

# Tracker Physics Labs

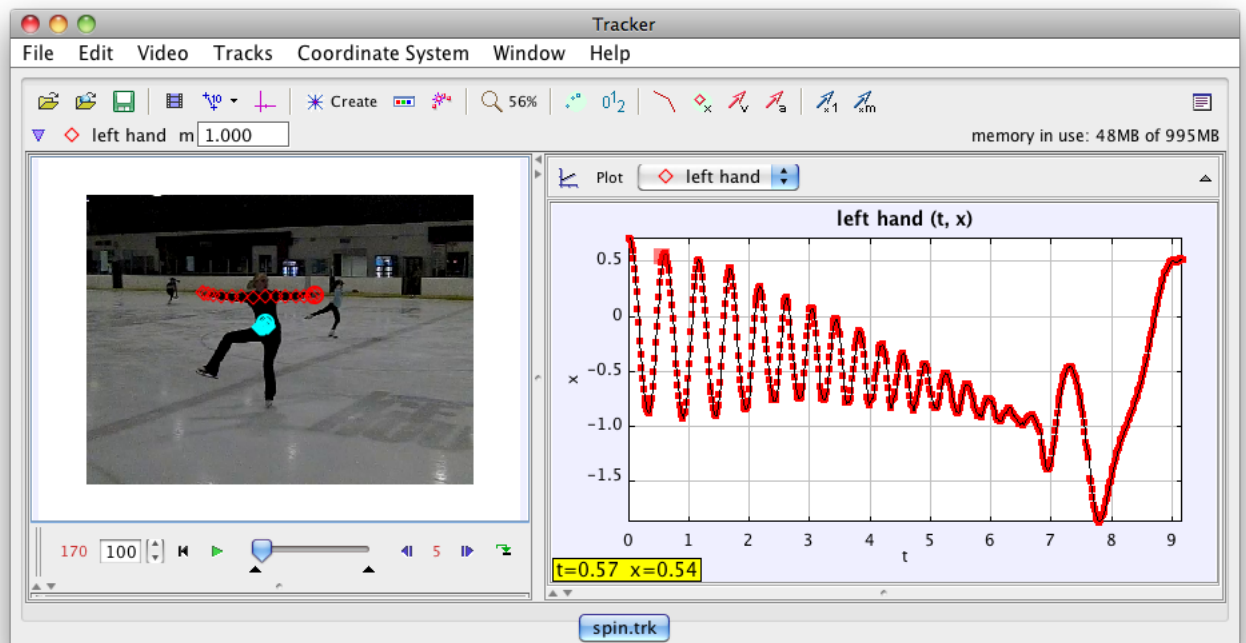
Lab Manual

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<http://physics.highpoint.edu/~atitus/>



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Tracker is developed by Doug Brown and uses the Open Source Physics library by Wolfgang Christian, et. al. Tracker is available from <http://www.cabrillo.edu/dbrown/tracker/>. Open Source Physics is at <http://www.compadre.org/osp/>.

Some videos were recorded by students or faculty at High Point University. LivePhoto Physics videos are used with permission from RIT and are available at <http://livephoto.rit.edu/>.

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# 1 Video Analysis of Uniform Motion

## Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)

video: `uniform-motion-ball-slow.mov` from <http://physics.highpoint.edu/~atitus/videos/>

video: `uniform-motion-ball-fast.mov` from <http://physics.highpoint.edu/~atitus/videos/>

## Goal

In this experiment, you will measure and graph the x-position of a rolling steel ball as a function of time. In addition, you will learn how to use video analysis software *Tracker* to measure position as a function of time for an object and find the best-fit curve to a graph.

## Introduction

A video is basically a set of images recorded at a rate of 30 frames per second, or in other words a time interval of  $1/30$  s between frames. (However, high-speed video has a higher frame rate.) To measure an object's position in a video, you need to:

1. define a coordinate system including an origin and x,y axes.
2. define a scale; in other words define a standard length, perhaps 1 m, in the video. For this, it helps to have an object of known length, such as a meterstick, in the video. This is called a *calibration*.

It's very important that the calibration instrument, like a meterstick, is in the same plane as the object's motion. If the meterstick is closer to the camera or further from the camera than the object you are studying, then your measurement of position will be inaccurate.

Consider the object in Fig. 1.1. To measure its x-position, draw a perpendicular line from the object to the x-axis. To measure its y-position, draw a perpendicular line from the object to the y-axis.

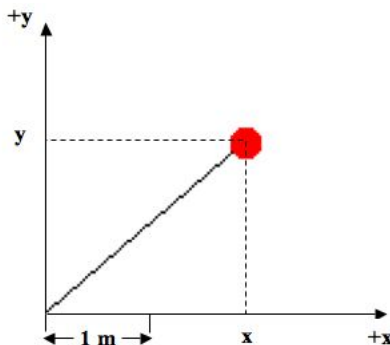


Figure 1.1: Position of an object depends on the origin and scale of the coordinate system.

Using the 1 m scale shown in the image, estimate the  $(x,y)$  coordinate of the object and use this to calculate the approximate distance of the object from the origin in Fig. 1.1

Video analysis software makes it easy to measure position coordinates (both  $x$  and  $y$ ) and time for an object. After defining the scale and the coordinate system, you click on the object. The software shows a dot where you clicked and advances the video to the next frame. The software measures the position of where you clicked in units of pixels and then uses the calibration and your definition of the coordinate system to convert this position in pixels to a position in meters (or whatever units are used in the calibration).

The software also measures time because it knows that the video is recorded at 30 frames per second (or perhaps higher for high-speed video). Thus, whenever you advance the video, time advances  $1/30$  s. With time and position measured by the software, you can calculate velocity by numerically calculating the derivative of the position with respect to time. Other variables can also be calculated and graphed. All of these calculations can be done by the software.

## Procedure

1. Download the file *constant-velocity-slow.mov* from the given web site by right-clicking on the link and choosing **Save As...** to save it to your desktop.
2. Open the *Tracker* software on your computer.
3. Use the menu **Video**→**Import...** to import your video, as shown in Figure 1.2.



Figure 1.2: Video→Import menu

4. To zoom in or out on the video, click on the toolbar's magnifying glass icon that is shown in Figure 1.3. When it appears with a  $\oplus$ , then clicking once on the video will zoom in (thus making it larger). Clicking the magnifying glass again will make it  $\ominus$ ; then clicking on the video will zoom out (thus making it smaller). Zoom in and out on the video to see how it works.



Figure 1.3: The icon used to expand the video.

5. At this point, it's nice to lay out the video and graphs so that you can clearly see everything. The middle border between panes, seen in Fig. 1.4, can be dragged left and right to make the video pane smaller and graphs larger. The same is true of any other bar that separates panes in the window.

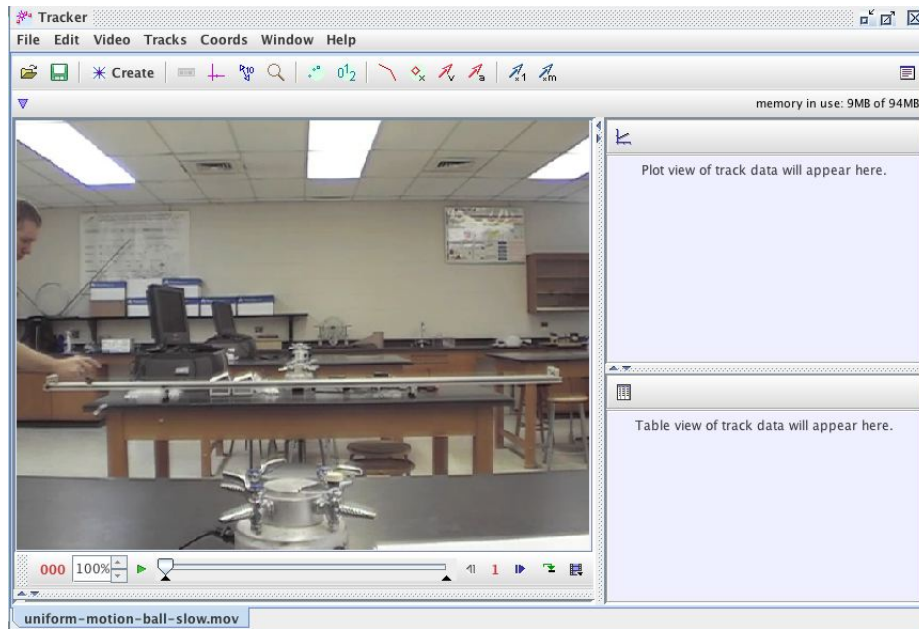


Figure 1.4: Drag the vertical or horizontal bars the make panes larger or smaller.

6. Note the video controls at the bottom of the video pane. Go ahead and play the video, step it forward, backward, etc. in order to learn how the video controls work. Note the counter that merely shows the frame number for any frame. Also, click on each of the icons in the video control bar to see what they are used for. Finally, use the left and right arrow keys on your keyboard, and note that they can be used to control the video as well.
7. Rewind to the first frame of the video. This is the instant that you will begin making measurements of the position of the moving object.
8. Since the ball moves fairly slowly on the track, we can skip frames between marking the ball and thus take fewer data points. Click on the **Step Size** button, as shown in Figure 1.5 and change it to **5**.

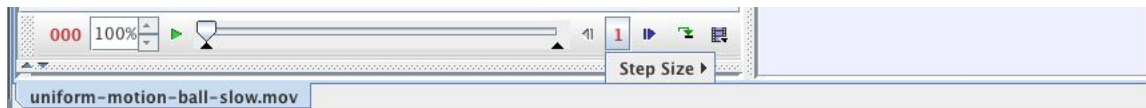


Figure 1.5: Change the step size in order to skip frames.

