlet decreases, applies also to a separation process.

3.3. Case 3: the specific exergy increases for one stream and decreases for the other material stream

For the exergy form Y we observe a different behavior of the specific exergy for material stream 1 (specific exergy increase) and 2 (specific exergy decrease). Since a specific exergy decrease belongs always to the fuel, we need for the exergy form Y to distinguish two subcases depending on whether the specific exergy increase is in accord with the purpose of the component.

In the first subcase, the specific exergy increase of stream 1 is desired and in accord with the purpose of the component, whereas in the second subcase this exergy increase is incidental to the purpose of the component. An example for the second subcase is the increase in specific chemical exergy for the air mass flow rate between inlet and outlet of a combustion chamber when the combustion is complete. This increase, which is accompanied by a decrease of the specific chemical exergy for the fuel mass flow rate between inlet and outlet, is incidental to the purpose of the combustion chamber, which is to increase the thermal exergy of both the air and fuel streams.

In the second subcase, compliance with the purpose of the component is achieved only when we consider the composite change in the exergy form Y between inlet and outlet. Thus, if the difference $\dot{E}_3^Y - \dot{E}_1^Y - \dot{E}_2^Y$ is negative, this difference belongs to the fuel and, according to the F principle, we formulate the auxiliary costing equation as

$$c_3^Y = \frac{\dot{m}_1 c_3^Y + \dot{m}_2 c_3^Y}{\dot{m}_1 + \dot{m}_2} \tag{15}$$

This equation is applied, for example, to the reactive exergy in a combustion chamber. Should the difference $\dot{E}_3^Y - \dot{E}_1^Y - \dot{E}_2^Y$ be positive, this difference must be considered with a negative sign at the fuel side since this exergy increase is not in accord with the component purpose. In other words the fuel of

the component is reduced by the difference $\Delta\dot{E}_F$

$$\Delta\dot{E}_F = \dot{E}_3^Y - \dot{E}_1^Y - \dot{E}_2^Y \tag{16}$$

It should be noted, however, that $\Delta\dot{E}_F$ being positive is a practically unknown but theoretically possible event in which again the auxiliary costing Eq. (15) should be used to calculate the variable c_3^Y.

We conclude that in the second subcase it is not necessary to work with separate streams at the outlet and it is sufficient to consider at the fuel side the overall change between inlet and outlet in the exergy form Y for the composite of streams 1 and 2.

Now, we move to the first subcase associated with the exergy form Y where the exergy increase for stream 1 is consistent with the purpose of the component. Here, we need to develop a model that is based on separate exergy streams (see streams at the inlet to the fictitious auxiliary device in Fig. 2a), separate exergy changes $(e_3^Y - e_1^Y)$ and $(e_2^Y - e_3^Y)$, and separate costs per exergy unit $(c_{3,1}^Y)$ and $(c_{3,2}^Y)$ for each material stream at the outlet. The imaginary state 3* is defined as the state in which the streams associated with the exergy form Y are hypothetically not merged, each one being associated with material streams 1 and 2, respectively. Both streams have at state 3* the same specific exergy (e_3^Y) but different costs per exergy unit ($c_{3,1}^Y$ and $c_{3,2}^Y$, respectively). The exergy difference $\dot{m}_2(e_2^Y - e_3^Y)$ is then associated with the fuel and the corresponding auxiliary costing equation derived from the F principle is

$$c_{3,2}^Y = c_2^Y \tag{17}$$

On the other hand, the exergy difference $\dot{m}_1(e_3^Y - e_1^Y)$ is part of the product and the following non-auxiliary equation is derived from the P principle:

$$\frac{\dot{m}_1(e_3^Y c_{3,1}^Y - e_1^Y c_1^Y)}{\dot{m}_1(e_3^Y - e_1^Y)} = c_P \tag{18}$$

The cost per exergy unit at the outlet (c_3^Y) is obtained through a cost balance for the fictitious auxiliary device:

$$c_3^Y = \frac{\dot{m}_1 e_3^Y c_{3,1}^Y + \dot{m}_2 e_3^Y c_{3,2}^Y}{(\dot{m}_1 + \dot{m}_2)e_3^Y} = \frac{\dot{m}_1 c_{3,1}^Y + \dot{m}_2 c_{3,2}^Y}{\dot{m}_1 + \dot{m}_2} \tag{19}$$

Thus, in the first subcase, Eqs. (17–19) are formulated for the exergy form Y to calculate the unknowns $c_{3,1}^Y$, $c_{3,2}^Y$ and c_3^Y. The model developed for this subcase uses an imaginary state (state 3* in Fig. 2a) explicitly. This state is used to derive the appropriate equations. In practical applications, however, an explicit consideration of this state is not needed if, as discussed above, the c_P value is calculated from the cost balance with the aid of the F principle (see also [29]). Then, the value c_3^Y is obtained by combining Eqs. (17–19):

$$c_3^Y = \frac{c_P \dot{m}_1(e_3^Y - e_1^Y) - \dot{C}_1^Y}{e_3^Y(\dot{m}_1 + \dot{m}_2)} + \frac{\dot{m}_2 c_2^Y}{\dot{m}_1 + \dot{m}_2} \tag{20}$$

3.4. Case 4: no exergy differences are used

For the exergy form Z in Fig. 2a the purpose of the component dictates that the exergy stream at the inlet is considered on the fuel side and the exergy stream at the outlet on the product side. In this case,

the P principle leads to

$$c_3^Z = c_P \tag{21}$$

No auxiliary equations are required in this case, which has already been covered in the discussion of Fig. 1.

To summarize the discussion in this section dealing with components in which the chemical exergy of material streams changes and separate exergy forms are considered, we conclude that we need to consider at the outlet two separate exergy streams for the same exergy form only when for this exergy form (e.g. Y) we have a decrease in the specific exergy of one mass flow rate and an increase in the specific exergy of the other mass flow rate between inlet and outlet, and this increase is consistent with the purpose of the component. In all other cases, it is sufficient to use one exergy stream for each exergy form at the outlet and to consider the overall exergy change between inlet and outlet.

In the remaining part of this section, the equations required to define fuel and product and to calculate all costs for the component shown in Fig. 2a are summarized. Assuming that the first subcase of case 3 applies to the exergy form Y, and the fictitious state 3* ($c_{3,1}^Y$ and $c_{3,2}^Y$) is used, we write for the fuel ($\dot{E}_F$) and product ($\dot{E}_P$) of the component shown in Fig. 2a

$$\dot{E}_F = \dot{E}_1^Z + \dot{E}_2^Z + (\dot{E}_1^X + \dot{E}_2^X - \dot{E}_3^X) + (\dot{E}_2^Y - \dot{E}_{3,2}^Y) \tag{22}$$

$$\dot{E}_P = \dot{E}_3^Z + (\dot{E}_3^V - \dot{E}_1^V - \dot{E}_2^V) + (\dot{E}_{3,1}^Y - \dot{E}_1^Y) \tag{23}$$

where $\dot{E}_{3,2}^Y = \dot{m}_2 e_3^Y$ and $\dot{E}_{3,1}^Y = \dot{m}_1 e_3^Y$. The cost balances are

$$\dot{C}_1^Z + \dot{C}_1^V + \dot{C}_1^X + \dot{C}_1^Y + \dot{C}_2^Z + \dot{C}_2^V + \dot{C}_2^X + \dot{C}_2^Y + \dot{Z} = \dot{C}_3^Z + \dot{C}_3^V + \dot{C}_3^X + \dot{C}_{3,1}^Y + \dot{C}_{3,2}^Y \tag{24}$$

$$\dot{C}_{3,1}^Y + \dot{C}_{3,2}^Y = \dot{C}_3^Y \tag{25}$$

The auxiliary costing equations derived from the F principle are (see also Eqs. (14) and (17))

$$\frac{\dot{C}_3^X}{e_3^X} = \frac{\dot{C}_1^X}{e_1^X} + \frac{\dot{C}_2^X}{e_2^X} \tag{26}$$

$$\frac{\dot{C}_{3,2}^Y}{\dot{E}_{3,2}^Y} = \frac{\dot{C}_2^Y}{\dot{E}_2^Y} \tag{27}$$

The auxiliary costing equations derived from the P principle (compare Eqs. (13), (18) and (21)) are

$$\frac{\dot{C}_3^V - \dot{C}_1^V - \dot{C}_2^V}{\dot{E}_3^V - \dot{E}_1^V - \dot{E}_2^V} = \frac{\dot{C}_{3,1}^Y - \dot{C}_1^Y}{\dot{E}_{3,1}^Y - \dot{E}_1^Y} = \frac{\dot{C}_3^Z}{\dot{E}_3^Z} \tag{28}$$

The unknown cost rates $\dot{C}_3^Z$, $\dot{C}_3^V$, $\dot{C}_3^X$, $\dot{C}_3^Y$, $\dot{C}_{3,1}^Y$ and $\dot{C}_{3,2}^Y$ are calculated by solving the system of linear Eqs. (24–28). The auxiliary variable c_P used in this paper to explain concepts does not necessarily need to be considered in the formulation of the system of linear equations to be solved.

