Laboratorio di Fisica Atomica CdL Fisica e Astrofisica

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Programma Lezioni

- 4 lezioni
  - Fasci Gaussiani — Mer. 6 Novembre

    Soluzione eq. Maxwell parassiale, principali proprietà, propagazione, formulazione matriciale
  - Polarizzazione onde e.m. — Mer. 13 Novembre

    Stati di polarizzazione, rappresentazione con vettore di Jones, birifrangenza ed ottiche polarizzanti (lamine di ritardo, polarizzatori)
  - Riflessione e rifrazione — Mer. 20 Novembre

    Applicazioni eq. di Fresnel, riflessione totale, dispersione, trattamenti (coatings) AR, HR, cubi
  - Propagazione guidata ed elementi elettro-ottici — Mer. 27 Novembre

    Fibre ottiche, Acusto-ottici, elettro-ottici

Fasci Gaussiani

- Testi/articoli di riferimento:

  - O. Svelto «Principles of Lasers»

  - A. E. Siegman «Lasers»

  - C.D. Davis «Lasers and Electro-Optics»

  - H. Kogelnik, T. Li «Laser beams and resonators», Applied Optics 5 (10), 1550 (1966)

Fasci Gaussiani

- Ottima soluzione approssimata eq. Maxwell per rappresentazione di fasci laser reali

- Eq. Maxwell nel vuoto

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{\jmath} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\rho = 0$$

$$\vec{\jmath} = 0$$

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

- Eq. delle onde

$$\Delta \vec{E} \equiv \nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

Fasci Gaussiani

- Eq. delle onde

$$\Delta \vec{E} \equiv \nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

- Polarizzazione lineare
- Rappresentazione complessa del campo (Ē->E)

$$E = \mathrm{Re}\left[\tilde{E}\right] = \frac{1}{2}\left(\tilde{E} + \tilde{E}^*\right)$$

$$E(\vec{r},t) = E(\vec{r})\exp[i\omega t]$$

- Eq. Di Helmholtz

$$\left(\nabla^2 + k^2\right) E(\vec{r}) = 0$$

$$k \equiv \frac{\omega}{c}$$

Onde piane

$$E(\vec{r},t) = E_0 \exp[\, i\,(\omega\, t - \vec{k}\cdot\vec{r})]$$

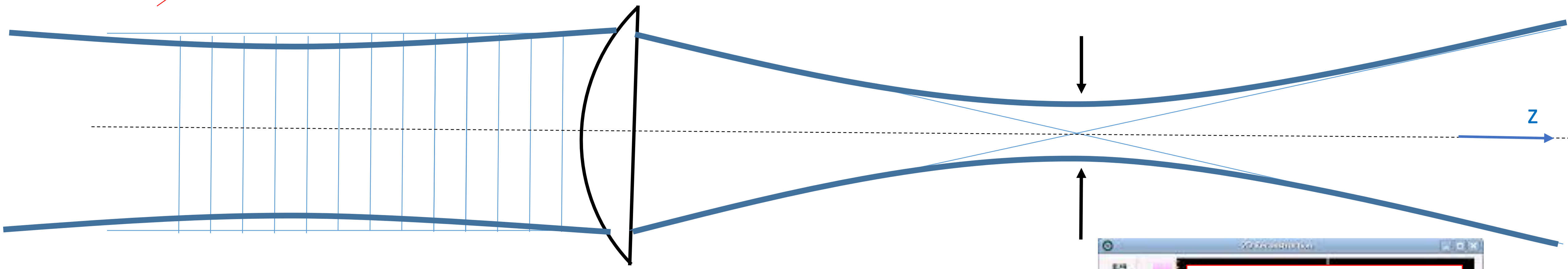
Onde sferiche

$$E(\vec{r},t) = \frac{E_0}{r}\exp[\, i\,(\omega\, t - k\, r)]$$

Onde piane/sferiche vs. fascio parassiale

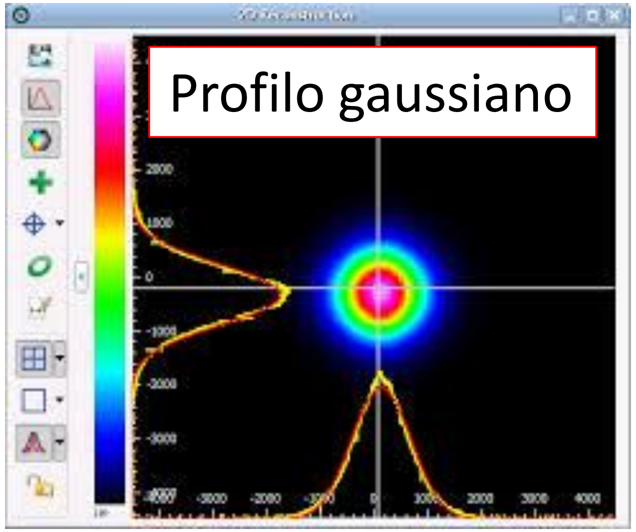
$$E(\vec{r},t) = E_0 \exp[\, i\,(\omega\, t - \vec{k}\cdot\vec{r})]$$

$$E(\vec{r},t) = \frac{E_0}{r}\exp[\, i\,(\omega\, t - k\, r)]$$



Soluzione particolare (onda *parassiale*):

- Onda confinata principalmente attorno ad un asse (asse di propagazione z) con piccole variazioni di ampiezza in dir. radiale.
- Onda piana lungo l'asse z con ampiezza dipendente solo da x e y