9. You now need to define the origin of the coordinate system. In the toolbar, click the **Axes** icon shown in Fig. 1.6 to show the axes of the coordinate system. (By now, you have probably noticed that you can hover the mouse over each icon to see what they do).



Figure 1.6: Icon used to set the coordinate system axes.

10. Click and drag on the video to place the origin of the coordinate system at the location where you would like to define (0,0), as shown in Figure 1.7. You can place the origin at any point you choose,



but in this case, it makes sense to put the origin at the location of the ball in the first frame being analyzed.

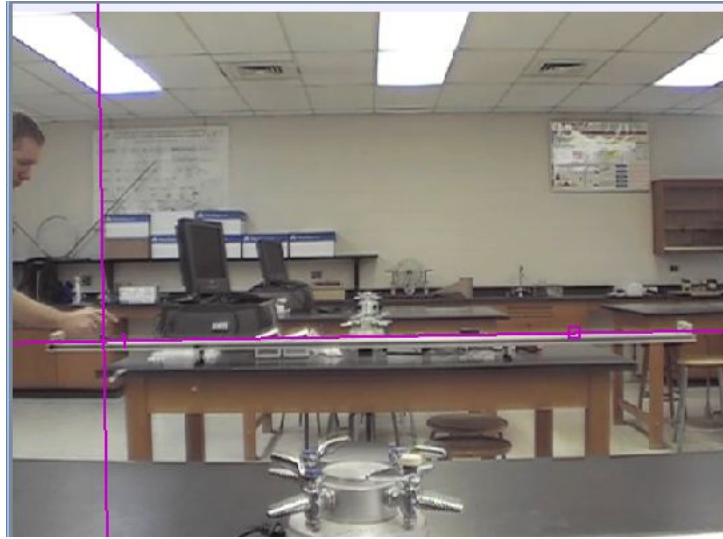


Figure 1.7: Click and drag to set the origin of the coordinate system

11. If you click the x-axis and drag, you can rotate the coordinate system. In this case, the video camera was not level; therefore, rotate the x-axis until it is parallel to the track.
12. Click the **Axes** tool again to hide the axes from the video pane. You can click this icon at any time to show or hide the axes.
13. Now, you must calibrate distances measured in the video. In the toolbar, click on the **Tape Measure** icon shown in Figure 1.8 to set the scale for the video.



Figure 1.8: Icon used to set the scale.

14. A blue double-sided arrow will appear. Move the left end of the arrow to the left end of the track, and move the right end of the arrow to the right end of the track. Double-click the number that is in the center of the arrow, and enter the length of the track, 2.2. (Our units are meters, but Tracker does not use units. You must remember that the number 2.2 is given in meters.) The scale will appear as shown in Figure 1.9.
15. Click the tape measure icon again to hide the blue scale from the video.
16. You are ready to add markers to the video to mark the position of the ball. Let's not show the coordinate system and scale. It's too distracting. So, make sure you've clicked the **Axes** and **Tape Measure** icons in the toolbar to hide them.
17. To add markers, click on the **Create** button and select **Point Mass** as shown in Figure 1.10. Then, **mass A** will be created, and a new  $x$  vs.  $t$  graph will appear in a different pane. You will now be able to mark the position of the ball which will be referred to as **mass A**.
18. We will want to set the frame step size to 5 so that it will skip 4 frames every time we step forward in the video. Double check that the step size is 5 in the video control toolbar.



Figure 1.9: Enter the length of the track.

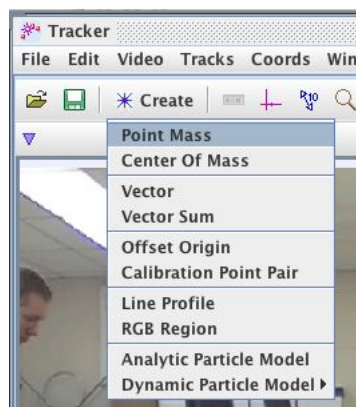


Figure 1.10: The Track Control toolbar.

19. **To mark the center of the ball, hold the SHIFT key down and click once on the center of the ball.** You should notice that a marker appears at the position of the ball where you clicked and that the video advances one step.
20. Again, shift-click on the ball to mark its position. You should now see two marks.
21. Continue marking the position of the ball until it reaches the right end of the track. Note that only a few of the marks are shown in the video pane. To display all of the marks or a few of the marks or none of the marks, use the **Set Trail Length** icon shown in Figure 1.11. Clicking this icon continuously will cycle through no trail, short trail, and full trail which will show you no marks, a few marks, or all marks.  
After marking the ball as it moves from the left end to the right end of the track, your video should look like the picture shown in Fig. 1.12 if you have set the trail length to show the full trail.
22. Tracker uses the frame and frame rate to calculate  $t$ , and it uses the scale and coordinates of the marks to calculate  $x$  and  $y$  coordinates for the ball. It uses numerical differentiation to calculate x-velocity and y-velocity.



Figure 1.11: The Set Trail icon is used to vary the number of marks shown.

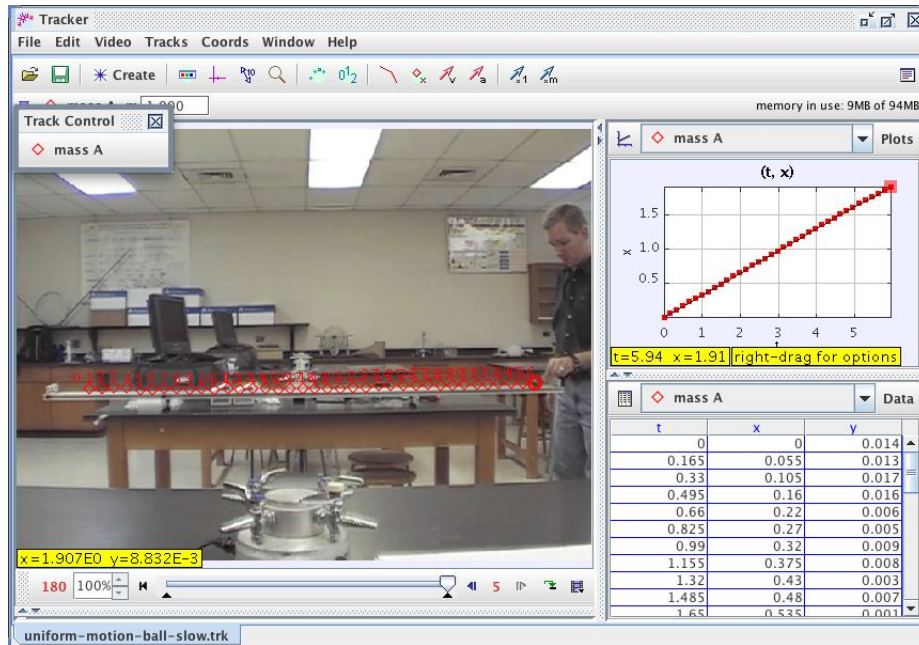


Figure 1.12: Marks showing the ball's position.

## Analysis

### $x$ vs. $t$ graph

1. We will now analyze the  $x$  vs.  $t$  graph. You can click and drag the border of the video pane to make it smaller so that you can focus on the graph.
2. Play the video. (You can hide the marks if you wish by clicking the **Show or hide positions** icon, and you can show the path by clicking the **Show or hide paths** icon. Both of these icons are in the toolbar.) Note how the graph and video are synced. Each video frame data point is shown in the graph using a filled rectangle.  
Also, when you click on a data point on the graph, the video moves to the corresponding frame.
3. Observe the  $x$  vs.  $t$  graph.

Describe in words the type of function that describes this graph of  $x$  vs.  $t$ ? (i.e. linear, quadratic, square root, sinusoidal, etc.)

4. Right-click (or ctrl-click) on the graph and select **Analyze...**. In the resulting window, check the checkbox for **Fit**, and additional input boxes will appear, as shown in Figure 1.13. The Fit Name should be "Line" and the equation will be  $x = a * t + b$  where  $a$  is the slope and  $b$  is the vertical intercept. Check the checkbox for **Autofit** and the best-fit line will appear in the graph.

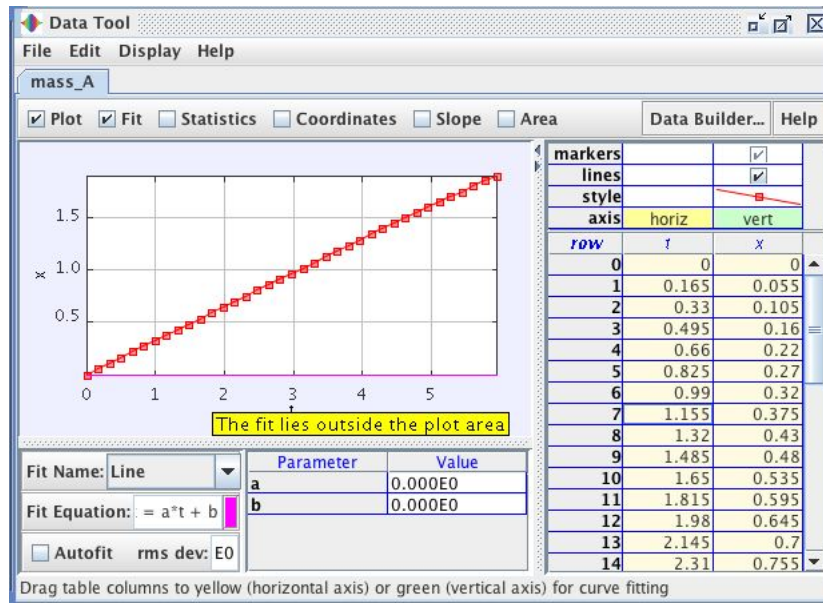


Figure 1.13: The Data Tool for finding the best-fit curve for the data.

