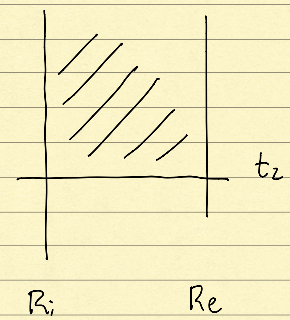
Metodo per la determinazione di b_w con misure di massa nella fase lineare

Pb.mo stadio 3

$$\begin{cases} \dfrac{\partial c}{\partial t} - D_w\left(\dfrac{\partial^2 c}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial c}{\partial r}\right) = 0 \\ c(r, t_2) = \text{dato dalla fase 2} \\ c(R_i, t) = c_s(T_i) \\ \dfrac{\partial c}{\partial r}(R_e, t) = 0 \end{cases}$$



$$c = c_s(T_i) + c_s(T_i)\tilde{c}$$

$$r = R_i \tilde{r}$$

$$t = \left(\frac{R_i^2}{D_w}\right)\tilde{t}$$

$$\Rightarrow \begin{cases} \dfrac{\partial \tilde{c}}{\partial \tilde{t}} - \left(\dfrac{\partial^2 \tilde{c}}{\partial \tilde{r}^2} + \dfrac{1}{\tilde{r}}\dfrac{\partial \tilde{c}}{\partial \tilde{r}}\right) = 0 \\ \tilde{c}(\tilde{r}, \tilde{t}_2) = \text{dato iniziale} \\ \tilde{c}(1, \tilde{t}) = 0 \\ \dfrac{\partial \tilde{c}}{\partial \tilde{r}}\left(\dfrac{R_e}{R_i}, \tilde{t}\right) = 0 \end{cases}$$

Cerco una soluzione a variabili separabili (ometto i ~)

$$c(r,t) = U(r)\, e^{-\alpha^2 (t - t_2)} \qquad c_r = U'(r)\, e^{-\alpha^2 (t - t_2)}$$

$$c_t = -\alpha^2 U(r)\, e^{-\alpha^2 (t - t_2)} \qquad c_{rr} = U''(r)\, e^{-\alpha^2 (t - t_2)}$$

$$\Rightarrow \quad c \text{ soddisfa} \quad \boxed{\begin{cases} -\alpha^2 U(r) + U''(r) + \dfrac{1}{r} U'(r) = 0 \quad \text{(Eq.ne di Bessel)} \\ U(1) = 0 \\ U'\left(R_e/R_i\right) = 0 \end{cases}}$$

La soluzione è fatta così

$$U(r;\alpha) = Y_0(\alpha) J_0(r\alpha) - J_0(\alpha) Y_0(r\alpha)$$

J_0, Y_0 sono funzioni di Bessel di ordine 0 del 1° e 2° tipo

$$J_0(z) = 1 - \frac{\left(\frac{z^2}{4!}\right)}{(1!)^2} + \frac{\left(\frac{z^2}{4!}\right)^2}{(2!)^2} - \frac{\left(\frac{z^2}{4!}\right)^3}{(3!)^2} + \dots$$

$$Y_0(z) = \frac{2}{\pi}\left[\ln\left(z/2\right) + \gamma\right] J_0(z) - \left(1 + \frac{1}{2}\right)\frac{\left(\frac{z^2}{4!}\right)^2}{(2!)^2} + \left(1 + \frac{1}{2} + \frac{1}{3}\right)\frac{\left(\frac{z^2}{4!}\right)^3}{(3!)^2} + \dots$$

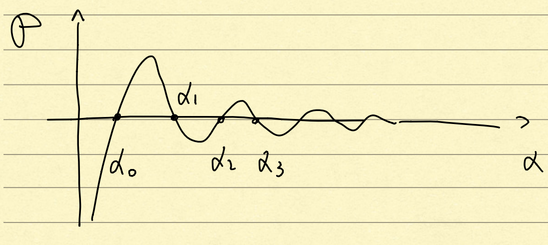
γ costante eulero $= \lim_{n \to +\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n} - \log n\right) \simeq 0.577$

$U(1) = 0$ è automaticamente soddisfatta

$$U'(r;\alpha) = \alpha Y_0(\alpha) J_0'(r\alpha) - \alpha Y_0'(r\alpha) J_0(\alpha)$$

$$U'\left(R_e/R_i ; \alpha\right) = 0 \quad \Rightarrow \quad \underbrace{Y_0(\alpha) J_0'\left(\frac{R_e}{R_i}\alpha\right) - J_0(\alpha) Y_0'\left(\frac{R_e}{R_i}\alpha\right)}_{P(\alpha)} = 0$$