It is important to note that for each exergy difference between inlet and outlet used in the definition of $\dot{E}_F$ (Eq. (22)) one auxiliary costing equation is derived from the F principle. In addition, the terms used in

the definition of $\dot{E}_P$ (Eq. (23)) appear with the same sign (and order) in the auxiliary costing equations derived from the P principle (Eqs. (28)). These practical observations provide an additional proof of the direct link between $\dot{E}_F$ and $\dot{E}_P$ on one hand and auxiliary costing equations on the other hand and assist in the formulation of the auxiliary equations once the fuel and product for a component have been defined.

4. Productive structure

In the productive structure the fuel and product of each component are illustrated graphically. This graphical representation of the fuel and product is an intrinsic part of the methodology in the functional approaches, in which it first appeared and was named 'functional diagram' [17], whereas it is not necessarily needed for applying the SPECO method. However, a productive structure may assist in visualizing and better understanding the definitions of fuel and product and consequently the auxiliary costing equations obtained from the F and P principles.

Fig. 1b shows how the component shown in Fig. 1a interacts with the entering exergy streams to provide the exiting exergy streams. The component uses exergy from the exergy streams associated with the definition of fuel (1, 2, 5, 6 and 9) to supply exergy to streams 7 and 8 and to generate the exergy of streams 3, 4 and 10 which is to supply the exergy associated with the definition of the product.

A so-called productive unit (indicated by a continuous line) is defined within the component boundaries, which are shown with a dotted line. Fictitious branches entering the productive unit are used to indicate removals of exergy (exergy differences in the fuel definition) from the fuel streams (e.g. streams 5 and 6 in Fig. 1a), whereas fictitious junctions leaving the productive unit are used to indicate additions of exergy (exergy differences in the product definition) to the product streams (e.g. streams 7 and 8 in Fig. 1a). The productive unit explicitly shows the fuel and product interactions between the component itself and the exergy streams by acting as a collector and distributor of fuel and product terms. Since the fictitious branches and junctions remain within the boundaries of the component, the interconnections among components remain in the productive structure associated with the SPECO method the same as in the flow diagram that represents the physical structure of the system (see also [26,27,32]). The SPECO productive structure graphically illustrates the exergy balance written in two different ways:

- A balance in terms of fuel and product can be written by considering the exergy streams crossing the boundaries of the productive unit (which correspond to the terms included in the fuel and product definitions).
- A balance in terms of input-output exergy streams can be written by observing the exergy streams crossing the boundaries of the system component (dotted line).

The SPECO productive structure for a component, in which the chemical composition changes, is shown in Fig. 2b. This productive structure is drawn by applying the same criteria already used for the component in Fig. 1a. The component uses (a) the exergy form X from both streams 1 and 2 (a branch appears in each of these streams upstream of the productive unit to indicate the exergy removal), (b) the exergy form Y from stream 2 (branch), and (c) the exergy form Z from both streams 1 and 2. In so doing it increases the exergy form V of both streams 1 and 2 (a junction appears in each of these streams

downstream from the productive unit to indicate the exergy addition), increases the exergy form Y of stream 1 (junction) and provides the exergy form Z of stream 3.

5. Application to components

In this section, we demonstrate the application of the SPECO method to obtain the fuel, product and the auxiliary costing equations for some components of thermal systems. Components in which the chemical composition of material streams does not change between inlet and outlet as the result of mixing, separation or chemical reaction include compressors, pumps, fans, turbines and heat exchangers. In all these components, the chemical exergy of each material stream entering a component remains unchanged. Since only changes of physical exergy of the respective material streams occur in these components, only exergy differences between inlet and outlet for every material stream are used in the definition of fuel and product. The equations related to fuel and product and the auxiliary costing equations for such components are given in Refs. [14,27,28] and [29].

Here, it should be mentioned that making a distinction between physical and chemical exergy in components within which the chemical exergy of each stream remains constant does not in any way affect the results, while it unnecessarily increases the required computational efforts. Small accuracy improvements are obtained, in general, when the physical exergy is split into thermal and mechanical exergy. However, this splitting might not be always meaningful because of the arbitrariness that might be involved in the separate calculation of mechanical and thermal exergies, particularly when working fluids that can change phase are used in the process being considered [14]. This arbitrariness and the increased computational efforts might outweigh in many cases the small accuracy improvements.

In the following, we consider some components within which the chemical exergy of material streams changes as a result of mixing or chemical reaction. Various equations have been suggested in the past for the efficiencies and auxiliary costing equations associated with these components. We believe that the equations given below are the most appropriate equations to be used in conjunction with the respective components. First, we present the equations to be applied when total exergies are used, and subsequently we deal with physical and chemical exergies associated with each material stream separately.

5.1. Mixing devices

From the thermodynamic viewpoint the only meaningful purpose for mixing two streams is to increase the physical (i.e. the thermal) exergy of a stream at the expense of (a) the physical exergy of the other stream, and (b) the sum of chemical exergies of both streams. From the economic viewpoint, some additional reasons for mixing two streams might be valid (e.g. to reduce piping costs). Thus, if the purpose from the thermodynamic viewpoint holds, and only total exergies are used, we obtain a case similar to the one discussed for exergy form Y in Section 3 first subcase (Eqs. (17)–(19)), with the only differences being that now the superscript Y refers to the total exergy (denoted without a superscript) and that the exergy forms V, X and Z do not exist. Therefore, for the mixing of ‘cold’ stream 1 with ‘hot’

stream 2, to obtain stream 3 (see Fig. 2a), we write for the exergetic efficiency

$$\varepsilon_{mix} = \frac{\dot{m}_1(e_3 - e_1)}{\dot{m}_2(e_2 - e_3)} = \frac{\dot{E}_{3,1} - \dot{E}_1}{\dot{E}_2 - \dot{E}_{3,2}} \tag{29}$$

and for the auxiliary costing equation based on the F principle (see also Eqs. (17))

$$c_{3,2} = c_2 \tag{30}$$

Thus, the cost stream associated with the fuel in the mixing process becomes

$$\dot{C}_{F,mix} = \dot{m}_2 c_2 (e_2 - e_3) \tag{31}$$

The cost per exergy unit at the outlet c_3 is calculated from the cost balance. The imaginary cost per exergy unit $c_{3,1}$ at state 3* at the outlet can be calculated (see also Eq. (19)) from

$$c_3 = \frac{\dot{m}_1 c_{3,1} + \dot{m}_2 c_{3,2}}{\dot{m}_1 + \dot{m}_2} \tag{32}$$