Record the function and the values of the constants for your curve fit. Write the function for  $x(t)$ , with the appropriate constants (also called fit parameters).

In general, what does the slope of the  $x$  vs  $t$  graph tell you? (Consider its units. Your answer should be a sentence, not a number.)

From the curve fit parameters, determine the  $x$ -velocity of the ball.

### $v_x$ vs. $t$ graph

5. Now, we will look at the  $x$ -velocity vs. time graph.

What do you expect the x-velocity vs. time graph to look like? Sketch your prediction below.

- Close the Data Tool window and return to the main window. Click once on the vertical axis label and change it from  $x$  to  $v_x$ , as shown in Figure 1.14.

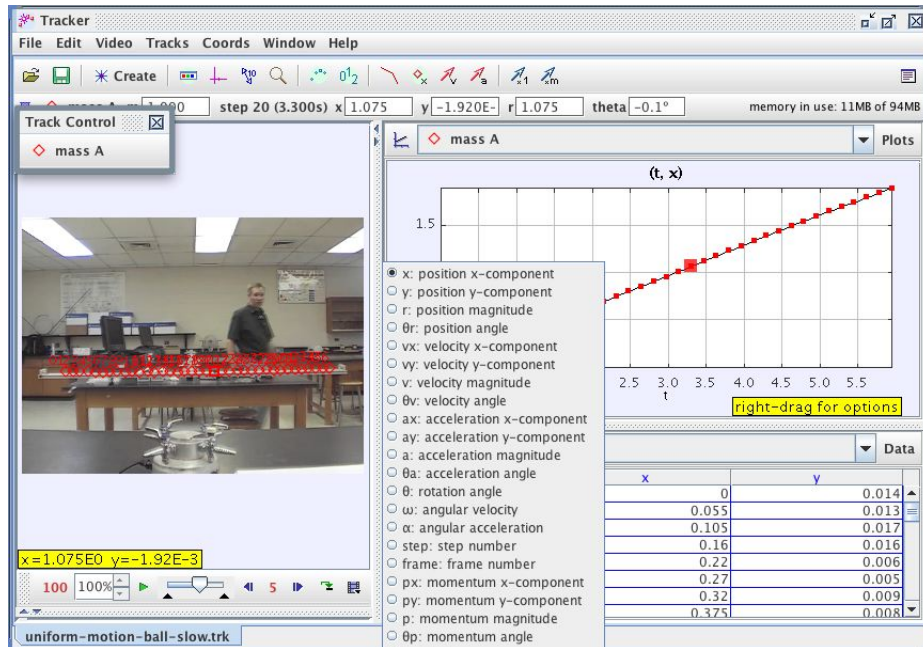


Figure 1.14: Changing the variables plotted on the graph.

Note that the data appears to be all over the place. That's because by default, the graph is "zoomed in" on the data. If you examine the numbers on the vertical axis, you'll notice that the data probably lies between 0.3 m/s and 0.35 m/s (though your data might also vary from mine). That's a very small variation in the velocity during the 5-second time interval that the ball is moving. And the variation is likely due to measurement error such as not clicking exactly on the center of the ball for every mark.

- Hover over the lowest part of the vertical axis (near the graph's origin, on the vertical axis). Click once and change the minimum on the vertical scale to 0. Do the same thing at the top of the scale and change the maximum to 0.5. The data now appears to be along a horizontal line, though there is some scattering in the data due to uncertainty in the measurements, as shown in Figure 1.15.
- Right-click (or ctrl-click) on the graph and select **Analyze...** in order to analyze the  $v_x$  vs.  $t$  graph. You may notice that the data for both  $x$  and  $v_x$  are displayed on the same graph. If this occurs, uncheck the checkboxes for  $x$  in the upper right corner of the window.
- Once again, change the minimum and maximum values on the scale so that the graph is not zoomed in on the data. Do an auto-fit.

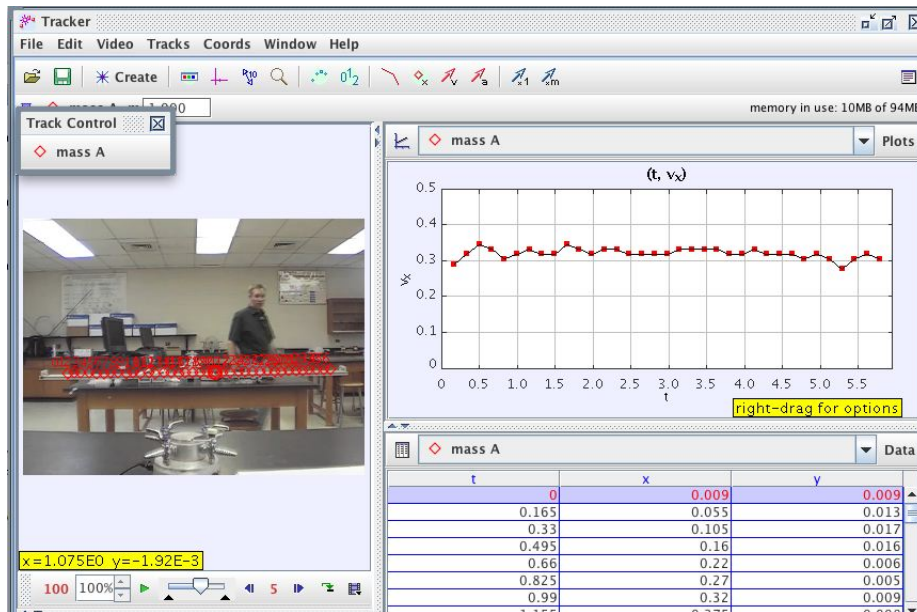


Figure 1.15: The x-velocity vs. time graph.

Record the fit constants and equation for the curve fit. From the curve fit, determine the initial x-velocity.

You will notice a slight downward slope of this graph. What does this tell you? Answer in a complete sentence.

10. Since the x-velocity is nearly constant, we would like to have an average of the x-velocity measured at each instant. Thus, check the checkbox for **Statistics** .

Record the mean x-velocity and the standard deviation.

Is the slope of the  $x$  vs.  $t$  graph within  $\bar{v}_x \pm \sigma$ ?

11. Be sure to save the Tracker file in the same folder as your video. Transfer this file and the video to a USB thumb drive for your records. In fact, it is best to save as you go. **Remember, save early and often!**

## Further Investigation

1. Analyze the motion of the ball in the video *constant-velocity-fast.mov* and answer the following questions.
  - (a) What is the x-velocity of the ball as determined from the  $x$  vs.  $t$  graph?
  - (b) What is the average x-velocity of the ball as determined from the  $v_x$  vs.  $t$  graph? Include the uncertainty in your measurement.
  - (c) If you were to only see the marks in the video and not see any graphs, how would you know that it is uniform motion (as opposed to non-uniform motion)?
2. Car A travels with a constant velocity of 30 mph for two hours, and Car B travels with a constant velocity of 60 mph for two hours. Each of them start at the origin at  $t = 0$ . On the same set of axes, sketch a graph of  $x$  vs.  $t$  for the two cars.
3. For the cars in the previous question, sketch a graph of  $v_x$  vs.  $t$  for the two cars on the same set of axes.
4. Suppose that the ball in the video started at the right end of the track and traveled with a constant velocity to the left. If the origin is set at the left end of the track with the  $+x$  axis pointing to the right (just as before), sketch a graph of  $x$  vs.  $t$  for the ball.
5. What would a graph of  $v_x$  vs.  $t$  look like for the ball in the previous question?
6. Suppose that the ball in the video started at the right end of the track and traveled with a constant velocity to the left. If the origin is set at the right end of the track with the  $+x$  axis pointing to the right, sketch a graph of  $x$  vs.  $t$  for the ball.
7. What would a graph of  $v_x$  vs.  $t$  look like for the ball in the previous question?

## 2 Video Analysis – Inertial and non-inertial reference frames

### Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)  
video: `two-carts.mov` from <http://physics.highpoint.edu/~atitus/videos/>

### Goal

In this experiment, you will measure the motion of two carts: (1) an accelerating fancart and (2) a cart moving with constant velocity. You will then describe the motion of the second cart as viewed from the reference frame of the fancart.

### Introduction

Newton's first law describes the motion of an object (or *system*) if the net force on the object is zero. It basically says that if the forces acting on an object balance (so that there is zero net force on an object), then its velocity will be constant (which may or may not be zero).

Newton's second law describes the relationship between the net force on an object (or *system*) and the object's acceleration.

$$(2.1) \quad \Sigma \vec{F} = m\vec{a}$$

Applying Newton's second law to one-dimensional motion in the x-direction gives:

$$(2.2) \quad \Sigma F_x = ma_x$$

If Newton's first and second laws are valid, then you are viewing the object from an *inertial* reference frame.

If the net force on an object is zero and yet the object accelerates, then Newton's laws are *not* valid, and you must be viewing the object from a *non-inertial* reference frame.

Another way to say this is if the object is accelerating, then you should be able to identify the force(s) causing the acceleration. If these forces do *not* exist, yet the object accelerates, then Newton's first and second laws are *not* valid, and you are viewing the object from a *non-inertial* reference frame. You have to either invent a force, called a *fictitious force*, to make Newton's laws work in the non-inertial reference frame, or you have to simply recognize that you are in a non-inertial reference frame and not apply Newton's laws.