Devo risolvere $P(\alpha) = 0$



Ci sono uno ∞ numerabile di soluzioni

$$U(r;\alpha_j)\, e^{-\alpha_j^2 (t - t_2)}$$

$$\boxed{C(r,t) = \sum_{j=0}^{\infty} U(r;\alpha_j)\, e^{-\alpha_j^2 (t-t_2)}}$$

Il termine dominante della serie per $t \gg t_2$

Per $t \gg t_2$ $\quad C(r,t) \simeq U(r;\alpha_0)\, e^{-\alpha_0^2 (t-t_2)}$

Per $t \gg t_2$ il fattore di decadimento (dimensionale) è

$$\exp\left\{-\frac{(t-t_2)\alpha_0^2}{\left(R_i^2/D_w\right)}\right\}$$

Posso scrivere la formula per la crescita di massa in questo modo

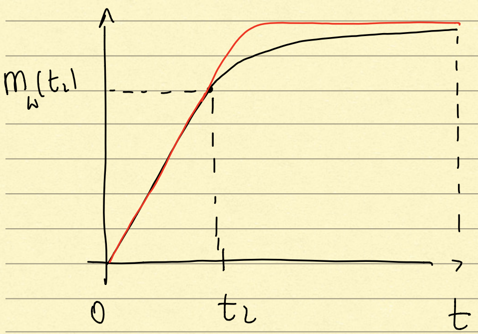
$t \gg t_2$

$$m_w(t) = m_w(t_2) + \underbrace{\frac{R_i^2}{D_w \alpha_0^2}}_{=\tau} \dot{m}_w(t_2)\left[1 - \exp\left\{-\frac{(t-t_2)}{\underbrace{\left(R_i^2/D_w\alpha_0^2\right)}_{=\tau}}\right\}\right]$$

questa formula approssima la "VERA FORMULA" di crescita della massa

$$m_w(t) = m_w(t_2) + \int_{t_2}^{t} D_w \frac{\partial c}{\partial r}(R_i,\tau)\,d\tau$$

(1) $$\boxed{m_w(t) = m_w(t_2) + \tau\, \dot{m}_w(t_2)\left[1 - \exp\left\{-\frac{(t-t_2)}{\tau}\right\}\right]}$$



" la formula approx è scelta così perché la fase lineare e asintotica si "raccordano con continuità C^1 "

$$m_w(t_2) = m_w(t_2)$$

$$\dot{m}_w(t) = \cancel{\tau}\,\dot{m}_w(t_2)\cdot \exp\left\{-\frac{t-t_2}{\tau}\right\}\cdot\frac{1}{\cancel{\tau}} \Longrightarrow \dot{m}_w(t_2) = \dot{m}_w(t_2)$$

Tramite la 1 ho che $\tau = \dfrac{R_i^2}{\alpha_0^2 D_w}$

$$\underbrace{M_w}(T_i) = m_w(t_2;T_i) + \tau\,\dot{m}_w(t_2) = m_w(t_2;T_i) + \frac{R_i^2}{\alpha_0^2 D_w}$$

massa asintotica
$t \to +\infty$

$$\dot{m}_w(t_2) = D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)}\frac{1}{R_i}$$

$$M_w(T_i) = m_w(t_2;T_i) + \frac{R_i^{\cancel{2}}}{\alpha_0^2 \cancel{D_w}}\cdot \cancel{D_w}\, b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)}\frac{1}{\cancel{R_i}}$$

$$M_w(T_i) = \left[C^*_{tot} - C_s(T_i)\right]\frac{(R_e^2 - R_i^2)\,\cancel{\pi H}}{2R_i\,\cancel{\pi H}} \qquad C^*_{tot} = C_s(T_{cloud})$$

$$M_w(T_i) = b_w(T_{cloud} - T_i)\,\frac{R_e^2 - R_i^2}{2R_i}$$

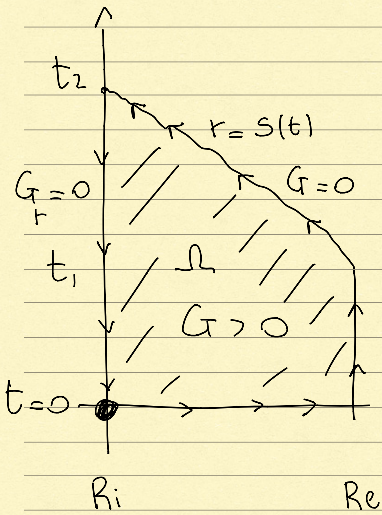
$$b_w(T_{cloud} - T_{i1})\,\frac{R_e^2 - R_i^2}{2R_i} = m_w(t_2; T_{i1}) + \frac{R_i\, b_w(T_e - T_{i1})}{\alpha_0^2 \ln(R_e/R_i)} \qquad (*)$$