Fasci Gaussiani

- Eq. di Helmholtz

$$\left(\nabla^2 + k^2\right) E(\vec{r}) = 0$$

$$E(\vec{r}, t) = E(\vec{r}) \exp[i\omega t]$$

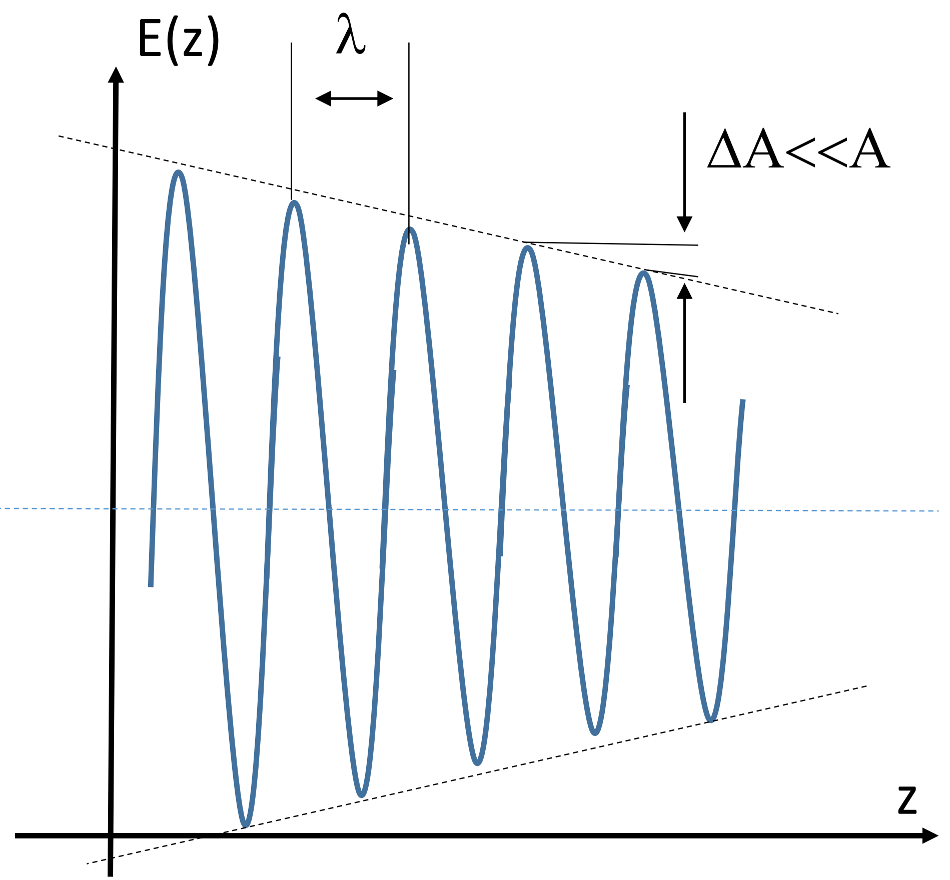
$$E(\vec{r}) = A(\vec{r}) \exp[-i\, k\, z\,]$$

Approx. Parassiale (propagazione confinata lungo z):

- Su scala $\Delta z \approx \lambda$ variazione lenta A(r):
  - $|\Delta A| \ll |A|$
  - $\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|\frac{\partial^2 A}{\partial x^2}\right|$
  - $\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|\frac{\partial^2 A}{\partial y^2}\right|$

$$\left|\frac{\partial A}{\partial z}\right| \ll k|A|$$

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll k\left|\frac{\partial A}{\partial z}\right|$$

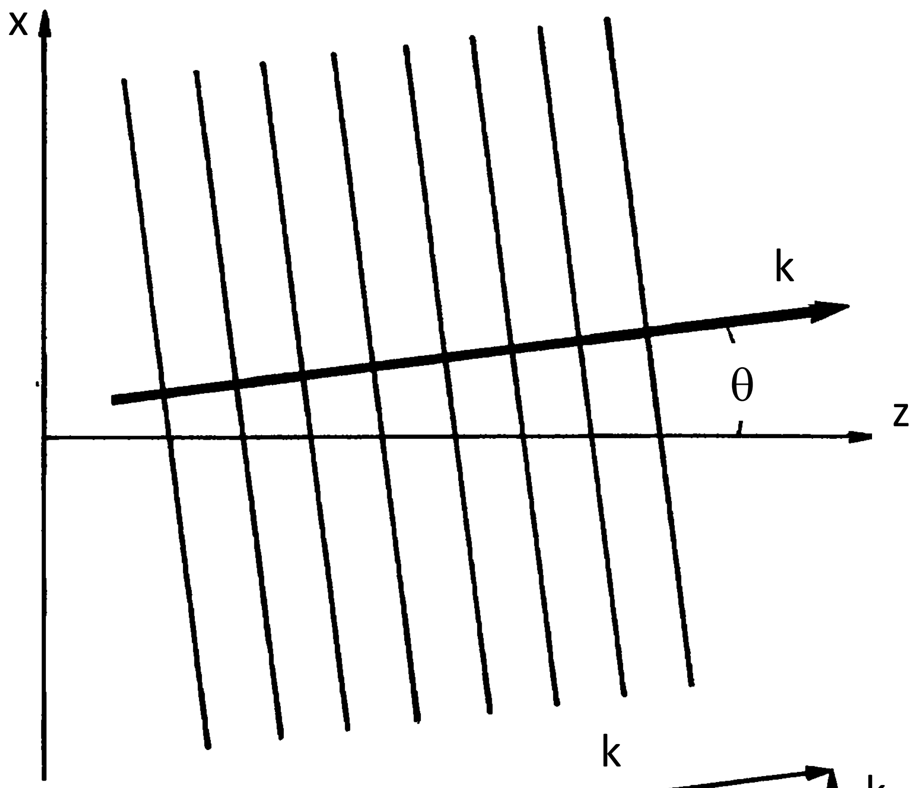


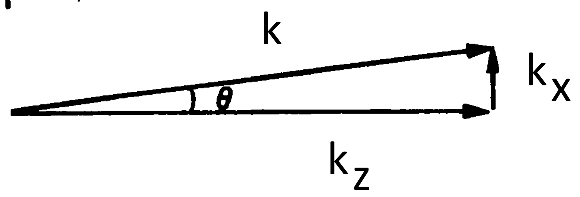
Fasci Gaussiani

- Validità approx. parassiale

$$\tilde{E}(x,z) = \exp[-jkx\sin\theta - jkz\cos\theta] = \tilde{u}(x,z)e^{-jkz}$$