### Procedure

In this experiment, you will analyze the video of two carts. Both carts are on low-friction, level tracks. (1) Cart 1 has a fan attached to it and (2) Cart 2 has no fan attached. There are no horizontal forces on Cart 2.



1. Download `two-carts.mov` by right-clicking on the link and choosing **Save As...** to save it to your desktop:
2. Open the *Tracker* software on your computer.
3. Go to the menu **Video→Import...** and import `two-carts.mov`, as shown in Figure 2.1.

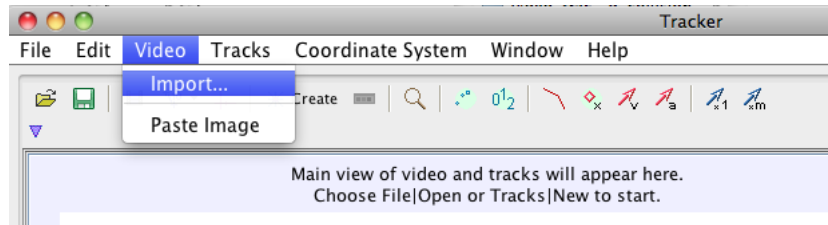


Figure 2.1: Video→Import menu

4. Play the video and watch the motion of the two carts.

Describe the motion of Cart 1.
List all of the forces on Cart 1, and sketch a free-body diagram. In what direction is the net force on Cart 1? Explain your reasoning.
Describe the motion of Cart 2.
List all of the forces on Cart 2, and sketch a free-body diagram. In what direction is the net force on Cart 2? Explain your reasoning.

Are your observations consistent with Newton's first and second laws?

5. Go to the video control bar and move the left black marker to the right until the video is on frame 001. (See Figure 2.2) You can also adjust this by clicking on the video settings icon (shown in Figure 2.3) and setting the start frame at 1.

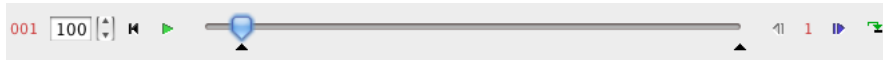


Figure 2.2: Use the video control bar to set the first frame for analysis to be frame 001.



Figure 2.3: Video settings icon in the toolbar.

6. There is a 1-m stick in the video. Use this to set the scale (i.e. calibrate distance) for the video.
7. Set the origin of the coordinate system to be at the position of Cart 2. In reality the choice of origin isn't important for measuring velocities and accelerations; but it does determine the intercept on the  $x$  vs.  $t$  graph for each cart.
8. Click the  button in the toolbar and create a new **Point Mass**.

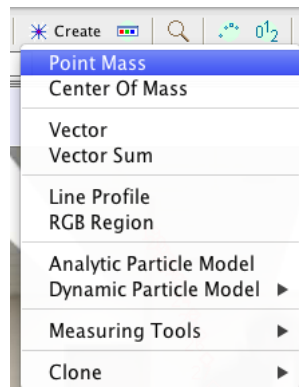


Figure 2.4: Create a new point mass and name it cart 1.

9. In the toolbar or in the Track Control pop-up window, click  , select **Name...**, and change its name to “cart 1”.
10. While holding down the Shift key, click on the yellow dot on Cart 1 to mark its position. Continue marking Cart 1 in all subsequent frames in the video.
11. Create a new point mass for Cart 2, change its name to “cart 2,” and mark its position in all subsequent frames of the video. (See Figure 2.5.)

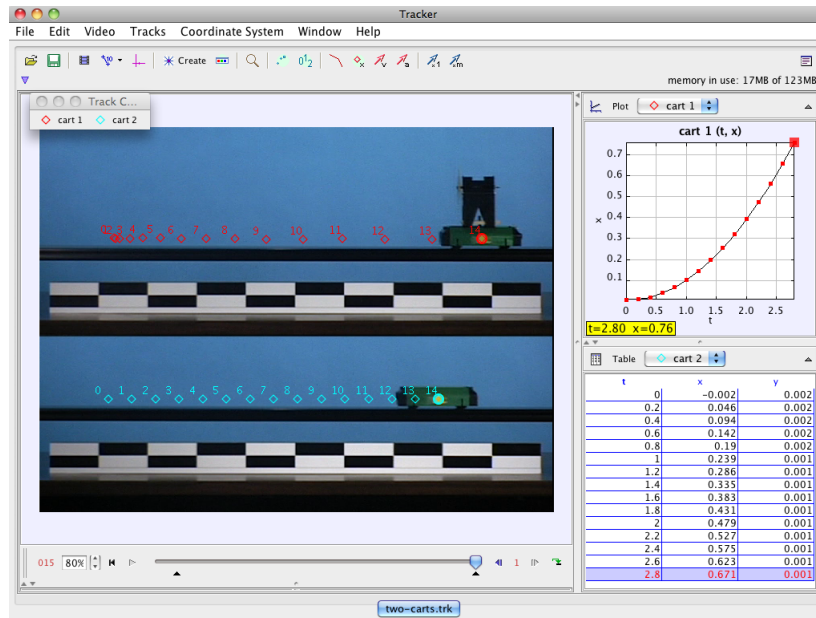


Figure 2.5: Trails for the two carts.

## Analysis

1. Fit a function to the graph of  $x$  vs.  $t$  for Cart 1. What is  $x(t)$ ?
2. Fit a function to the graph of  $v_x$  vs.  $t$  for Cart 1. What is  $v_x(t)$ ?
3. What is the initial velocity of Cart 1? What is the acceleration of Cart 1?
4. Fit a function to the graph of  $x$  vs.  $t$  for Cart 2. What is  $x(t)$ ?
5. What is the velocity of Cart 2?
6. What is the acceleration of Cart 2?
7. Do Newton's laws explain the motion of each cart? Sketch free-body diagrams for each cart and explain your answer.
8. Is the "ground" (i.e. the video camera) an inertial reference frame?
9. At what time (i.e. clock reading) does Cart 1 pass Cart 2? **Answer this question algebraically.**
10. Now measure the clock reading on the video when Cart 1 passes Cart 2. Compare your measured value with the calculated result.

Now imagine that these are cars and you are riding in Cart 1 that is accelerating to the right, as measured by a person on the ground. Describe the motion of Cart 2 from your reference frame (i.e. the reference frame of Cart 1).

11. **Answer all of the following questions from the reference frame of Cart 1.** Note that these are all predictions. You will make the measurements later.
- What is the initial velocity of Cart 2 (at  $t = 0$ )? Be sure to consider both direction and magnitude.
  - Before you pass Cart 2, will Cart 2 appear to speed up or slow down or both?
  - What is the acceleration of Cart 2 (in your reference frame)?
  - Sketch a graph of  $x$  vs.  $t$  for Cart 2 in your reference frame.
  - Sketch a graph of  $v_x$  vs.  $t$  for Cart 2 in your reference frame.
12. Now, go to **Coordinate System**→**Reference Frame** and select “cart 1” as shown in Figure 2.6. Answer the following questions.

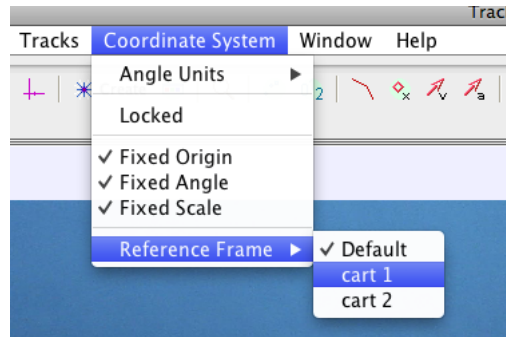


Figure 2.6: Change the reference frame to Cart 1.

- View the  $x$  vs.  $t$  graph for Cart 2. How is it similar or different to what you predicted?
- View the  $v_x$  vs.  $t$  graph for Cart 2. How is it similar or different to what you predicted?
- What is the initial velocity of Cart 2?
- What is the acceleration of Cart 2?
- Do Newton’s first and second laws describe Cart 2 in your reference frame? Explain your answer.
- Is Cart 1 an inertial reference frame? Explain your answer.

Now imagine that these are cars and you are riding in Cart 2 that has a constant velocity to the right, as measured by a person on the ground. Describe the motion of Cart 1 from your reference frame (i.e. the reference frame of Cart 2).

13. **Answer all of the following questions from the reference frame of Cart 2.** Note that these are all predictions. You will make the measurements later.
- What is the initial velocity of Cart 1 (at  $t = 0$ )? Be sure to consider both direction and magnitude.
  - Before Cart 2 passes Cart 1, will Cart 1 appear to speed up or slow down?
  - What is the acceleration of Cart 1 (in your reference frame)?
  - Sketch a graph of  $x$  vs.  $t$  for Cart 1 in your reference frame.

- (e) Sketch a graph of  $v_x$  vs.  $t$  for Cart 1 in your reference frame.
14. Now, go to **Coordinate System**→**Reference Frame** and select “cart 2” to change the reference frame to Cart 2. Answer the following questions.
- (a) View the  $x$  vs.  $t$  graph for Cart 1. How is it similar or different to what you predicted?
  - (b) View the  $v_x$  vs.  $t$  graph for Cart 1. How is it similar or different to what you predicted?
  - (c) What is the initial velocity of Cart 1?
  - (d) What is the acceleration of Cart 1?
  - (e) Do Newton’s first and second laws describe Cart 1 in your reference frame? Explain your answer.
  - (f) Is Cart 1 an inertial reference frame? Explain your answer.