$$b_w(T_{cloud} - T_{i2})\,\frac{R_e^2 - R_i^2}{2R_i} = m_w(t_2; T_{i2}) + \frac{R_i\, b_w(T_e - T_{i2})}{\alpha_0^2 \ln(R_e/R_i)} \qquad (**)$$

$$b_w(T_{i2} - T_{i1})\,\frac{R_e^2 - R_i^2}{2R_i} = m_w(t_2; T_{i1}) - m_w(t_2; T_{i2}) + b_w\,\frac{(T_{i2} - T_{i1})\, R_i}{\alpha_0^2 \ln(R_e/R_i)}$$

$$b_w = \frac{m_w(t_2; T_{i1}) - m_w(t_2; T_{i2})}{(T_{i2} - T_{i1})} \cdot \left[\frac{R_e^2 - R_i^2}{2R_i} - \frac{R_i}{\alpha_0^2 \ln(R_e/R_i)} \right]^{-1}$$

STIME (dal basso e dall'alto di D_w coeff. di diffusività)



In Ω $\quad G(r,t)$ (conc. cero segregato)

soddisfa

$$\frac{\partial G}{\partial t} - D_G\left(\frac{\partial^2 G}{\partial r^2} + \frac{1}{r}\frac{\partial G}{\partial r}\right) = 0$$

$$\frac{\partial G}{\partial t} - \frac{D_G}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right) = 0$$

$$\iint_\Omega \underbrace{\left[\frac{\partial}{\partial t}(rG) - \frac{\partial}{\partial r}\left(D_G r\frac{\partial G}{\partial r}\right)\right]}_{=0} dr\,dt = 0$$

Per le formule di Gauss-Green

$$\oint_{\partial\Omega} (rG)\,dr + \left(D_G r\frac{\partial G}{\partial r}\right)dt = 0$$

$$\int_{R_i}^{R_e} r G_0(r)\,dr + \underbrace{\int_0^{t_1} D_G R_e \frac{\partial G}{\partial r}(R_e,t)\,dt} + \int_{t_1}^{t_2} D_G s\frac{\partial G}{\partial r}(s,t)\,dt = 0$$

$$D_G\frac{\partial G}{\partial r}(R_e,t) = -D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)}\frac{1}{R_e} \qquad \text{su } r = R_e$$

$$D_G \frac{\partial G}{\partial r}(s,t) + D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)} \frac{1}{s} = D_w \frac{\partial c}{\partial r}(s,t) \quad \text{su } r = s$$

$$\int_{R_i}^{R_e} r G_0(r)\,dr - \int_0^{t_1} D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)} dt + \int_{t_1}^{t_2} D_w s \frac{\partial c}{\partial r}(s,t) - \int_{t_1}^{t_2} D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)} dt = 0$$

$$\int_{R_i}^{R_e} r G_0(r)\,dr = \int_0^{t_2} D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)} dt - \underbrace{\int_{t_1}^{t_2} D_w s \frac{\partial c}{\partial r}(s,t)}_{>0} < \underbrace{\int_0^{t_2} D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)} dt}$$

$$D_w b_w \frac{(T_e - T_i)\, t_2}{\ln(R_e/R_i)} > \int_{R_i}^{R_e} r G_0(r)\,dr$$

$$D_w > \frac{\int_{R_i}^{R_e} r G_0(r)\,dr}{b_w (T_e - T_i)} \cdot \ln(R_e/R_i)$$ Stima dal basso

Stima dall'alto $\mathcal{M}$ (massa totale di deposito di cera) $[\mathcal{M}] = kg$

$$\mathcal{M}(t_2) = D_w b_w \frac{(T_e - T_i)}{\ln(R_e/R_i)\, R_i} \frac{t_2}{\,} \frac{2\pi R_i H}{1} \rightarrow \text{sup. del CF}$$

$$m_{in} = (\text{massa totale di cera al tempo } t=0 \text{ nell'olio}) = C^*_{tot} \cdot \underbrace{\pi H (R_e^2 - R_i^2)}_{\text{volume CF}}$$

$$m_{in} > m(t_L)$$

$$C^*_{tot}\, \cancel{\pi}\cancel{H}(R_e^2 - R_i^2) > D_w b_w \cdot \frac{(T_e - T_i)}{\cancel{R_i}\ln(R_e/R_i)}\, 2\cancel{\pi}\cancel{R_i}\cancel{H}$$