$$\tilde{u}(x,z) = \exp[-jkx\sin\theta - jkz(1-\cos\theta)] \approx \exp\left[-jk\theta x + jk\frac{\theta^2 z}{2}\right]$$





$$-j\frac{2k}{\tilde{u}}\frac{\partial\tilde{u}}{\partial z} = +2k^2(1-\cos\theta) \approx k^2\theta^2 \quad \rightarrow \quad \frac{1}{\tilde{u}}\frac{\partial^2\tilde{u}}{\partial z^2} = -k^2(1-\cos\theta)^2 \approx -\frac{k^2\theta^4}{4}$$

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|\frac{\partial^2 A}{\partial x^2}\right| \qquad \theta^2/4 \ll 1 \quad \Rightarrow \quad \begin{array}{l} \theta \leq 0.5\ rad \\ \theta \leq 30^\circ \end{array}$$

Fasci Gaussiani

Approx. Parassiale

$$\left|\frac{\partial A}{\partial z}\right| \ll k|A|$$

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll k\left|\frac{\partial A}{\partial z}\right|$$

- Eq. di Helmholtz

$$\left(\nabla^2 + k^2\right) E(\vec{r}) = 0$$

$$\begin{aligned}
0 &= \left(\nabla^2 + k^2\right) A(\vec{r})\boldsymbol{e}^{-i\,k\,z} \\
&= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) A(\vec{r})\boldsymbol{e}^{-i\,k\,z} \\
&= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) A(\vec{r})\boldsymbol{e}^{-i\,k\,z} + \frac{\partial}{\partial z}\left(\frac{\partial A(\vec{r})}{\partial z}\boldsymbol{e}^{-i\,k\,z} - i\,k A(\vec{r})\boldsymbol{e}^{-i\,k\,z}\right) + k^2 A(\vec{r})\boldsymbol{e}^{-i\,k\,z} \\
&= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) A(\vec{r})\boldsymbol{e}^{-i\,k\,z} + \underbrace{\frac{\partial^2 A(\vec{r})}{\partial z^2}\boldsymbol{e}^{-i\,k\,z}}_{\text{neglect this term}} - 2\,i\,k\frac{\partial A(\vec{r})}{\partial z}\boldsymbol{e}^{-i\,k\,z} + (-k^2 + k^2)A(\vec{r})\boldsymbol{e}^{-i\,k\,z}
\end{aligned}$$

$$\approx \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) A(\vec{r})\boldsymbol{e}^{-i\,k\,z} - 2\,i\,k\frac{\partial A(\vec{r})}{\partial z}\boldsymbol{e}^{-i\,k\,z}$$

Fasci Gaussiani

- Eq. parassiale per ampiezza A(**r**)

$$\left(\nabla_T^2 - 2\,i\,k\frac{\partial}{\partial z}\right) A(\vec{r}) \approx 0$$

$$\nabla_T^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Soluzione

$$A(\vec{r}) = A(\rho, z) = \frac{A_1}{q(z)} \exp\left[-ik\frac{\rho^2}{2q(z)}\right]$$

$$\rho^2 \equiv x^2 + y^2$$

$$q(z) \equiv z + i\,z_0$$

Raggio di curvatura complesso

z_0 Lunghezza di Rayleigh

$$E(\rho, z, t) = \frac{A_1}{z + iz_0} \exp\left[-ik\frac{\rho^2}{2(z + iz_0)}\right] \exp\left[i(\omega t - kz)\right]$$

Fasci Gaussiani

$$E(\rho, z, t) = \frac{A_1}{z + iz_0} \exp\left[-ik\frac{\rho^2}{2(z + iz_0)}\right] \exp\left[i(\omega t - kz)\right]$$

Nell'origine $z = 0$

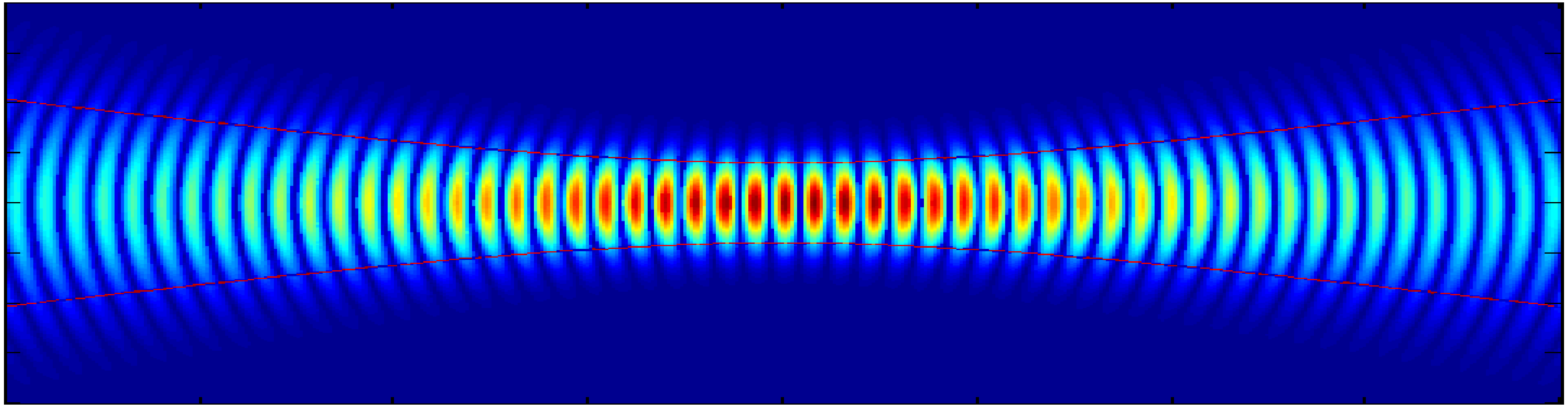
$$E(\rho, z, t) = \frac{A_1}{iz_0} \exp\left[-k\frac{\rho^2}{2z_0}\right] \exp\left[i\omega t\right]$$

Onda Piana!!