# 3 Video Analysis of a Person Landing After a Jump

## Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)  
video: `landing-long.mov` from <http://physics.highpoint.edu/~atitus/videos/>  
video: `landing-short.mov` from <http://physics.highpoint.edu/~atitus/videos/>

## Goal

In this experiment, you will measure the force of impact by the floor on a person as he is landing on the floor after he jumps from a table. You will compare the impact force in two different cases: (1) he bends his knees as much as possible and (2) he bends knees a small amount.

## Introduction

Newton's second law describes the relationship between the net force on an object (i.e. *system*) and the object's acceleration.

$$(3.1) \quad \Sigma \vec{F} = m\vec{a}$$

Suppose that a gymnast is landing on a floor during a dismount in the vault or uneven bars events. Or suppose she is landing on the floor while tumbling during the floor exercise. Or suppose she is landing on the beam during the balance beam exercise. Regardless of the event, when landing there is a large upward force of the floor (or beam) on the gymnast as she slows down during the landing.



Figure 3.1: Nastia Liukin performs on the balance beam.

A force diagram showing the forces on a gymnast as she is landing on the mat is shown in Figure 3.2.

Applying Newton's second law to the person in the y-direction gives:

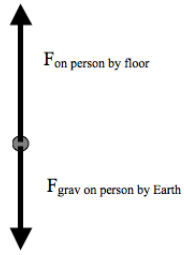


Figure 3.2: Forces on a gymnast when she is landing.

$$(3.2) \quad \Sigma F_y = ma_y$$

$$(3.3) \quad F_{on\ person\ by\ floor} - F_{grav\ on\ person\ by\ Earth} = ma_y$$

$$(3.4) \quad F_{on\ person\ by\ floor} - mg = ma_y$$

Let's consider three different parts of her motion:

1. while she is in the air, before she touches the floor.
2. while she is touching the floor and slowing down.
3. after she comes to a stop and she is at rest and remains at rest.

### Before the gymnast touches the floor

While she is falling, the floor is NOT touching her. Therefore, the floor exerts no force on her. Then,

$$(3.5) \quad \Sigma F_y = ma_y$$

$$(3.6) \quad F_{on\ person\ by\ floor} - F_{grav\ on\ person\ by\ Earth} = ma_y$$

$$(3.7) \quad 0 - mg = ma_y$$

$$(3.8) \quad -mg = ma_y$$

$$(3.9) \quad a_y = -g$$

Her acceleration is equal to  $-9.8\text{ m/s}^2$ .

### While she is touching the floor and slowing down

When she is landing, she is moving downward and slowing down. Therefore, her y-acceleration is upward (positive). Then,

$$(3.10) \quad F_{on\ person\ by\ floor} = ma_y + mg$$

which shows that the force on the gymnast by the floor is *greater* than her weight ( $mg$ ). As a result, it is the large upward force of the floor on the gymnast that is causing her to slow down.

### While she remains at rest on the floor

After stops, she is at rest and remains at rest. Therefore, her velocity is constant and her acceleration is zero. Then,

$$(3.11) \quad \Sigma F_y = ma_y$$

$$(3.12) \quad F_{on\ person\ by\ floor} - F_{grav\ on\ person\ by\ Earth} = ma_y$$

$$(3.13) \quad F_{on\ person\ by\ floor} - mg = 0$$

$$(3.14) \quad F_{on\ person\ by\ floor} = mg$$

The force on the gymnast by the floor is *equal to* her weight ( $mg$ ).

In which of the three cases described above is the force by the floor on her the greatest?

### How Time Interval affects the force by the floor on her while landing

As you deduced from Newton's second law, the force of the floor on the gymnast is largest while she is landing. In this case, it is

$$(3.15) \quad F_{on\ person\ by\ floor} = ma_y + mg$$

By measuring her acceleration, you can calculate the force by the floor on the person. The greater her acceleration, the greater the force by the floor and the more she will "feel" the impact.

The average acceleration of the gymnast while landing is:

$$(3.16) \quad a_y = \frac{v_{yf} - v_{yi}}{\Delta t}$$

where  $v_{yi}$  is her initial velocity when first touching the mat and  $v_{yf}$  is her final velocity when she comes to rest on the mat. Her final y-velocity is zero. Therefore,

$$(3.17) \quad a_y = \frac{0 - v_{yi}}{\Delta t}$$

$$(3.18) \quad a_y = \frac{-v_{yi}}{\Delta t}$$

The gymnast's initial velocity when first touching the mat depends on her peak height. The higher she is when she falls, the faster she is moving when she hits the mat and the greater her acceleration and the greater the force on her by the floor during her landing.

The time interval is the duration of time that she is slowing down during her landing. If she bends her knees a lot during her landing, then the time interval will be larger and her acceleration and the force by the floor on her during her landing will be smaller.

If she bends her knees very little during the landing, then the time interval is smaller as she comes to a stop. As a result, her acceleration is greater and the force on her by the floor is greater during the landing.



## Procedure

In this experiment, you will analyze the video of a student as he is landing on the floor. You will measure the force by the floor on him during the landing.

1. Download the following files by right-clicking on the link and choosing **Save As...** to save it to your desktop:
  - (a) `landing-long.mov`
  - (b) `landing-short.mov`
2. Open the *Tracker* software on your computer.
3. Go to the menu **Video**→**Import...** and import `landing-long.mov` , as shown in Figure 3.3.

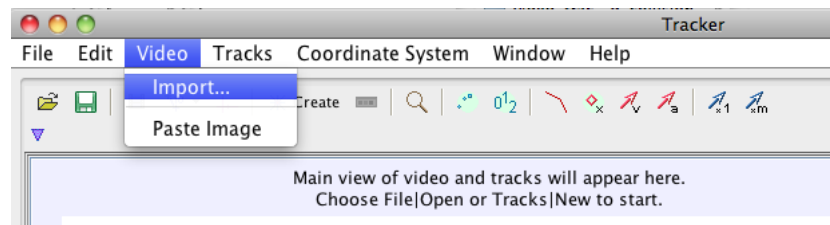


Figure 3.3: Video→Import menu

4. Play the video and watch the person's motion.
5. There is a 2-m stick in the frame of the video. Use this to set the scale (i.e. calibrate distance) for the video.
6. Set the origin of the coordinate system to be at the base of the meterstick. As you will use, this point is useful for aligning the coordinate system, but in reality the choice of origin isn't important for measuring velocities and accelerations.
7. After setting the origin of the coordinate system, click and drag up or down on the x-axis to rotate the coordinate system so that the y-axis is parallel to the stick. This will compensate for the fact that the camera is not level.
8. Advance the video to the first frame that his feet are no longer touching the table. Click the "Create" button in the toolbar and select **Point Mass** . Hold the shift key down and start marking his center-of-mass position. Assume that his center of mass is at the center of his belt.
9. View the y vs. t graph. Right-click on the graph and select **Analyze** in order to bring up the Data Tool.
10. Select the portion of the data that represents his motion before his feet hit the floor. You can count the number of dots in the video before his feet hit the floor, and select the same number of data.
11. Fit a curve to this data.



Figure 3.4: Rotate the coordinate system so that the y-axis is parallel to the 2-m stick.

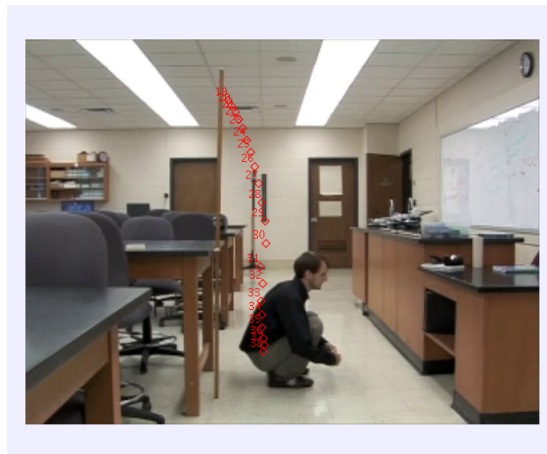


Figure 3.5: Marks for the person at approximately his center of mass.

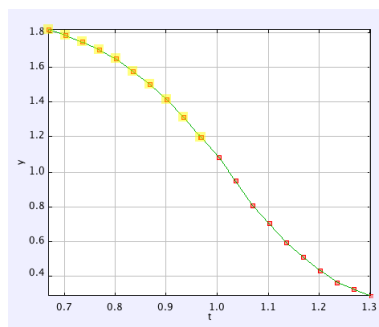


Figure 3.6: Selected data on the y-t graph for the person while falling

Note: he is in free-fall. What function should fit this portion of the curve? Record the best-fit function and the parameters of the curve fit. From the curve fit, calculate and record his y-acceleration as he is falling.

12. Now, select the portion of the graph where his feet are in contact with the floor and he is slowing down.

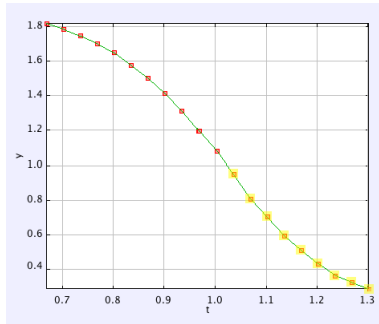


Figure 3.7: Selected data on the y-t graph for the person while landing

13. Fit a curve to this data, *assuming* constant acceleration.

What is his acceleration as he is landing and slowing down?

14. View the y-velocity vs. time graph. You will notice the part of the motion where he is accelerating downward and the part where he is accelerating upward.
15. Do a linear fit to each part of the graph.