The variable $c_{3,1}$ is used in defining the cost stream associated with the product of the mixing process $\dot{C}_{P,mix}$

$$\dot{C}_{P,mix} = \dot{m}_1 (e_3 c_{3,1} - e_1 c_1) \tag{33}$$

It should be noted, however, that the imaginary state 3* and the specific costs $c_{3,1}$ and $c_{3,2}$ do not necessarily need to be considered explicitly in the analysis of a mixing device because after the term $\dot{C}_{F,mix}$ has been calculated from Eq. (31), the term $\dot{C}_{P,mix}$ can be obtained directly from the cost balance:

$$\dot{C}_{P,mix} = \dot{C}_{F,mix} + \dot{Z}_{mix} \tag{34}$$

When the total exergy is split into physical and chemical exergies, indicated with the respective superscripts PH and CH, separate exergy streams for each mass flow rate at the outlet (state 3*) need to be considered only for the physical exergy (see Eqs. (17)–(19)). By taking into account that the overall chemical exergy decreases between inlet and outlet, we obtain for the

- efficiency

$$\varepsilon_{mix} = \frac{\dot{E}_{3,1}^{PH} - \dot{E}_1^{PH}}{(\dot{E}_2^{PH} - \dot{E}_{3,2}^{PH}) + (\dot{E}_1^{CH} + \dot{E}_2^{CH} - \dot{E}_3^{CH})} \tag{35}$$

- cost balances

$$\dot{C}_1^{PH} + \dot{C}_1^{CH} + \dot{C}_2^{PH} + \dot{C}_2^{CH} + \dot{Z}_{mix} = \dot{C}_{3,1}^{PH} + \dot{C}_{3,2}^{PH} + \dot{C}_3^{CH} \tag{36}$$

$$\dot{C}_{3,1}^{PH} + \dot{C}_{3,2}^{PH} = \dot{C}_3^{PH}, \tag{37}$$

and

- auxiliary costing equations

$$\frac{\dot{C}_{3,2}^{PH}}{\dot{E}_{3,2}^{PH}} = \frac{\dot{C}_{2}^{PH}}{\dot{E}_{2}^{PH}} \tag{38}$$

$$\frac{\dot{C}_{3}^{CH}}{e_{3}^{CH}} = \frac{\dot{C}_{1}^{CH}}{e_{1}^{CH}} + \frac{\dot{C}_{2}^{CH}}{e_{2}^{CH}} \tag{39}$$

The unknowns $\dot{C}_3^{PH}$, $\dot{C}_3^{CH}$, $\dot{C}_{3,1}^{PH}$ and $\dot{C}_{3,2}^{PH}$ are calculated from Eqs. (36)–(39).

5.2. Combustion chamber (gaseous fuel)

In the following discussion, we refer to Fig. 2a where material stream 2 represents the fuel stream, material stream 1 the oxidizer, and stream 3 refers to the combustion products. The purpose of a combustion chamber is to increase the physical exergy of the entering streams at the expense of their chemical exergies. When only total exergies are used, no auxiliary costing equation is required and the exergetic efficiency is defined according to the purpose of the combustion chamber as

$$\varepsilon_{comb} = \frac{\dot{E}_3 - \dot{E}_1}{\dot{E}_2} \tag{40}$$

since the main exergy form in streams 1 and 3 is the physical exergy, whereas in stream 2 it is the chemical exergy.

When physical and chemical exergies are considered separately, only differences between inlet and outlet are used and the following relations apply:

Exergetic efficiency

$$\varepsilon_{comb} = \frac{\dot{E}_3^{PH} - (\dot{E}_1^{PH} + \dot{E}_2^{PH})}{(\dot{E}_1^{CH} + \dot{E}_2^{CH}) - \dot{E}_3^{CH}} \tag{41}$$

Cost balance

$$\dot{C}_1^{PH} + \dot{C}_1^{CH} + \dot{C}_2^{PH} + \dot{C}_2^{CH} + \dot{Z}_{comb} = \dot{C}_3^{PH} + \dot{C}_3^{CH} \tag{42}$$

Auxiliary equation for the chemical exergy (see Eq. (15))

$$\frac{\dot{C}_{3}^{CH}}{e_{3}^{CH}} = \frac{\dot{C}_{1}^{CH}}{e_{1}^{CH}} + \frac{\dot{C}_{2}^{CH}}{e_{2}^{CH}} \tag{43}$$

The unknown variables $\dot{C}_3^{PH}$ and $\dot{C}_3^{CH}$ are calculated from Eqs. (42) and (43).

5.3. Gasification reactor

We refer to Fig. 3 where material streams 1, 2, 3 and 4 represent the oxidant, fuel, gasification products, and ash, respectively. The purpose of a gasification reactor is mainly to convert the chemical

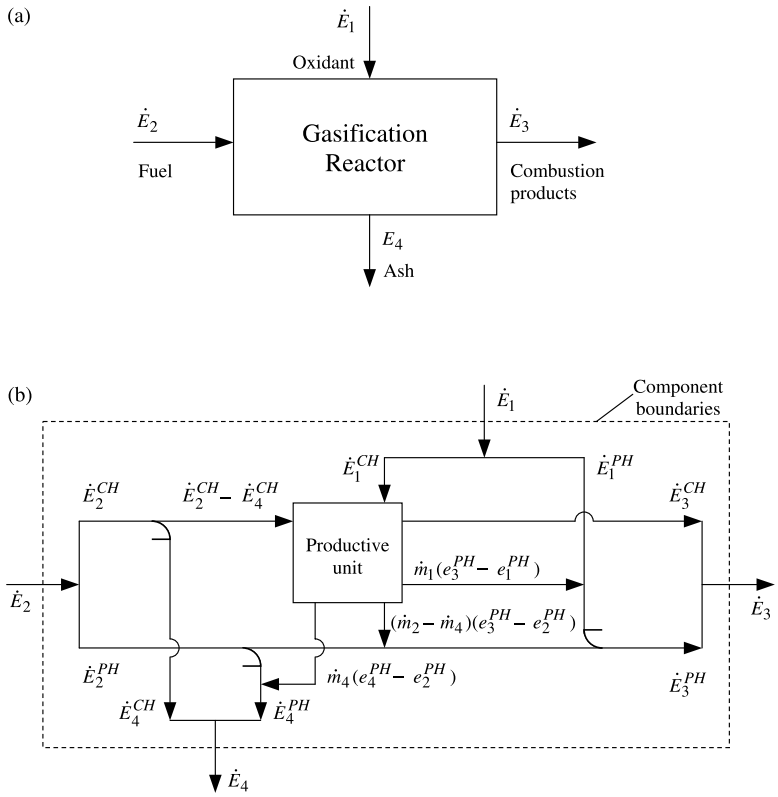

Fig. 3. Schematic of a gasification reactor (a) and the corresponding productive structure (b) when physical and chemical exergies are considered separately.

exergy (more precisely the reactive exergy) of a solid fuel into chemical exergy (reactive exergy) of a gaseous fuel. As a secondary purpose we define the increase of physical exergy of the material streams between inlet and outlet. Both purposes are achieved at the expense of the chemical exergies of streams 1 and 2. It should be noted that, when total exergies are considered, the treatment of $\dot{E}_1$ is determined by the treatment of the physical exergy of stream 1, whereas the treatment of $\dot{E}_2$, $\dot{E}_3$ and $\dot{E}_4$ depends on the treatment of chemical exergy of streams 2, 3 and 4. In each case the dominating exergy form determines the treatment of total exergy. When only total exergies are used, the exergetic efficiency is given by

$$\varepsilon_{gas} = \frac{\dot{E}_3 - \dot{E}_1}{\dot{E}_2 - \dot{E}_4} \tag{44}$$

and only one auxiliary costing equation, derived from the F principle, is required:

$$\frac{\dot{C}_4}{\dot{E}_4} = \frac{\dot{C}_2}{\dot{E}_2} \tag{45}$$

When physical and chemical exergies are considered separately, we write

$$\varepsilon_{gas} = \frac{\dot{E}_3^{CH} + (\dot{E}_3^{PH} + \dot{E}_4^{PH} - \dot{E}_1^{PH} - \dot{E}_2^{PH})}{\dot{E}_1^{CH} + \dot{E}_2^{CH} - \dot{E}_4^{CH}} \tag{46a}$$

$$\varepsilon_{gas} = \frac{\dot{E}_3^{CH} + [\dot{m}_1(e_3^{PH} - e_1^{PH}) + (\dot{m}_2 - \dot{m}_4)(e_3^{PH} - e_2^{PH})] + \dot{m}_4(e_4^{PH} - e_2^{PH})}{\dot{E}_1^{CH} + \dot{E}_2^{CH} - \dot{E}_4^{CH}} \tag{46b}$$

or

$$\varepsilon_{gas} = \frac{\dot{E}_3^{CH} + [(\dot{m}_1 + \dot{m}_2 - \dot{m}_4)e_3^{PH} - \dot{E}_1^{PH} - (\dot{m}_2 - \dot{m}_4)e_2^{PH})] + \dot{m}_4(e_4^{PH} - e_2^{PH})}{\dot{E}_1^{CH} + \dot{E}_2^{CH} - \dot{E}_4^{CH}} \tag{46c}$$