A grande distanza $z \gg z_0$

$$E(\rho, z, t) = \frac{A_1}{z} \exp\left[-ik\frac{\rho^2}{2z}\right] \exp\left[i(\omega t - kz)\right]$$

Onda sferica !!



Fasci Gaussiani - definizioni

Definiamo due nuove funzioni reali:

r(z) raggio di curvatura
w(z) dim. radiale del fascio (spot size - **raggio 1/e²**)

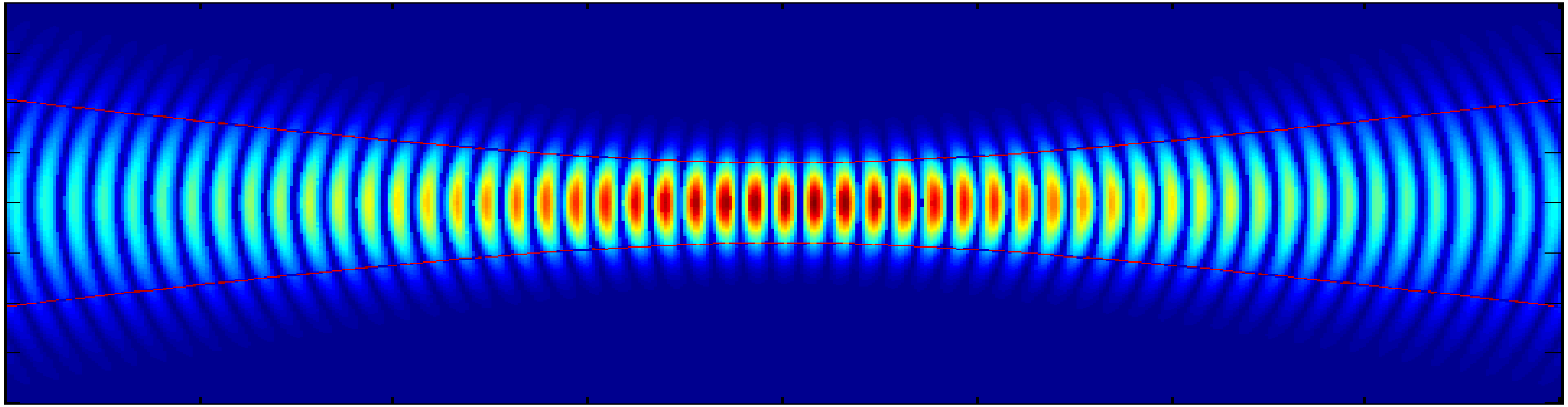
$$\frac{1}{q(z)} \equiv \frac{1}{r(z)} - i\frac{\lambda}{\pi\, w(z)^2}$$

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$r(z) = z\left(1 + \left(\frac{z_0}{z}\right)^2\right)$$

dove

$$w_0^2 \equiv \frac{\lambda\, z_0}{\pi}$$



Fasci Gaussiani - conti

Patendo dalla definizione $$\frac{1}{q(z)} \equiv \frac{1}{r(z)} - i\frac{\lambda}{\pi\, w(z)^2}$$ dove $$\frac{1}{q(z)} = \frac{1}{z + i z_0} = \frac{z - i z_0}{z^2 + z_0^2}$$

$$w(q)^2 = -\frac{\lambda}{\pi\,\mathrm{Im}\big[1/q(z)\big]} = \frac{\lambda\,\big|q(z)\big|^2}{\pi\,\mathrm{Im}\big[q(z)\big]}\ ,$$

$$w(q) = \sqrt{\frac{\lambda}{\pi}\frac{\big|q(z)\big|^2}{\mathrm{Im}\big[q(z)\big]}}$$

$$r(q) = \frac{1}{\mathrm{Re}\big[1/q(z)\big]} = \frac{\big|q(z)\big|^2}{\mathrm{Re}\big[q(z)\big]}\ .$$

Fasci Gaussiani – conti2

Patendo dalla definizione $$\frac{1}{q(z)} \equiv \frac{1}{r(z)} - i\frac{\lambda}{\pi\, w(z)^2}$$ dove $$\frac{1}{q(z)} = \frac{1}{z + i z_0} = \frac{z - i z_0}{z^2 + z_0^2}$$

$$w(z) = \sqrt{\frac{\lambda}{\pi}\frac{\left|q(z)\right|^2}{\mathrm{Im}\left[q(z)\right]}} = \sqrt{\frac{\lambda\,(z^2 + z_0^2)}{\pi\, z_0}} = \sqrt{\frac{\lambda\, z_0}{\pi}}\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$= w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

dove $$w_0^2 \equiv \frac{\lambda\, z_0}{\pi}$$

$$r(z) = \frac{\left|q(z)\right|^2}{\mathrm{Re}\left[q(z)\right]} = \frac{z^2 + z_0^2}{z} = z\left(1 + \left(\frac{z_0}{z}\right)^2\right).$$

→

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$r(z) = z\left(1 + \left(\frac{z_0}{z}\right)^2\right)$$