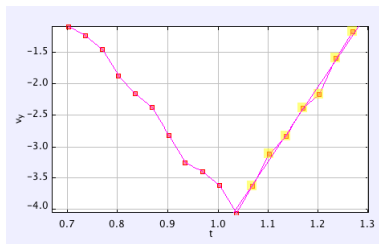


Figure 3.8: The y-velocity vs. time graph for the person while falling and landing

Using this graph, what is his during each part of his motion? Record your best-fit functions, and record  $a_{free\ fall}$  and  $a_{landing}$ .

Is your determination of the acceleration during each part of the motion consistent with what you measured using the quadratic curve fit on the  $y$  vs.  $t$  graph?

Now, apply Newton's second law (Eq. 11) to the landing. Calculate the force on the person by the floor as he is landing.

16. Suppose that the person bends his knees less while landing. Will his acceleration be greater or less than in this case? Will the force by the floor on him be greater or less (i.e. will it be a more "jarring" impact or less "jarring" impact)?

Download and analyze the movie `landing-short.mov`. Repeat the analysis that you did for the first video. Record all measurements and calculations below for his motion in the new video. In which case did the floor exert a larger force on the person while he was landing? What caused the force to be larger in this case?

# 4 Velocity of the Center of Mass of a System

## Apparatus

Tracker video analysis software

Video: *collision-pucks.mov* from <http://physics.highpoint.edu/~atitus/videos/>

## Goal

The purpose of this experiment is to measure the velocity of the center of mass of two pucks that make a collision on an air hockey table. You will measure the center-of-mass velocity before the collision and after the collision, and you will compare the results.

## Introduction

The location of the center of mass of a system of two particles is

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

This is a vector equation that must be true for both the x and y directions (for two dimensions).

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

Likewise, the center-of-mass velocity is

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Again, this equation must hold true for both the x and y components of the center-of-mass velocity.

$$v_{cm,x} = \frac{m_1v_{1x} + m_2v_{2x}}{m_1 + m_2}$$

$$v_{cm,y} = \frac{m_1v_{1y} + m_2v_{2y}}{m_1 + m_2}$$

## Procedure

It is expected that you have completed the other video analysis experiments, so these instructions do not include details about how to use the *Tracker* software.

1. Download the video *collision-pucks.mov* .
2. Open *Tracker* and insert the video.
3. Record the mass of each puck.

$m_{blue} =$

$m_{red} =$

4. Set the origin of your coordinate system.
5. Set the calibration using the meterstick on the left side of the video. You will have greater accuracy if you use 5 of the 10-cm segments for your calibration. In other words, stretch the calibration tool across 5 segments for a total length of 0.5 m.
6. Mark the blue puck for each frame of the video.
7. Mark the red puck for each frame of the video.
8. Using the graphs of  $x$  vs.  $t$  and  $y$  vs.  $t$  for each puck, measure the following quantities:

Table 4.1: default

	$v_{xi}$	$v_{yi}$	$v_{xf}$	$v_{yf}$
blue				
red				

## Analysis

Calculate the x-component of the center-of-mass velocity *before* the collision,  $v_{cm,ix}$ .

Calculate the y-component of the center-of-mass velocity *before* the collision,  $v_{cm,iy}$ .

Calculate the x-component of the center-of-mass velocity *after* the collision,  $v_{cm,fx}$ .

Calculate the y-component of the center-of-mass velocity *after* the collision,  $v_{cm,fy}$ .

What is  $\vec{v}_{cm,i}$ ? Write and sketch the vector.

What is  $\vec{v}_{cm,f}$ ? Write and sketch the vector.

*Tracker* can track the center of mass for you. In the following steps, you will learn how to calculate the track the center of mass of the system.

1. We need to define the masses of the pucks. Click the tab for **mass A** in the Track Control toolbar. In the drop-down menu, select **Define...** . In the resulting pop-up window, enter the mass of the puck for the parameter **m** as shown in Figure 4.1.
2. Repeat the previous step for **mass B** and enter its mass.
3. Click the  button and select **Center of Mass** , as shown in Figure 4.2.
4. You will see a new tab in the Track Control toolbar named **cm**. Click  to get the menu for the cm object shown in Figure 4.3. Click the **Select Masses...** menu item.
5. An additional window will pop up. Select both masses “mass A” and “mass B” in this window and click  as shown in Figure 4.4.
6. You will see a track for the center of mass and you will see a graph of  $x$  vs.  $t$ .



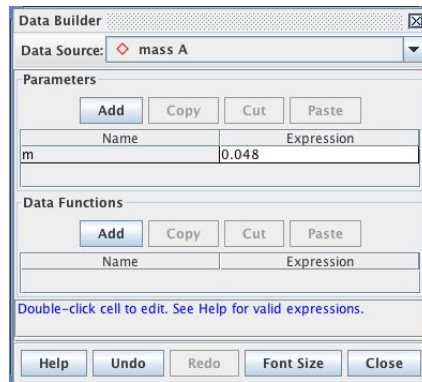


Figure 4.1: Enter of the mass of the puck.

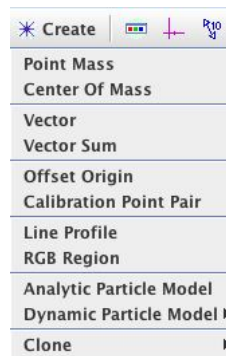


Figure 4.2: Select **Center of Mass** from the menu.

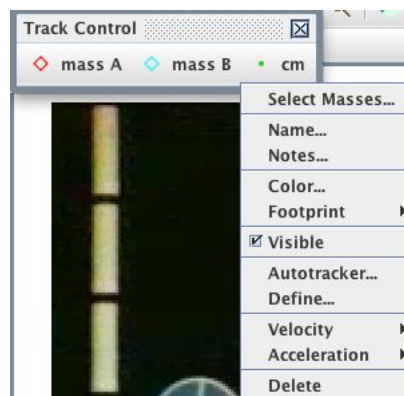


Figure 4.3: Click on **Select Masses...** from the menu.

By observing the  $cm$  track and by analyzing the graphs of  $x(t)$  and  $y(t)$ , what can you say about the center-of-mass velocity? Is it constant or non-constant?

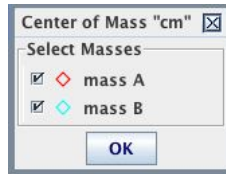


Figure 4.4: Check both masses (i.e. pucks) in this window.

Using the graphs of  $x(t)$  and  $y(t)$ , measure  $v_{cm,x}$  and  $v_{cm,y}$ .

## Further Investigation

1. Is the center-of-mass velocity constant or non-constant?
2. What is the net force on the system of pucks?
3. The collision occurs in approximately  $1/30$  of a second. What is the force by the red puck on the blue puck during the collision?
4. What is the force by the blue puck on the red puck during the collision?



# 5 Video Analysis of a Bicycle Wheel in Uniform Circular Motion

## Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)  
video: `bicycle-wheel.mov` from <http://physics.highpoint.edu/~atitus/videos/>

## Goal

In this experiment, you will measure and graph the x-position, y-position, and angle as a function of time for a sticker on a bicycle wheel rotating in a circle at constant speed. You will use the data to determine the sticker's linear speed, angular speed, period, and radius.

## Introduction

Suppose an object moves counterclockwise along a circular path. Figure 5.1 shows an object at intervals of  $1/30$  s between the first image A and the last image G.

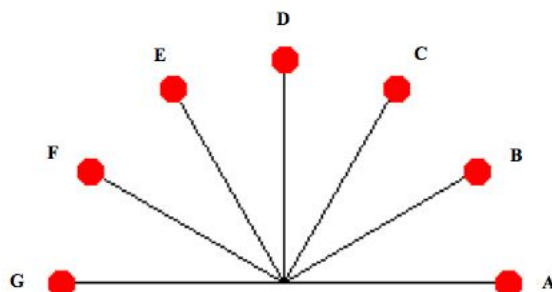


Figure 5.1: An object moves in a circular path.

Suppose that we define  $t = 0$  to occur at the first position of the ball. On the picture, label the time  $t$  for each subsequent position of the ball.

If you were to graph the x-position of the object as a function of time, what do you think the graph would look like? (No numbers are needed, just a qualitative sketch.)

The x-position of the object for each image is shown in Figure 5.2.

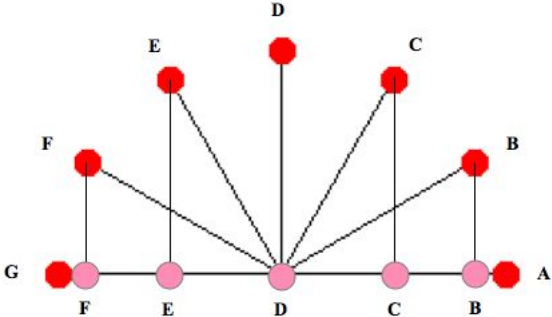


Figure 5.2: The x-position of an object moving in a circular path.

If you just look at the x-position of the object from A to G, what does its motion remind you of?

The radius of the circle is  $R$ . We can calculate the object's x and y position at any instant. Let's look at the object when it is at position C. The triangle showing the object, its x-position, and its y-position is shown below.

Using this angle, the x-position and y-position can be calculated as

$$\begin{aligned}
 x &= R \cos(\theta) \\
 y &= R \sin(\theta)
 \end{aligned}$$

For an object moving in circular motion with a constant speed, the angle  $\theta$  that the object makes with the  $+x$  axis changes at a constant rate. The rate that the angle changes is called the *angular speed*. To calculate angular speed, you measure how much it turns ( $\Delta\theta$ ) and divide by the time interval. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t}$$

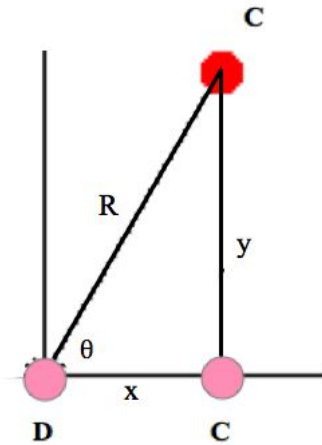


Figure 5.3: The x and y components of the object's position.