Here, we have considered that the entire mass stream 4 (and its chemical exergy) is derived from stream 2. The cost balance is

$$\dot{C}_1^{PH} + \dot{C}_1^{CH} + \dot{C}_2^{PH} + \dot{C}_2^{CH} + \dot{Z}_{gas} = \dot{C}_3^{PH} + \dot{C}_3^{CH} + \dot{C}_4^{PH} + \dot{C}_4^{CH} \tag{47}$$

One auxiliary costing equation is obtained from the F principle for the chemical exergy of stream 4, which appears on the fuel side:

$$\frac{\dot{C}_4^{CH}}{\dot{E}_4^{CH}} = \frac{\dot{C}_2^{CH}}{\dot{E}_2^{CH}} \tag{48}$$

For each exergy stream exiting the component and used in the definition of product (here $\dot{E}_3^{CH}$, $\dot{E}_3^{PH}$ and $\dot{E}_4^{PH}$), a term is used in the equations derived from the P principle

$$\frac{\dot{C}_3^{CH}}{\dot{E}_3^{CH}} = \frac{\frac{\dot{m}_1}{\dot{m}_3}\dot{C}_3^{PH} - \dot{C}_1^{PH} + \frac{\dot{m}_2}{\dot{m}_3}\dot{C}_3^{PH} - \frac{\dot{m}_4}{\dot{m}_3}\dot{C}_3^{PH} - \dot{C}_2^{PH} + \frac{\dot{m}_4}{\dot{m}_2}\dot{C}_2^{PH}}{\frac{\dot{m}_1}{\dot{m}_3}\dot{E}_3^{PH} - \dot{E}_1 + \frac{\dot{m}_2}{\dot{m}_3}\dot{E}_3^{PH} - \frac{\dot{m}_4}{\dot{m}_3}\dot{E}_3^{PH} - \dot{E}_2^{PH} + \frac{\dot{m}_4}{\dot{m}_2}\dot{E}_2^{PH}} = \frac{\dot{C}_4^{PH} - \frac{\dot{m}_4}{\dot{m}_2}\dot{C}_2^{PH}}{\dot{E}_4^{PH} - \frac{\dot{m}_4}{\dot{m}_2}\dot{E}_2^{PH}} \tag{49a}$$

$$\frac{\dot{C}_3^{CH}}{\dot{E}_3^{CH}} = \frac{\left(\frac{\dot{m}_1+\dot{m}_2-\dot{m}_4}{\dot{m}_3}\right)\dot{C}_3^{PH} - \dot{C}_1^{PH} - \left(\frac{\dot{m}_2-\dot{m}_4}{\dot{m}_2}\right)\dot{C}_2^{PH}}{\left(\frac{\dot{m}_1+\dot{m}_2-\dot{m}_4}{\dot{m}_3}\right)\dot{E}_3^{PH} - \dot{E}_1^{PH} - \left(\frac{\dot{m}_2-\dot{m}_4}{\dot{m}_2}\right)\dot{E}_2^{PH}} = \frac{\dot{C}_4^{PH} - \frac{\dot{m}_4}{\dot{m}_2}\dot{C}_2^{PH}}{\dot{E}_4^{PH} - \frac{\dot{m}_4}{\dot{m}_2}\dot{E}_2^{PH}} \tag{49b}$$

The above equations are given to demonstrate the similarities in the formulation of exergetic efficiency and auxiliary costing equations (compare terms in square brackets in Eqs. (46b) and (46c) with Eqs. (49a) and (49b), respectively). The unknowns $\dot{C}_3^{PH}$, $\dot{C}_3^{CH}$, $\dot{C}_4^{PH}$ and $\dot{C}_4^{CH}$ are calculated from Eqs. (47), (48) and either (49a) or (49b).

6. Dissipative components and exergy losses

6.1. Dissipative components

The discussion so far has focused on components for which a product is readily defined when the component is considered in isolation (as discussed above in step 2 of Section 2), because these

components fulfil a productive purpose. We may call these components productive components. There are, however, other components in which exergy is destroyed without gaining something thermodynamically useful directly from the same component. Examples of such components include gas cleaning units as well as throttling valves and coolers both operating at temperatures above the ambient temperature. We call these components dissipative components. A product cannot be defined when these components are considered in isolation because their operation appears to be meaningless from the thermodynamic viewpoint. In other words, the purpose of a component from the thermodynamic viewpoint cannot be to remove exergy from a stream without getting a thermodynamic use out of this removal in the same component.

The operation of dissipative components becomes meaningful only when they are considered in the context of the overall thermal system: Dissipative components serve productive components in a system (i.e. they help reduce the exergy destruction in at least one of the remaining system components), assist in reducing the investment costs of the overall system or enable the system to fulfil the required emission standards. For example, a cooler immediately preceeding a compressor assists in reducing the power required for operating the compressor.

When a dissipative component is involved, we can define a meaningful exergetic efficiency only if this component is considered together with all components it serves. In the above mentioned example, we can define a meaningful exergetic efficiency only if the cooler is considered together with the compressor it serves ([13,14]).

In costing applications, all costs associated with owning and operating a dissipative component must be charged directly to the component(s) served by it. Fig. 4 shows a dissipative component with entering and exiting streams associated with the main working fluid (e.g. gas in a gas cleaning unit) and with an auxiliary working fluid (e.g. cooling water). The exergy of the main working fluid at the outlet is lower than at the inlet due to exergy destruction within the dissipative component and to exergy transfer

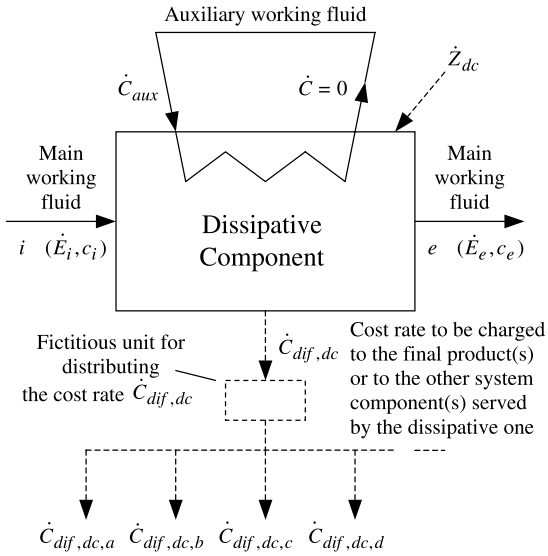

Fig. 4. Schematic used to illustrate the cost balance for a dissipative component.

through the auxiliary working fluid

$$\Delta\dot{E} = \dot{E}_i - \dot{E}_e \tag{50}$$

According to the F principle, the cost per exergy unit of the main working fluid remains constant between inlet and outlet

$$c_e = c_i \tag{51}$$

If the rate of the total charges associated with the use of the auxiliary working fluid is $\dot{C}_{aux}$ and the contribution of investment cost and operating and maintenance expenses is $\dot{Z}_{dc}$, the cost balance for the dissipative component becomes

$$\dot{C}_e + \dot{C}_{dif,dc} = \dot{C}_i + \dot{C}_{aux} + \dot{Z}_{dc} \tag{52}$$