Fasci Gaussiani – fase di Guoy

$$E(\rho, z, t) = \frac{A_1}{z + iz_0} \exp\left[-ik\frac{\rho^2}{2(z + iz_0)}\right] \exp\left[i(\omega t - kz)\right]$$

$$\frac{i}{q(z)} = \left|\frac{i}{q(z)}\right| e^{i\zeta(z)} = \frac{1}{|z + i\, z_0|} e^{i\zeta(z)}$$

$$= \frac{1}{z_0\sqrt{1 + (\frac{z}{z_0})^2}} e^{i\zeta(z)} = \frac{w_0}{z_0\, w(z)} e^{i\zeta(z)} \; .$$

dove $\tan[\zeta(z)] \equiv \frac{z}{z_0}$

fase di Guoy

Fasci Gaussiani - definizioni

$$\frac{1}{q(z)} \equiv \frac{1}{r(z)} - i\frac{\lambda}{\pi\, w(z)^2}$$

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$$

$$r(z) = z\left(1+\left(\frac{z_0}{z}\right)^2\right)$$

dove $w_0^2 \equiv \frac{\lambda\, z_0}{\pi}$

$$E(\vec{r}) = A_0\frac{w_0}{w(z)}\exp\left[-\frac{\rho^2}{w(z)^2}\right]\exp\left[i\left(-k\,z - k\,\frac{\rho^2}{2r(z)} + \zeta(z)\right)\right]$$

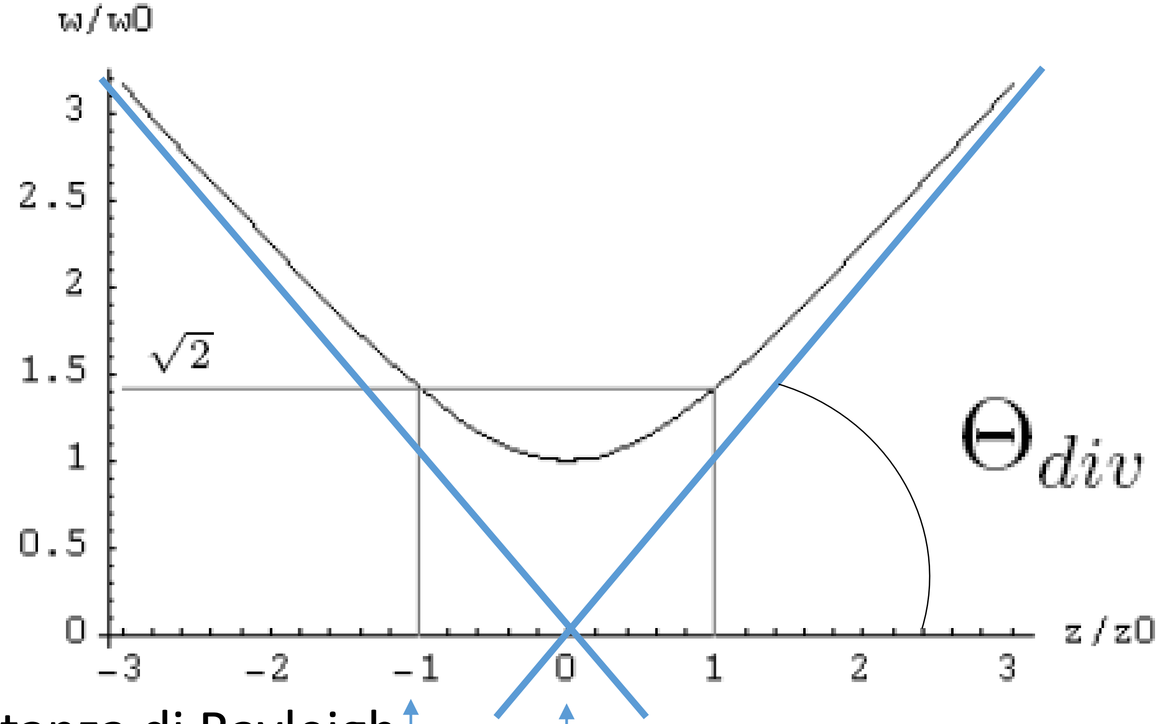
con $A_0 \equiv A_1/(iz_0)$

$$\zeta(z) = \arctan\left[\frac{z}{z_0}\right]$$

Fasci Gaussiani - definizioni

- w(z) dim. radiale del fascio

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$



Distanza di Rayleigh
(Rayleigh range)

Beam waist (z=0)