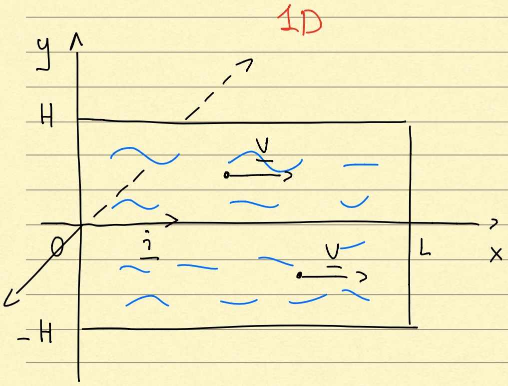
$$D_w < \frac{C^*_{tot}(R_e^2 - R_i^2)}{b_w(T_e - T_i)\,2t_L}\ln(R_e/R_i)$$

FLUSSO FLUIDO NEWTONIANO in un CANALE

$$\begin{cases} \operatorname{div} \underline{v} = 0 \\ \rho\left(\frac{\partial \underline{v}}{\partial t} + \underbrace{(\nabla \underline{v})\underline{v}}_{\substack{=0 \\ 1D}}\right) = -\nabla p + \mu \Delta \underline{v} + \rho \underline{f} \end{cases} \quad (NS)$$

NAVIER STOKES

(incomprimibile $\rho = cost$



Moto 1D (trascuro z

$\underline{v} = v(y,t)\,\underline{i}$

suppongo questo campo di velocità

$\text{div}\, \underline{v} = \frac{\partial v}{\partial x} = 0$ (automat. soddisfatta)

$\bar{\nabla} \underline{v} = \begin{pmatrix} 0 & v_y \\ 0 & 0 \end{pmatrix}$ $\qquad (\bar{\nabla}\underline{v})\underline{v} = \begin{pmatrix} 0 & v_y \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ 0 \end{pmatrix} = \underline{0}$

$$\begin{cases} \rho \cancel{\frac{\partial v}{\partial t}} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v}{\partial y^2} + \cancel{\rho f_x} \\ 0 = -\frac{\partial p}{\partial y} \end{cases}$$

Cerco la soluz. stazionaria impongo le cond. al contorno

$v(\pm H) = 0$ $\quad \Big| \quad \begin{matrix} v(H) = 0 \\ v_y(0) = 0 \end{matrix}$ SIMMETRIA

$p_y = 0 \implies p = p(x) \implies p_x = \text{Costante} = A$

$p = Ax + B$ Impongo la pressione in $x = 0$ e $x = L$

p_{in} $\qquad p_{out}$

$$\begin{cases} B = p_{in} \\ AL + p_{in} = p_{out} \end{cases}$$

$\Delta P = p_{in} - p_{out} > 0$

$A = -\left(\frac{\Delta P}{L}\right) \implies p_x = -\frac{\Delta p}{L} < 0$

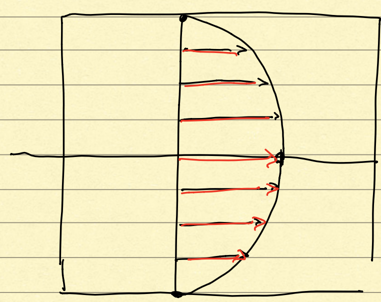
$$\begin{cases} \mu V_{yy} = p_x = -\left(\Delta p / L\right) \\ V(H) = 0 \quad (\text{No\_slip}) \\ V_y(0) = 0 \end{cases} \quad \rightsquigarrow \quad \mu V_y = -\frac{\Delta p}{L} y$$

$$\mu V = -\frac{\Delta p}{L}\frac{y^2}{2} + C_1$$

Impongo $V(H) = 0 \Rightarrow 0 = -\frac{\Delta p}{2L} H^2 + C_1$

$$\mu V = -\frac{\Delta p}{L}\frac{y^2}{2} + \frac{\Delta p}{2L} H^2$$

$$V(y) = \frac{\Delta p}{2L\mu}\left(H^2 - y^2\right)$$



$$\mathbb{T} = -p\mathbb{I} + \mu\left(\nabla \underline{v} + \nabla \underline{v}^T\right)$$

$T_{12} = \mu \frac{\partial V}{\partial y}$ (verificare)

T_{12} è massimo alla parete $y = \pm H$

T_{12} è 0 nel centro $y = 0$

Stress di taglio (shear)