Here, $\dot{C}_{dif,dc}$ is a fictitious cost rate associated with the use of the dissipative component being considered. This cost rate is calculated from the cost balance. Combining Eqs. (50)–(52) we obtain

$$\dot{C}_{dif,dc} = c_i\Delta\dot{E} + \dot{C}_{aux} + \dot{Z}_{dc} \tag{53}$$

The cost rate $\dot{C}_{dif,dc}$ needs to be (a) apportioned to the system component(s) served by the dissipative component, (b) charged to the productive component(s) being responsible for the use of the dissipative component, or (c) apportioned to the final product(s) of the overall system. In a steam power plant, for example, the cost rate $\dot{C}_{dif,cond}$ derived from the condenser could be apportioned among all remaining plant components by using the entropy increase in each component as a weighting factor. This approach leads to results similar to those obtained when the negentropy concept is applied [17,18]. In a process in which pollutants have to be removed from a combustion gas or a gasification gas, for example, the term $\dot{C}_{dif,gas\ clean}$ derived from the gas cleaning unit should be charged directly to the combustion chamber or gasification reactor (case b). The cost rate $\dot{C}_{dif,th\ v}$ associated with a throttling valve used to control the mass flow rate in a pump should be charged to the pump (case a). The cost rate $\dot{C}_{dif,th\ v}$ associated with the mass flow rate control in a steam turbine should be charged directly to the electricity generated by the turbine generator (case c).

When a part $\dot{C}_{dif,dc,a}$ of the cost rate $\dot{C}_{dif,dc}$ derived from a dissipative component needs to be charged to another component, the term $\dot{Z}$ of the latter can be expanded to include the term $\dot{C}_{dif,dc,a}$

$$\dot{Z} = \dot{Z}^{CI} + \dot{Z}^{OM} + \dot{C}_{dif,dc,a} \tag{54}$$

In this way, the equations given in the previous sections do not need to be modified.

6.2. Exergy losses

Exergy loss is the transport of exergy from the overall system to its surroundings. Exergy loss is associated with the rejection of heat and streams of matter to the surroundings. These streams are not further used in any system.

In the approach presented here the boundaries needed for formulating balances are chosen to be at ambient temperature. Thus, the exergy loss associated with heat transfer to the surroundings is always zero. Exergy losses associated with material streams should be included among the losses when the efficiency of the overall system is evaluated, but not when evaluating the efficiencies of the system

components that the streams last exit in the overall system. In a component efficiency, such streams should be included with a minus sign in the fuel term. If such a stream would be considered as exergy loss for a component K that the stream last exits, we would obtain two different efficiencies for the same operation of component K depending on whether this material stream is rejected to the surroundings immediately after leaving component K or (through a design change) after supplying some exergy to another component L. The efficiency of component K, however, should always be independent of what is happening downstream with the streams exiting this component.

The best approach to costing of an exergy loss $\dot{E}_{L,k}$ exiting last the kth component is either to apply the F principle (if this is possible), or to set [14]

$$\dot{C}_{L,k} = c_{F,k}\dot{E}_{L,k} \tag{55}$$

Here $c_{F,k}$ is the cost per exergy unit of the fuel of the kth component. This equation assumes that the exergy loss is covered through the supply of additional fuel to the kth component and that the average cost ($c_{F,k}$) of supplying a unit of fuel exergy remains approximately constant with varying exergy loss in the kth component. The cost rate $\dot{C}_{L,k}$ associated with the exergy loss $\dot{E}_{L,k}$ needs to be charged to the final product(s) generated by the overall system. If more than one product is generated by this system, some subjective decisions are required with respect to what part of $\dot{C}_{L,k}$ should be charged to each product.

An alternative approach to costing of $\dot{E}_{L,k}$ is to set

$$\dot{C}_{L,k} = 0 \tag{56}$$

This equation avoids the subjectivity of apportioning $\dot{C}_{L,k}$ to the final products but does not allow an estimation of the cost consequences of rejecting $\dot{E}_{L,k}$ to the surroundings. In addition, it violates the F principle.

If a treatment is required for a material stream before it is rejected to the surroundings, then the cost rate of this stream at the exit of the kth component which is also inlet to the treatment unit is

$$\dot{C}_{L,k} = -\dot{Z}_{tr} + \dot{C}^{*}_{L,k} \tag{57}$$

Here $\dot{Z}_{tr}$ is the cost rate associated with the investment and operating and maintenance expenses of the treatment unit and $\dot{C}^{*}_{L,k}$ is the cost rate of the stream exiting the treatment unit and being rejected to the surroundings. The term $\dot{C}^{*}_{L,k}$ can be calculated according to either Eq. (55) or Eq. (56). The cost rate $\dot{C}_{L,k}$ in Eq. (57) is usually negative. This means that the cost flows in the direction opposite to the material stream. In this way, the costs of treating the material stream are charged upstream to the component in which this stream was generated.

7. Matrix formulation for calculating costs

In this section, we consider a general example of a thermal system for which the analyst decided to separate the thermal, mechanical and chemical components of the exergy associated with each material stream. Then, for a system consisting of n components and t exergy streams, if s represents the total number of material streams and q represents the total number of energy streams,

$$t = 3s + q \tag{58}$$

For calculating the costs associated with each exergy stream in the system we need to solve a $t \times t$ system of linear equations. As suggested in Ref. [8] for total exergies, these equations can be written in the form

$$\boldsymbol{A} \times \boldsymbol{X} = \boldsymbol{B} \tag{59}$$

as shown in Fig. 5.

The vector $\boldsymbol{X}$ of the unknown variables consists of (a) the cost flow rates ($\dot{C}$) associated with thermal exergy (s variables with the superscript T: $\dot{C}^T$), mechanical exergy (s variables with the superscript M:$\dot{C}^M$), and chemical exergy (s variables with the superscript CH: $\dot{C}^{CH}$) of all material streams, and (b) the cost flow rates associated with all energy streams (q variables) in the overall system. The matrix $\boldsymbol{A}$ consists of the sub-matrices $\boldsymbol{A}_m$, $\boldsymbol{A}_{en}$, $\boldsymbol{A}^T_{0,m}$, $\boldsymbol{A}^M_{0,m}$, $\boldsymbol{A}^{CH}_{0,m}$, $\boldsymbol{A}_{0,en}$, $\boldsymbol{A}^T_F$, $\boldsymbol{A}^M_F$, $\boldsymbol{A}^{CH}_F$, $\boldsymbol{A}_{F,en}$ and $\boldsymbol{A}_P$. These sub-matrices are discussed in the following.

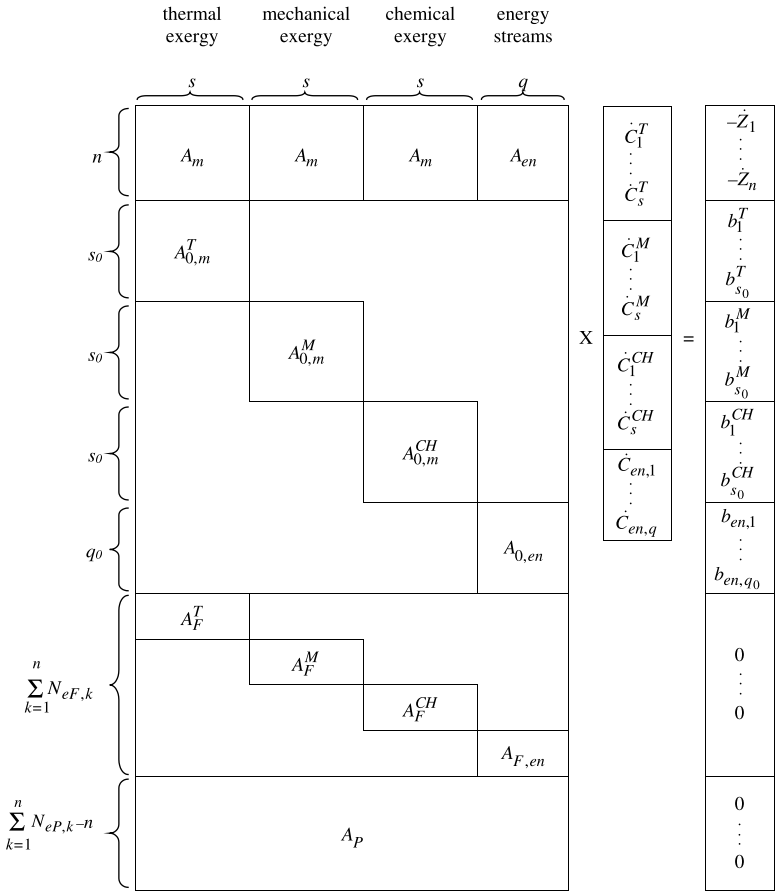

Fig. 5. Representation of the cost equations in a matrix form when separate forms of exergy are used.