If  $\theta$  is the angle at any instant  $t$  and if  $\theta_0$  is the initial angle at  $t = 0$ , then

$$\omega = \frac{\theta - \theta_0}{t}$$

$$\theta = \omega t + \theta_0$$

Thus, the x-position and y-position of the object at the clock reading  $t$  is

$$x = R \cos(\omega t + \theta_0)$$

$$y = R \sin(\omega t + \theta_0)$$

As a result, the x-motion and y-motion each resemble simple harmonic motion.

The *linear speed* of the object is distance traveled per second. It's easiest to consider one complete rotation. The distance traveled around a circle is the *circumference*. The time for one revolution is the *period*. Thus, the linear speed of the object is

$$|\vec{v}| = \frac{2\pi R}{T}$$

The angular speed during one revolution is  $\omega = 2\pi/T$ . Therefore, we can write the linear speed as

$$v = \omega R$$

where  $\omega$  is in units of rad/s. It is typical to drop the magnitude symbol and vector symbol and write the speed  $|\vec{v}|$  more simply as  $v$ .

In Figure 1, the time interval between positions is  $1/30$  s, and the angle it turns, between successive positions, is  $30^\circ$ . What the object's angular speed in degrees per second?

Often, we use units of radians instead of degrees when measuring angles.  $180^\circ$  is  $\pi$  radians. What is the angular speed of the object in Figure 5.1 in radians per second?

## Procedure

1. Download the video `bicycle-wheel.mov`. This video was recorded at 300 fps, though it plays back at 30 fps and appears in slow motion.
2. Open *Tracker* and import the video.
3. Open the Clip Settings window by clicking on the **Clip Settings** icon (shown in Figure 5.4) which is part of the video control toolbar that is below the video clip.



Figure 5.4: The Clip Settings icon.

4. In the **Clip Settings** pop-up window, enter a frame rate of 300 fps, as shown in Figure 5.5

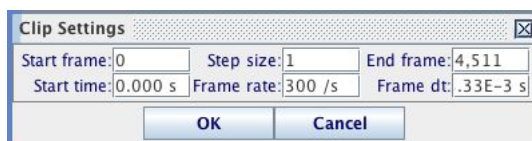


Figure 5.5: The Clip Settings icon.

5. Go to the first frame of the video and use the meterstick to calibrate distance in the video.
6. Advance to the next frame (frame number 001).
7. Place the origin at the axle (at the center of the white disk at the hub of the wheel in the video).
8. Since there are 300 frames of video recorded per second, the time interval between frames is quite small. As a result, we can skip frames between marking the ball and thus take fewer data points. Click on the **Step Size** button, as shown in Figure 5.6 and change it to **5**.



Figure 5.6: Change the step size in order to skip frames.

9. Click the  button in the toolbar and create a new point mass.
10. Make sure the video is on the first frame.
11. Hold the shift key down and click once on the green sticker that is on the bicycle tire. The video will then advance 5 frames. Continue to mark the green sticker for a total of 4 revolutions. If you are only displaying the last few marks, then it will look something like Figure 5.7.



Figure 5.7: Marks for the green sticker on the rotating bicycle wheel.

## Analysis

1. Right-click on the  $x$  vs.  $t$  graph and select **Analyze**.
2. Check the **Fit** checkbox.
3. In the curve fit pane, Click the , click the button , and select "Sinusoid." Click  to add a parameter  $d$  with value 0. Edit the Sinusoid1 function to be  $a * \cos(b * t + c) + d$ , as shown in 5.8. The additive constant  $d$  will shift the curve fit up and down by a constant as needed, though it should be 0.
4. The Autofit algorithm does not do well with sinusoidal curves. Thus, you will need to adjust each parameter manually. When you get these values fairly close, then you can do an Autofit. It helps to start with some reasonable values. So make some approximate guesses for the following functions based on the curve:



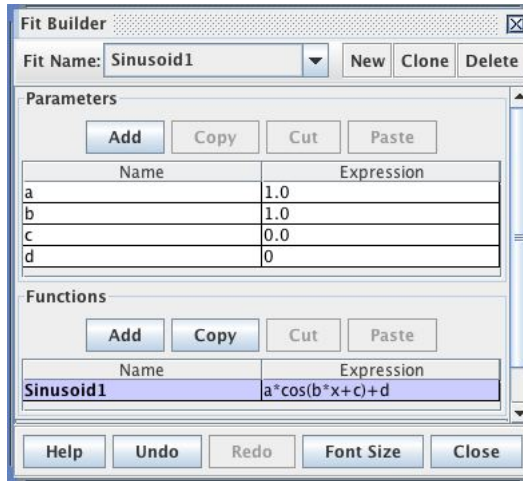


Figure 5.8: The Fit Builder is used to fit a curve to a user-defined function.

$d$  shifts the curve up or down. It should be zero (or close to zero).

$a$  is the maximum value on the curve. What is approximately the maximum value of  $a$ ? Enter this into the box for  $a$ .

$b$  is the angular velocity,  $2\pi/T$ . What is the approximate period of the oscillation? Use it to calculate an approximate value of  $b$ , and enter this value into the box.

$c$  is the phase. Once you set  $a$  and  $b$ , it will be easy to determine  $c$ . It shifts the curve right or left. You can adjust this manually without knowing the initial value.

If you begin with approximate values, you can click in the Parameter Values box and click the up and down arrows to make small adjustments. You can also change the step size as you hone in on the best value for the parameter. Adjust each parameter by very small amounts (steps of 0.1%) until you get the best curve-fit possible.

An example curve-fit is shown in Figure 5.9.

What are the curve parameters and the fit equation for the best-fit curve?

From the curve fit, determine the radius of the sticker  $R$ .

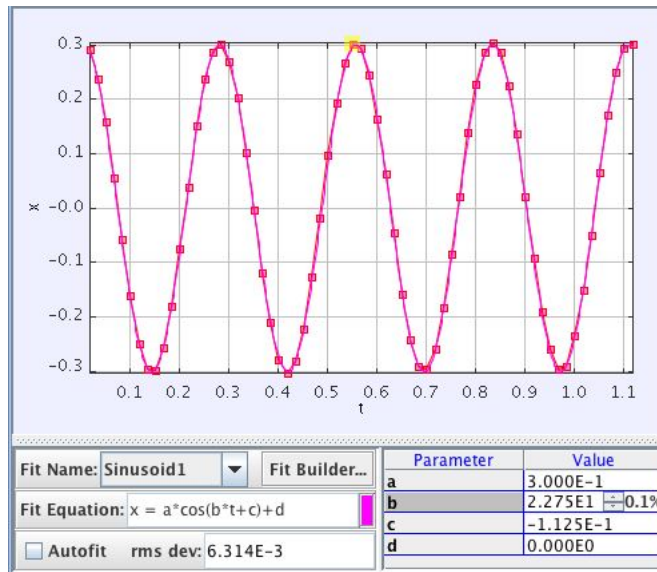


Figure 5.9: The best-fit curve for a point on the wheel.

How does your result compare to the measured radius of the sticker which is 30 cm?

From the curve fit, determine the angular velocity of the wheel  $\omega$ .

From the curve fit, determine the initial angle of the green sticker  $\theta_0$ .

What is the period  $T$  of the wheel?

What is the linear speed  $v$  of the green sticker?

5. Close the Data Tool window and return to the main Tracker window. Change the graph to plot  $y$  vs.  $t$ . Right-click the graph and select **Analyze**. Repeat the same procedure as before, but this time fit a curve to  $y = a * \sin(b * t + c) + d$ .

Record the values for the curve fit and compare to what you found when analyzing  $x(t)$ .

6. Close the Data Tool window and return to the main Tracker window. Change the graph to plot  $\theta$  (the rotation angle) as a function of  $t$ . Right-click the graph and select **Analyze**. Your graph of  $\theta$  vs.  $t$  will look like Figure 5.10.

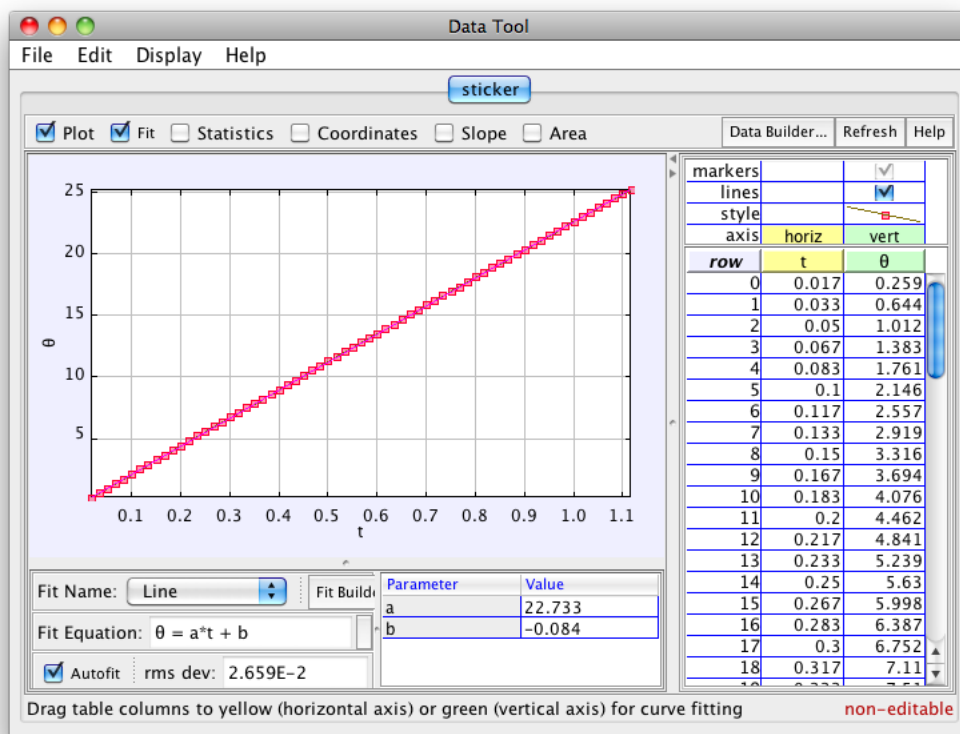


Figure 5.10:  $\theta$  vs.  $t$ .