- Nel limite $z \gg z_0$

$$w(z) \approx \frac{\omega_0}{z_0} z = \Theta_{div} z$$

$$w_0^2 \equiv \frac{\lambda\, z_0}{\pi}$$

$$\Theta_{div} = \frac{\lambda}{\pi\, w_0}$$

Fasci Gaussiani - Esempi

- Dimensioni fascio laser su luna

- frequency doubled Nd:YAG laser (λ=532nm)
- w_0=10 cm
- z = 3.84×10^8 m

$$z_0 = \frac{\pi\,\omega_0^2}{\lambda} = 5.9\times10^4\,\mathrm{m}$$

$$\Theta_{div} = \frac{\lambda}{\pi\,w_0}$$

$$\theta_{div} = 1.7\,\mu\mathrm{rad}$$

$$w(z) \approx \frac{\omega_0}{z_0} z = \Theta_{div} z$$

$$r_{luna} = \omega(z) \approx 650\,\mathrm{m}$$

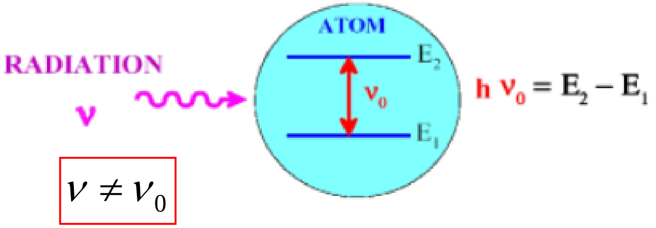
$$d_{luna} = 1.3\,\mathrm{km}$$

Fasci Gaussiani - Esempi

- Intrappolamento ottico

$$U(r, z) = -\vec{p} \cdot \vec{E}(r,z) \propto I(r,z)$$

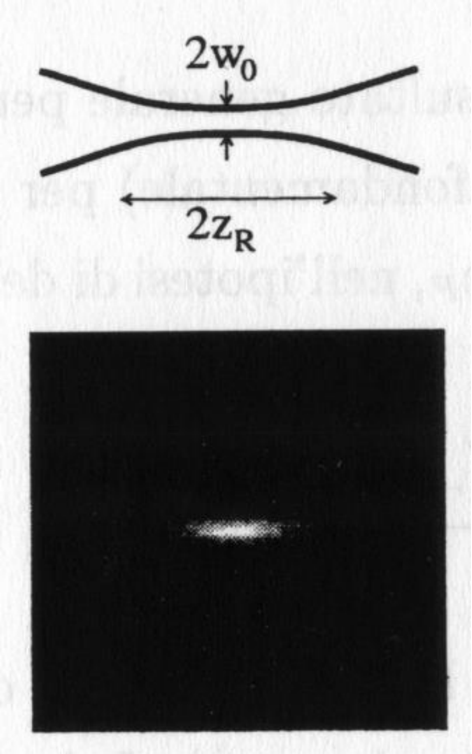
$$\tilde{p} = \alpha \tilde{E}$$ Dipolo-dipolo indotto



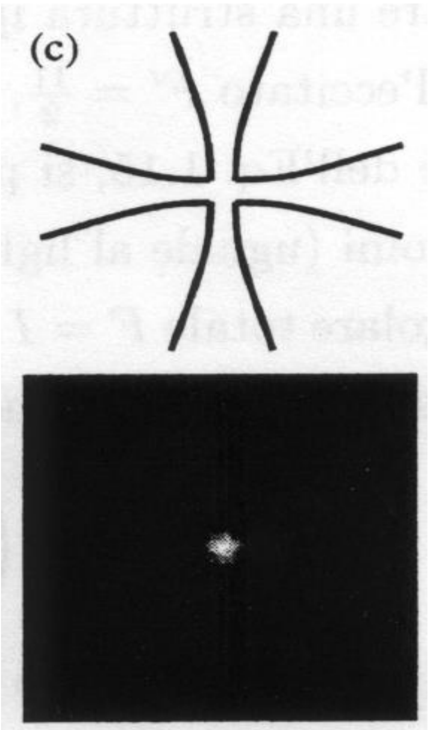
$$I(r,z) = \frac{2P}{\pi\omega^2(z)} \exp\left(-\frac{2r^2}{\omega^2(z)}\right)$$

$$\mathbf{F}_{\rm dip}(\mathbf{r}) = -\nabla U_{\rm dip}(\mathbf{r}) = = \frac{1}{2\epsilon_0 c} \mathrm{Re}(\alpha) \nabla I(\mathbf{r})$$