7.1. Cost balances (matrices $\boldsymbol{A}_m$ and $\boldsymbol{A}_{en}$)

$\boldsymbol{A}_m$ $(n\times s)$ is the incidence matrix representing the interconnections between components and exergy streams associated only with material streams. $\boldsymbol{A}_{en}$ $(n\times q)$ is the incidence matrix representing the interconnections between components and exergy streams associated only with energy streams. The elements of the matrices $\boldsymbol{A}_m$ and $\boldsymbol{A}_{en}$ are $+1$ (for an exergy stream entering the component being considered), -1 (for a stream exiting the component being considered), or 0 (when there is no interconnection between the stream and the component).

The equations associated with the matrices $\boldsymbol{A}_m$ and $\boldsymbol{A}_{en}$ are the cost balances formulated for each component of the system. The general form of the cost balance for the kth component is

$$\sum_i \dot{C}^T_{i,k} + \sum_i \dot{C}^M_{i,k} + \sum_i \dot{C}^{CH}_{i,k} + \sum_i \dot{C}_{en,i,k} - \left(\sum_e \dot{C}^T_{e,k} \sum_e \dot{C}^M_{e,k} + \sum_e \dot{C}^{CH}_{e,k} + \sum_e \dot{C}_{en,e,k} \right) = \dot{Z}_k \tag{60}$$

The first three terms in the above equation are the cost flow rates associated with the thermal, mechanical and chemical exergy of all material streams entering the kth component. The fourth term denotes the cost flow rates associated with all energy streams entering the component. The terms in parenthesis refer to the exergy streams exiting the kth component. The term on the right side of Eq. (60) is the cost flow rate associated with capital investment and operating and maintenance expenditures.

A total of n equations of the form of Eq. (60) is obtained since a cost balance is written for each of the n components.

7.2. Costs of the streams entering the overall system (matrices $\boldsymbol{A}^T_{0,m}$, $\boldsymbol{A}^M_{0,m}$, $\boldsymbol{A}^{CH}_{0,m}$ and $\boldsymbol{A}_{0,en}$)

The matrices $\boldsymbol{A}^T_{0,m}$, $\boldsymbol{A}^M_{0,m}$, $\boldsymbol{A}^{CH}_{0,m}$ and $\boldsymbol{A}_{0,\mathrm{en}}$ are associated with the exergy streams X (X=T, M, CH) supplied with a material stream to the overall system from outside. Their dimensions are $(s\times s_0)$ where s_0 is the total number of material streams entering the overall system. Each equation associated with these matrices is of the form

$$\dot{C}^X_l = b^X_l \quad l = 1, \ldots, s_0 \tag{61}$$

where b^X_l is the known value of the cost flow rate $\dot{C}^X_l$ of the exergy stream X associated with the material stream l supplied to the overall system from outside.

The matrix $\boldsymbol{A}_{0.\mathrm{en}}$ refers to the exergy streams associated with energy streams supplied to the overall system from outside. The equations associated with this matrix are of the form

$$\dot{C}_{en,r} = b_{en,r} \quad r = 1, \ldots, q_0 \tag{62}$$

where $b_{\mathrm{en},r}$ is the known value of the cost flow rate of the exergy associated with the energy stream r supplied to the overall system from outside, and q_0 is the total number of energy streams entering the overall system.

The values of b^X_l and $b_{\mathrm{en},r}$ in the previous two equations are known from the cost rates at which all external resources are purchased. The coefficients of the matrices $\boldsymbol{A}^T_{0,m}$, $\boldsymbol{A}^M_{0,m}$, $\boldsymbol{A}^{CH}_{0,m}$, $\boldsymbol{A}_{0,\mathrm{en}}$ are either $+1$ (for all exergy streams supplied to the overall system from outside) or 0 (for all remaining exergy

streams). These matrices are placed within the large matrix $\boldsymbol{A}$ diagonally one below the other since each row of each matrix supplies one equation (Eq. (61) or (62)). The total number of Eqs. (61) and (62) is equal to $3s_0+q_0$.

7.3. Auxiliary equations associated with the F principle (matrices $\boldsymbol{A}_F^T$, $\boldsymbol{A}_F^M$, A_F^{CH}, $\boldsymbol{A}_{F,en}$)

Every time exergy is removed from an exergy stream in a system component, an auxiliary equation based on the F principle is formulated. The matrices $\boldsymbol{A}_F^T, \boldsymbol{A}_F^M, \boldsymbol{A}_F^{CH}$ refer to the removals of exergy from thermal, mechanical and chemical exergy streams, respectively. The equations associated with these matrices are either of the form (see, for example, Eqs. (8), (9), (17) and (27))

$$\frac{\dot{C}_i^X}{\dot{E}_i^X}-\frac{\dot{C}_e^X}{\dot{E}_e^X}=0 \tag{63}$$

or of the form of Eqs. (14), (15) and (26), which, after some re-arrangements, becomes

$$\dot{C}_{i,1}^X\frac{1}{e_{i,1}^X}+\dot{C}_{i,2}^X\frac{1}{e_{i,2}^X}-\dot{C}_e^X\frac{1}{e_e^X}=0 \tag{64}$$

In Eqs. (63) and (64), the subscript i refers to an entering exergy stream ($i,1$ is the first and $i,2$ the second entering exergy stream) and e denotes the exiting exergy stream after the exergy removal. In the matrices $\boldsymbol{A}_F^T$, $\boldsymbol{A}_F^M$, $\boldsymbol{A}_F^{CH}$ the coefficient associated with the variable $\dot{C}_j^X$ is either $\pm 1/\dot{E}_j^X$, when Eq. (63) applies, or $\pm 1/e_j^X$ when Eq. (64) is used. The matrix $\boldsymbol{A}_{F,en}$ refers to the rare case of partial exergy removal from energy streams. The equations associated with this matrix are of the form

$$\frac{\dot{C}_i}{\dot{E}_i}-\frac{\dot{C}_e}{\dot{E}_e}=0 \tag{65}$$

The elements of matrix $\boldsymbol{A}_{F,en}$ are of the form $1/\dot{E}_j$. The matrices $\boldsymbol{A}_F^T$, $\boldsymbol{A}_F^M$, $\boldsymbol{A}_F^{CH}$ and $\boldsymbol{A}_{F,en}$ are placed within the large matrix $\boldsymbol{A}$ diagonally one below the other, since each row of each matrix supplies one auxiliary costing equation (Eqs. (63), (64) or (65)). The total number of Eqs. (63)–(65) is equal to $\sum_{k=1} N_{eF,k}$ where $N_{eF,k}$ is the number of exergy streams that exit the kth component and belong to the fuel.