Do a linear curve fit as shown in Figure 5.10. From this curve fit, record the angular velocity  $\omega$ .

$\omega =$

### Further Investigation

- (a) If the wheel is instead rotating clockwise, how would it affect the graph of  $\theta$  vs.  $t$ ? How would  $\omega$  be different?
- (b) In the video, you will notice a white sticker on a spoke that is at a radius of a little less than half the radius of the green sticker. If you measure the motion of this white sticker, which of these variables will be the same as that of the green sticker:  $R$ ,  $\omega$ ,  $\theta_0$ ,  $v$ ,  $T$ ? (There may be more than one.)



# 6 Conservation of Angular Momentum of a Spinning Figure Skater

## Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)  
video: `figure-skater.mov` from <http://physics.highpoint.edu/~atitus/videos/>

## Goal

In this experiment, you will measure the initial and final angular velocity of a figure skater doing a “scratch spin” and you will calculate her change in moment of inertia.

## Introduction

The “scratch spin” is a classic figure skating maneuver in which the skater draws her arms and leg in, which causes her angular speed to increase. This is the result of conservation of angular momentum. As the skater reduces her moment of inertia by pulling her arms and legs in, closer to the axis of rotation, her angular speed increases to order to maintain constant angular momentum.



Figure 6.1: A skater executing a scratch spin.

Newton’ second law for rotation (sometimes referred to as The Angular Momentum Principle) is

$$\tau_{net} = \frac{d\vec{L}}{dt}$$

where  $\tau_{net}$  is the net torque on the system about some point  $P$  and  $\vec{L}$  is the angular momentum of the

system about the same point. If the system is a rigid body rotating about an axis, then

$$\vec{L} = I\omega\vec{e}_a$$

where  $I$  is the moment of inertia of the body about the rotation axis and  $\vec{\omega}$  is the angular velocity about the same axis.

If the net torque on a system is zero, then its angular momentum is constant. This principle is called **Conservation of Angular Momentum**. For a figure skater spinning on the ice, the net torque on her is nearly zero, and her angular momentum is constant, and

$$\vec{L}_i = \vec{L}_f$$

If we define her rotation axis to be the  $z$ -axis, then

$$I_i\omega_i = I_f\omega_f$$

If she is spinning and she pulls her arms and leg inward, then her moment of inertia decreases. Since  $I\omega$  is constant, then this results in an increase in her angular velocity  $\omega$  and she rotates faster.

If she is spinning and she brings her arms and leg outward, then her moment of inertia increases. Since  $I\omega$  is constant, then this results in a decrease in her angular velocity  $\omega$  and she rotates slower.

## Procedure

The following instructions assume that you are familiar with using Tracker.

1. Import the video *figure-skater.mov* into Tracker.
2. Use the skater's height to calibrate distance in the video. Her height from the bottom of her skates to the top of her head is 1.7 m.
3. Set the frame step size to 5 in order to skip 4 frames between data points. You will still have plenty of data to analyze her motion.

**Note that this is still a lot of data. If you want to further reduce your data set, you can increase the step size to 10, though you'll sacrifice some precision and accuracy perhaps.**

4. Create a point mass and mark the skater's left hand. It is best to mark the base of her hand, closer to her wrist. When her hand disappears behind her body, do not skip the frame. Rather, guess at where the hand might be and mark it there.
5. Look at the graph of  $x(t)$ . You'll notice the oscillatory motion that is indicative of uniform circular motion; however, the data drifts "downward" on the graph in the  $-x$  direction. What I mean is that it doesn't oscillate about a constant  $x$  value.

Why does the data drift in the  $-x$  direction?

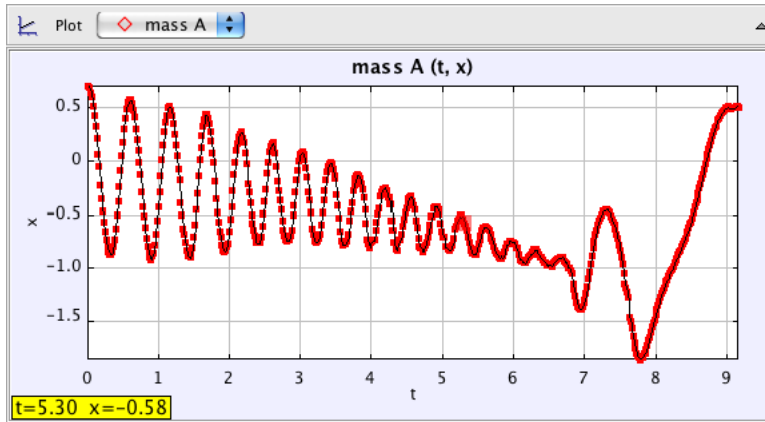


Figure 6.2: A graph of  $x(t)$  for the skater's left hand.

So that we can measure her rotation relative to her center of mass, we have to mark her center of mass. Make the approximation that her center of mass is at the center of her waist.

6. Create a new point mass and name it *center of mass*. Mark the center of her waist in all of the frames.
7. Look at the graph of  $x(t)$  for her center of mass. In what direction is she moving?

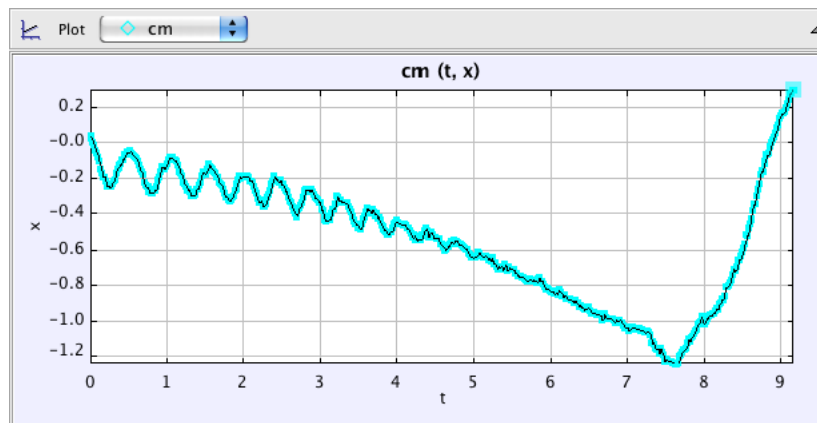


Figure 6.3: A graph of  $x(t)$  for the skater's center of mass.

We would like to analyze her rotational motion (relative to her center of mass). Thus, we must subtract off her center-of-mass motion. Tracker makes this very easy for us.

8. Go to the menu **Coordinate System**→**Reference Frame** and select “center of mass” in order to measure the motion of all objects relative to the center of mass.
9. Change the graph to show  $x(t)$  for her left hand.

Does her hand now rotate around  $x = 0$ ?



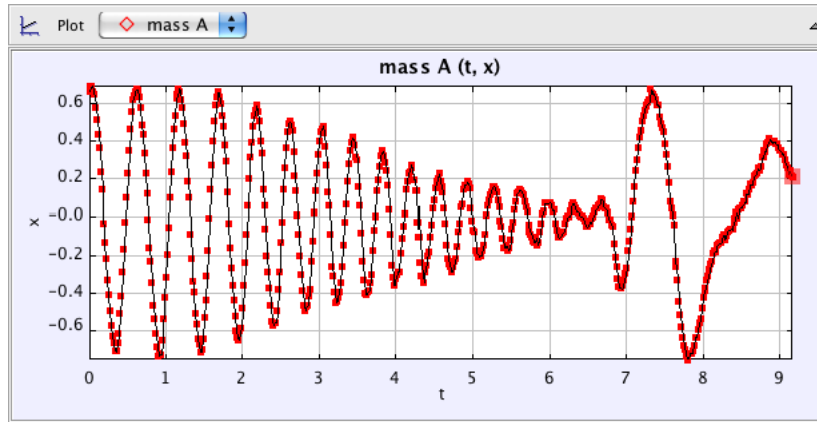


Figure 6.4: A graph of  $x(t)$  for the skater's left hand relative to her center of mass.

## Analysis

1. Right-click the graph of  $x(t)$  for the skater's left hand and select **Analyze...**
2. In the Data Tool, highlight one cycle of data when her arms are fully extended. Fit a sinusoidal curve to the data.

What is the resulting curve fit? From this equation, determine the radius of the circle traveled by her hand and her angular velocity  $\omega$

3. Highlight one cycle of data when her arms are over her head. Fit a sinusoidal curve to the data.

What is the resulting curve fit? From this equation, determine the radius of the circle traveled by her hand and her angular velocity  $\omega$

4. What is the ratio  $\frac{\omega_f}{\omega_i}$ ?
5. What is the ratio  $\frac{I_f}{I_i}$ ?