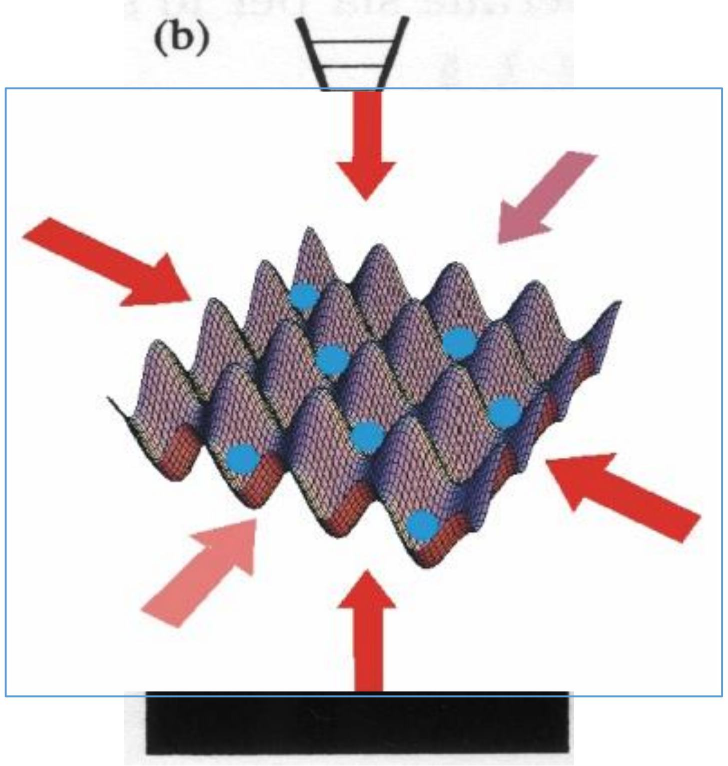
Trappole a singolo fascio focalizzato



Trappole a fasci incrociati



Reticoli ottici



Fasci Gaussiani - Esempi

- Intrappolamento ottico

- frequency doubled Nd:YAG laser (λ=532nm)
- w_0= 50 μm
- P = 1W

$$z_0 = \frac{\pi\,\omega_0^2}{\lambda} = 15\,\text{mm}$$

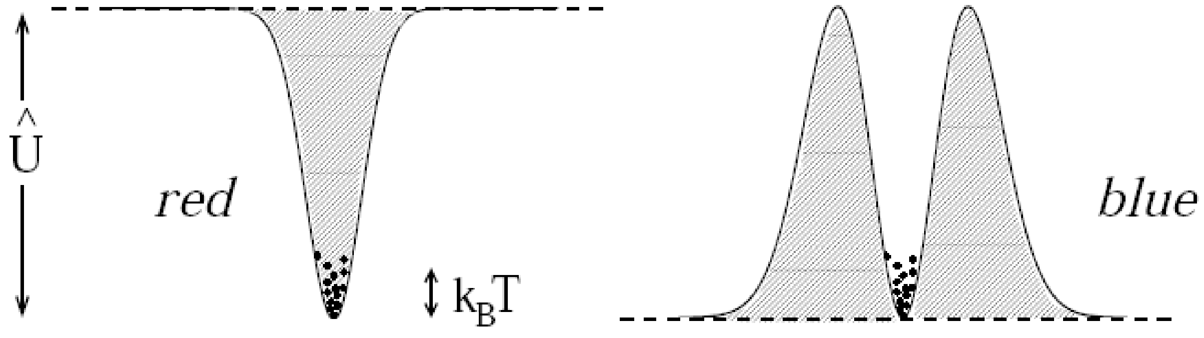
$$\theta_{div} = 3.4\,\text{mrad}$$

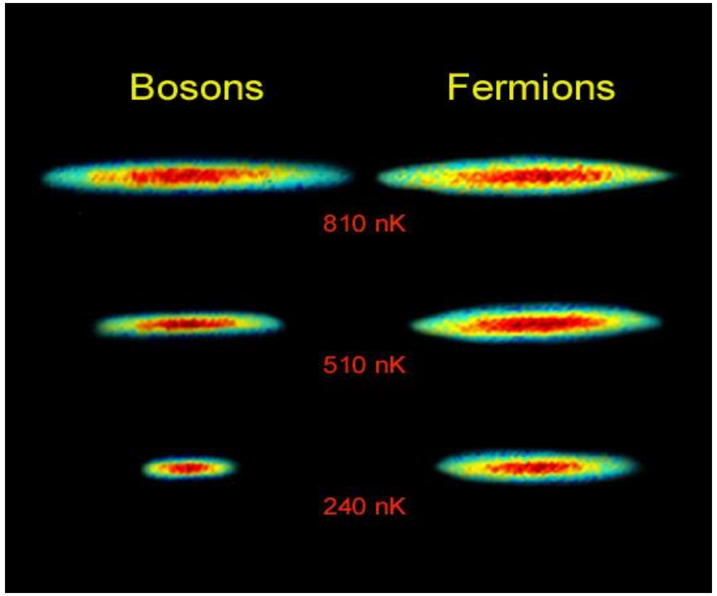
$$U_{\text{dip}}(\mathbf{r}) = \frac{3\pi c^2}{2\omega_0^3}\,\frac{\Gamma}{\Delta}\,I(\mathbf{r}) \qquad U_0 = 42\,\mu\text{K}$$

$$\simeq -\hat{U}_0\left[1 - 2\left(\frac{r}{w_0}\right)^2 - \left(\frac{z}{z_R}\right)^2\right]$$

$$\omega_r = 2\pi \times 400\,\text{Hz}$$

$$\omega_z = 2\pi \times 0.96\,\text{Hz}$$





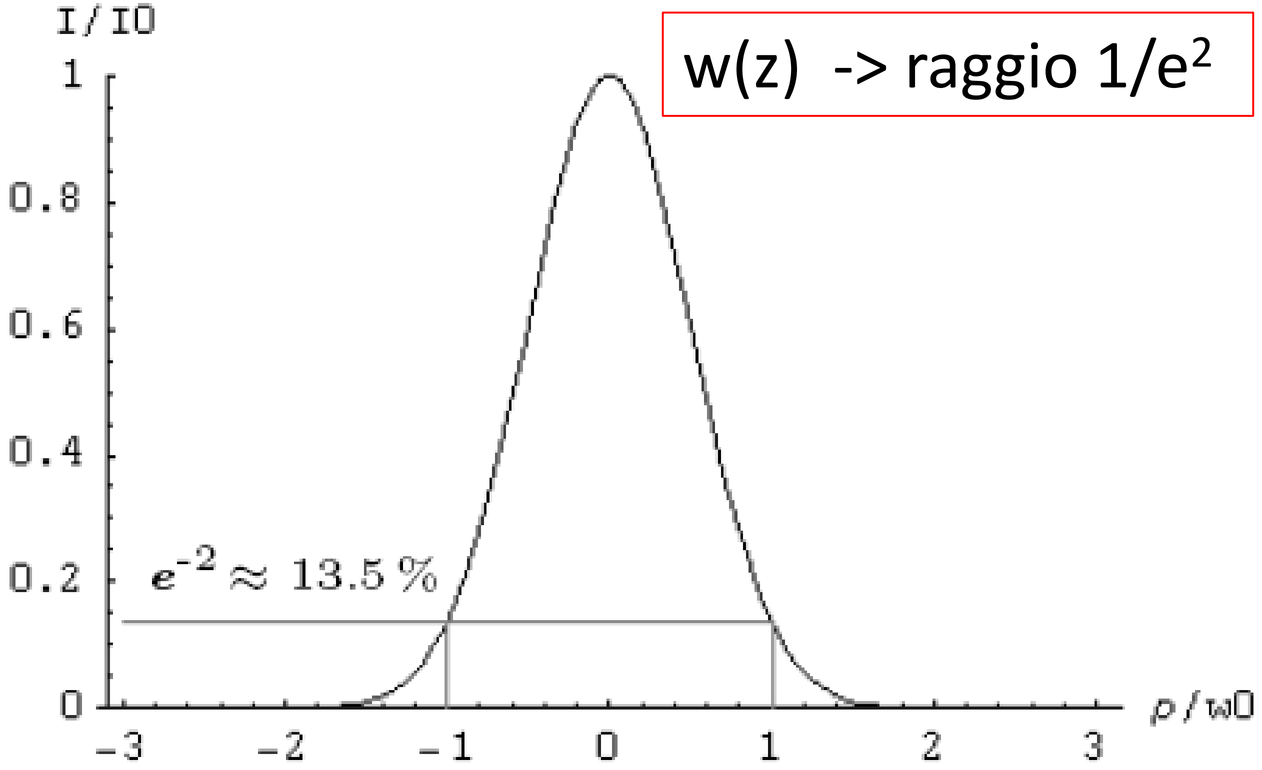
Fasci Gaussiani - definizioni

Intensità di un'onda $I(z,t) \equiv \epsilon_0\, c\, \left\langle Re(E(z,t))^2 \right\rangle_T$