7.4. Auxiliary equations associated with the P principle (matrix $\boldsymbol{A}_P$)

When the total number of exergy streams associated with the product of a component is larger than one, the P principle is used for calculating the costs of these streams. The matrix $\boldsymbol{A}_{\mathrm{P}}$, having $\sum_{k-1} N_{eP,k}-n$ rows, consists of the coefficients of the equations obtained from the P principle. When the chemical exergy does not change in the component being considered, these equations can take one of

the following forms (see, for example, Eq. (10)):

$$\dot{C}^X_{e,1}\frac{1}{\dot{E}^X_{e,1}} - \dot{C}^X_{e,2}\frac{1}{\dot{E}^X_{e,2}} = 0 \tag{66}$$

$$\frac{\dot{C}^X_{e,1} - \dot{C}^X_{i,1}}{\dot{E}^X_{e,1} - \dot{E}^X_{i,1}} - \frac{\dot{C}^X_{e,2} - \dot{C}^X_{i,2}}{\dot{E}^X_{e,2} - \dot{E}^X_{i,2}} = 0 \tag{67}$$

$$\frac{\dot{C}^X_{e,1} - \dot{C}^X_{i,1}}{\dot{E}^X_{e,1} - \dot{E}^X_{i,1}} - \dot{C}^X_{e,2}\frac{1}{\dot{E}^X_{e,2}} = 0 \tag{68}$$

where the subscripts i and e refer to an entering and exiting exergy stream, respectively and the subscripts 1 and 2 refer to the first and second exergy stream.

For components in which the chemical exergy changes, the auxiliary equations associated with the P principle can take, in addition to Eqs. (66)–(68), one of the following forms (see, for example, Eqs. (16), (18) and (28)):

$$\frac{\dot{C}^X_{e,(1+2)} - \dot{C}^X_{i,1} - \dot{C}^X_{i,2}}{\dot{E}^X_{e,(1+2)} - \dot{E}^X_{i,1} - \dot{E}^X_{i,2}} - \dot{C}^X_{e,3}\frac{1}{\dot{E}^X_{e,3}} = 0 \tag{69}$$

$$\frac{\dot{C}^X_{e,(1+2)} - \dot{C}^X_{i,1} - \dot{C}^X_{i,2}}{\dot{E}^X_{e,(1+2)} - \dot{E}^X_{i,1} - \dot{E}^X_{i,2}} - \frac{\dot{C}^X_{e,3} - \dot{C}^X_{i,3}}{\dot{E}^X_{e,3} - \dot{E}^X_{i,3}} = 0 \tag{70}$$

$$\frac{\dot{C}^X_{e,(1+2)} - \dot{C}^X_{i,1} - \dot{C}^X_{i,2}}{\dot{E}^X_{e,(1+2)} - \dot{E}^X_{i,1} - \dot{E}^X_{i,2}} - \frac{\dot{C}^X_{e,(3+4)} - \dot{C}^X_{i,3} - \dot{C}^X_{i,4}}{\dot{E}^X_{e,(3+4)} - \dot{E}^X_{i,3} - \dot{E}^X_{i,4}} = 0 \tag{71}$$

Here the subscripts $(1+2)$ or $(3+4)$ refer to the exiting mass flow rates that result from the mixing of the entering mass flow rates 1 and 2, or 3 and 4, respectively. Eqs. (69)–(71) have been written for mixing processes. If a separation process is involved, in the respective terms of the equations both the subscripts i and e and the signs '+' and '−' should be exchanged in all terms.

In accord with Eqs. (66)–(71), the coefficients of the matrix $\boldsymbol{A}_P$ are of one of the following forms: $\pm 1/\dot{E}^X_j$, $\pm 1/(\dot{E}^X_j - \dot{E}^X_l)$ or $\pm 1/(\dot{E}^X_j - \dot{E}^X_l - \dot{E}^X_r)$.

The above discussion in this section refers only to material streams. If energy streams are associated with the product, terms of the form $\pm\dot{C}_q/\dot{E}_q$ or $\pm\dot{C}_w/\dot{E}_w$ would appear in Eqs. (66), (68) or (69). In this case, the coefficients of the matrix $\boldsymbol{A}_{\mathrm{P}}$ are $\pm 1/\dot{E}_q$ or $\pm 1/\dot{E}_w$ for the variables $\dot{C}_q$ or $\dot{C}_w$, respectively.

7.5. Total number of equations

The total number of equations required to calculate the variables included in the vector $\boldsymbol{X}$ is equal to the total number t of exergy streams in the system. These equations are provided by the following:

- n equations from the cost balances;
- $3s_0 + q_0$ equations from the known costs of the exergy streams entering the overall system;

- $\sum_{k=1}^{n} N_{eF,k}$ equations from the F principle; and
- $\sum_{k=1}^{n} N_{eP,k} - n$ equations from the P principle.

Thus, the total number of equations N is equal to

$$N = 3s_0 + q_0 + \sum_{k=1}^{n} N_{eF,k} + \sum_{k=1}^{n} N_{eP,k} \tag{72}$$

Since an exergy stream exiting the kth component is automatically included either in the term $N_{eF,k}$ (if the stream is part of the fuel) or in the term $N_{eP,k}$ (if the stream is part of the product), the last two terms in the above equation represent the total number of streams exiting all the components. Thus,

$$N = 3s_0 + q_0 + \sum_{k=1}^{n} N_{e,k} \tag{73}$$

The total number of streams in a system is always equal to the total number of streams exiting all the components plus the total number of streams supplied from outside. Therefore $N=t$ and the number of linear equations is equal to the number of variables included in the vector of the unknowns $\boldsymbol{X}$.

In the above discussion we excluded the special case of a mixing process, in which one fictitious device and two fictitious streams need to be introduced (see Section 2) for cost calculating purposes. Thus, for each mixing process in the overall system, the number of unknowns t in the above system of equations will increase by two (i.e. the two fictitious streams entering the fictitious auxiliary device shown in Fig. 2a. The two additional equations per mixing process required to calculate these unknowns are provided by the cost balance for the fictitious auxiliary component and the F equation for the exergy form Y of material stream 2 (see Eqs. (17) and (19)).

If a dissipative component is present in the overall system, the number of unknowns and equations must be increased by the number of all fictitious cost rates (such as the cost rates $\dot{C}_{dif,dc}$ and $\dot{C}_{dif,dc,a}$ through $\dot{C}_{dif,dc,d}$ shown in Fig. 4) used to charge the cost associated with a dissipative component to the final product(s) or to the other system component(s) served by the dissipative one. The equations needed are (a) the cost balances for the dissipative component and for the fictitious unit that distributes the cost rate $\dot{C}_{dif,dc}$ in Fig. 4, (b) an auxiliary equation for the main working fluid derived from the F principle (Eq. (51)), and (c) all the equations used to charge $\dot{C}_{dif,dc}$ to the final product(s) or to the component(s) served by the dissipative one. Since this cost apportionment is subjective, these equations can have different forms. The matrix resulting from dissipative components is not shown in Fig. 5. It should be added at the end of the large matrix $\boldsymbol{A}$.

8. Comparison of the SPECO method with other approaches

This section presents a brief discussion of the major differences between the SPECO method and previous exergy-based approaches for calculating efficiencies and costs in thermal systems.

These approaches include the

– Exergy Economics Approach (EEA) ([5,6]);
– First Exergoeconomic Approach (FEA) ([1,2,7]);

– Exergetic Cost Theory (ECT) ([8]);
– Thermoeconomic Functional Analysis (TFA) ([17]);
– Engineering Functional Analysis (EFA) ([18]);
– Last-In–First-Out Approach (LIFOA) ([25,26]); and
– Structural Analysis Approach (SAA) ([30]).

In the following we discuss the differences in the definition of exergetic efficiencies, the development of auxiliary costing equations, the productive structure and the formulation of matrix $\boldsymbol{A}$.

8.1. Exergetic efficiencies

The SPECO method presents a general, systematic, simple and unambiguous approach for developing the exergetic efficiencies of a thermal system and its components. The approaches followed in EEA, TFA, FEA, ECT, EFA and SAA use to some extent more flexible decisions with respect to the exergy streams that should be included in the fuel and product. In addition, in the SPECO method there are no exergy losses associated with material streams at the component level since all exergy streams associated with material streams exiting a component are considered either on the fuel or on the product side. All other approaches, with the exception of LIFOA, consider some exergy losses associated with material streams at the component level.

8.2. Productive structure

A productive structure is part of the application of TFA and EFA, whereas it is optional for the remaining approaches. In the SPECO method, in agreement with the criterion of considering the actual exergy exchanges between the component and the rest of the system, the productive interactions (productive structure) remain within the component boundaries. Each component acts on the existing physical (material and energy) flows that cross its boundaries (by adding or removing exergy) and, apparently, these exergy exchanges cannot modify the physical interconnections among components. Thus, the streams representing the interactions among the components remain in the productive structure the same as in the process flow diagram of the system, whereas the productive structures associated with other approaches (a) introduce states that do not correspond to any real state, and (b) may lead to complex diagrams that often have little resemblance with the corresponding flow diagrams ([27]).

8.3. Auxiliary costing equations

Since the number of streams in a system is, in general, larger than the number of components (i.e. the number of cost balances that can be formulated), all exergy based costing approaches explicitly (EEA, FEA, ECT and LIFOA) or implicitly (TFA, EFA and SAA) require auxiliary equations for calculating the costs of streams. In the latter the auxiliary equations are implied in the formulation of the productive structure. In general, for a given definition of fuel and product, all methods result in similar auxiliary costing equations. However, in the TFA and EFA, the introduction of fictitious states into the productive structure may lead to different cost values within the system.

The SPECO method provides general criteria for developing auxiliary costing equations associated with any system. These criteria are consistent with common business practice and, thus, increases acceptance of these equations.

8.4. Matrix formulation

A matrix formulation for the system of linear equations was presented in the literature only in conjunction with ECT, SAA and LIFOA. ECT [8] presents a general and well structured matrix formulation for equations based on both exergetic costs and average monetary costs when total exergies are considered in the analysis. The SAA matrix was initially proposed [30] for exergetic costs or monetary costs associated only with exergy streams. Although not explicitly mentioned by the authors, the matrix cost equations shown in [36] for a gas turbine cogeneration plant can be considered as an application of SSA where also the amortization and maintenance costs of the devices are included. Based on [8], the LIFOA matrix was presented for both total exergies and separate forms of exergy in [26]. A distinct feature of this approach is that in the vector $\boldsymbol{X}$, component-based variables (e.g. c_P) are used in addition to stream-based variables. The general matrix presented here for the SPECO approach extends the application of the ECT matrix formulation to the calculation of average costs in the case in which separate components of exergy are considered. A specific application to gas turbine cogeneration plants was previously presented in [36].

In the TFA and EFA no matrix such as matrix $\boldsymbol{A}$ is needed because the marginal cost equations are obtained by solving an optimization problem using the Lagrangian approach. However, these methods can also be used to calculate average costs [see e.g. 31]. In this case costing is performed for any design or operation point. Although not explicitly presented in the literature, a matrix formulation similar to those of the other methods could be developed.

9. On the need for an unambiguous procedure for cost evaluation

The idea of a ‘systematic and unambiguous’ procedure for fuel and product definition and for cost evaluation is obviously in contrast with the more ‘flexible’ idea of having different definitions of fuel, products, and consequently cost values. This flexible approach is more or less implicitly applied by all the methodologies that, depending on the purpose of the analysis, define different ‘productive structures’ of the system. We strongly believe that there is a significant need for using at the component level an unambiguous exergoeconomic procedure that is independent of the purpose of the analysis and independent of the system configuration. Having said that, we should add that more flexible approaches could also be used when they do not lead to misleading conclusions from the thermodynamic and economic viewpoints. The problem is that misleading definitions of fuel and product cannot always be identified easily and, are, therefore, often adopted by the so-called flexible approaches.

To understand the need of unambiguity, let us consider the three main fields of application of a thermoeconomic analysis: the optimization of the structure and the design variables of the system, the optimization of the system operation, and the diagnosis of malfunctions in system operation. There is no doubt that in the first two cases we need an unambiguous procedure for identifying the inefficiencies and the associated costs, in order to correctly minimize the total costs and obtain the optimum design or operation point. Conversely, some space for flexibility might exist in the diagnosis of malfunctions for

an operating system. In this case, the two main objectives consist in locating the origins of the anomalies and quantifying their cost, both at the component level and the system level. If we want to quantify correctly the cost of the malfunctions we have to be very strict in following the development of the process in terms of exergy consumption. Thus, the more we reduce the 'logical interpretation' of the process, the more precise the cost evaluation becomes. One may argue that this is not true when the objective is to locate the causes of inefficiencies, in the sense that a 'flexible' interpretation of the cost formation process could be used, in order to better detect and locate the causes of inefficiencies. We believe that costs are more suitable for quantifying effects than for locating causes of malfunctions (a thorough discussion on this point can be found in [37,38]), and, therefore, a reliable and unambiguous procedure for cost evaluation is in general to be preferred to other more flexible procedures.

10. Closure

This paper presents a final form of the SPECO method, a method that is based on specific exergies and costs per exergy unit, to define exergetic efficiencies and to calculate the auxiliary costing equations for components of thermal systems. The approach discussed here is the simplest and most general approach among all versions of the SPECO method presented in the past. The main features of the generalized and simplified SPECO method are the following:

- The exergetic efficiency and the auxiliary costing equations are obtained by systematically registering, along a material stream, exergy and cost additions to and removals from each exergy stream considered in the analysis.
- Exergy differences between inlet and outlet are considered for most cases of chemical, reactive and nonreactive exergies and in all cases involving physical, thermal and mechanical exergies.
- When only total exergies are used and the purpose of the component dictates that no difference of chemical exergies between inlet and outlet should be considered for a material stream (for example, in a gasification reactor), then no difference between the total exergies of the same material stream should be used either.
- When the chemical exergies of material streams change between inlet and outlet as a result of mixing, separation or chemical reaction and exergy differences are considered for an exergy form, then it is sufficient to calculate the overall change of exergy between inlet and outlet for the respective exergy form. The only exception to this rule is provided when in a mixing or separation process the specific exergy of one mass flow rate increases and of the other mass flow rate decreases between inlet and outlet and the increase of specific exergy is consistent with the purpose of the component. In this case an imaginary state may be used (e.g. state 3* in Fig. 2) to visualize the costing process, improve understanding and facilitate calculations. Each mass flow rate associated with this state is considered separately and has the same specific exergy but different costs per exergy unit.
- The F principle is applied to each exiting exergy stream used in the definition of the fuel and, in conjunction with the AVCO method and single unmixed mass streams, states that the average costs per exergy unit remain unchanged between inlet and outlet for each exergy stream. With the aid of the F principle, we calculate the cost associated with each exiting exergy stream used in the fuel definition.

- The cost balance and the equations derived from the F principle provide the value of the average cost per unit of product exergy (c_P).
- The P principle is applied to each exiting exergy stream associated with the definition of the product and states that each exergy unit is supplied to each such stream at the same average cost (c_P). With the aid of the P principle we calculate the cost associated with each exiting exergy stream used in the product definition.

The above guidelines generalize and simplify the definitions of exergetic efficiencies and the costing procedures. These guidelines significantly reduce the arbitrariness in applications of exergy costing. All discussions in this paper refer to the so-called productive components [12]. For dissipative components, a meaningful exergetic efficiency cannot be defined. All costs associated with a dissipative component should be appropriately apportioned among all other components served by it.

Although we recognize that the concept of 'cost' is inevitally subjective, the idea of founding this concept on exergy additions and removals along exergy streams, or, in other words, on exergy consumption and supply 'evaluated' along these streams, is, in our opinion, the criterion that keeps the concept of 'cost' closer to the concept of 'consumption' and generates, therefore, a strict link between cost and physical development of the process. In this sense, we believe that the SPECO approach expresses the strongest possible effort in the direction of 'validating' the calculated cost values.

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