$$\begin{aligned} I(z,t) &= \frac{\epsilon_0\, c}{4} \left\langle (E(z,t) + E^*(z,t))^2 \right\rangle_T \\ &= \frac{\epsilon_0\, c}{4} \left(\left\langle E^2(z,t) \right\rangle_T + \left\langle E^{*2}(z,t) \right\rangle_T + 2 \left\langle E(z,t)\, E^*(z,t) \right\rangle_T \right) \\ &= \frac{\epsilon_0\, c}{2} \left\langle \left| E(z,t) \right|^2 \right\rangle_T \ , \end{aligned}$$

$$I(\vec{r}) = I_0\, \frac{w_0^2}{w(z)^2} \exp\left[-\frac{2\, \rho^2}{w(z)^2} \right]$$

$$I_0 \equiv \frac{\epsilon_0\, c}{2} \left| A_0 \right|^2$$



w(z) -> raggio $1/e^2$

Fasci Gaussiani - definizion

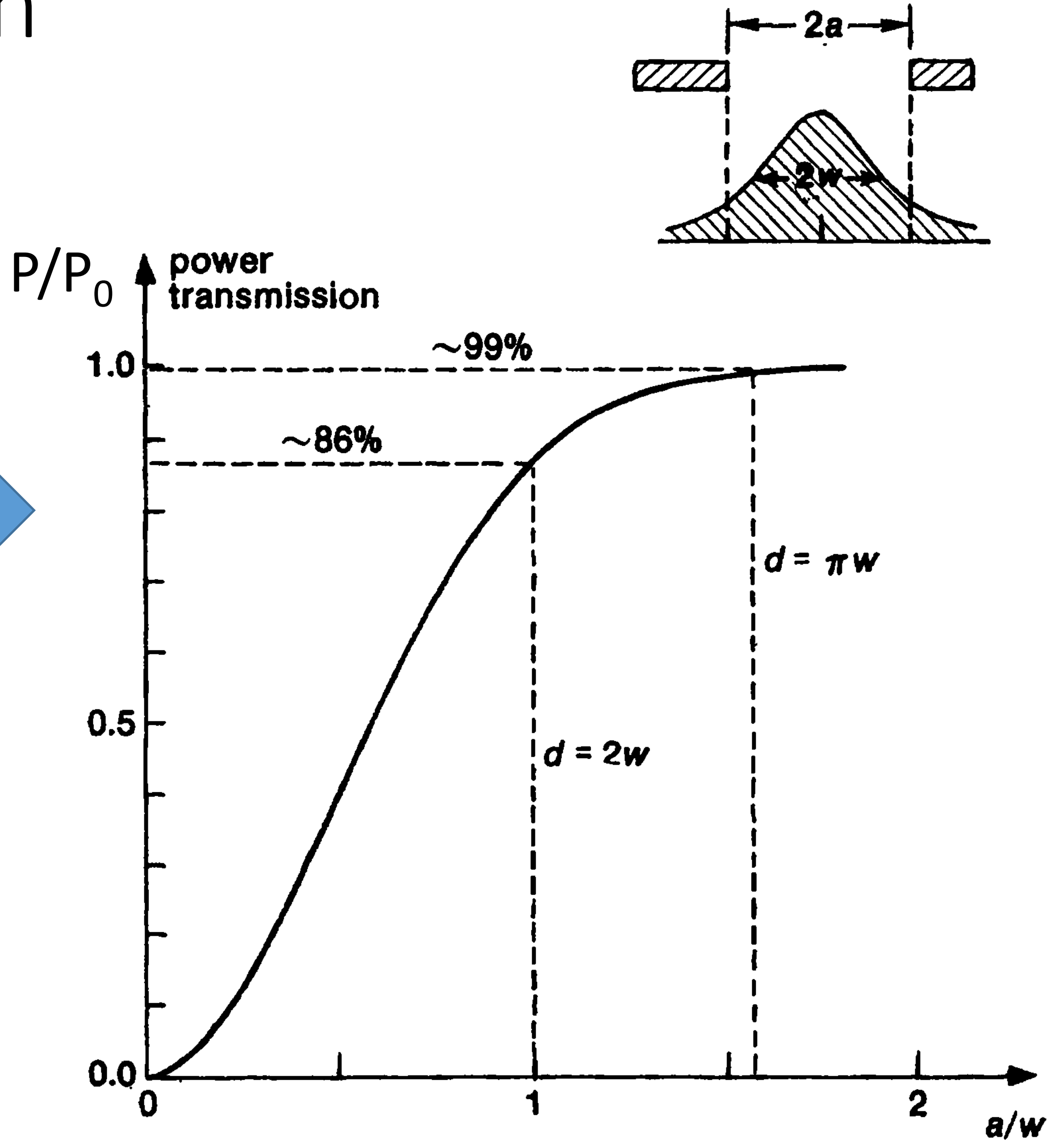
Potenza complessiva

$$P(\rho) = \int_0^{\rho} I(r')\, 2\pi \rho'\, d\rho'$$

$$P_0 \equiv \frac{I_0\, \pi\, w_0^2}{2}$$

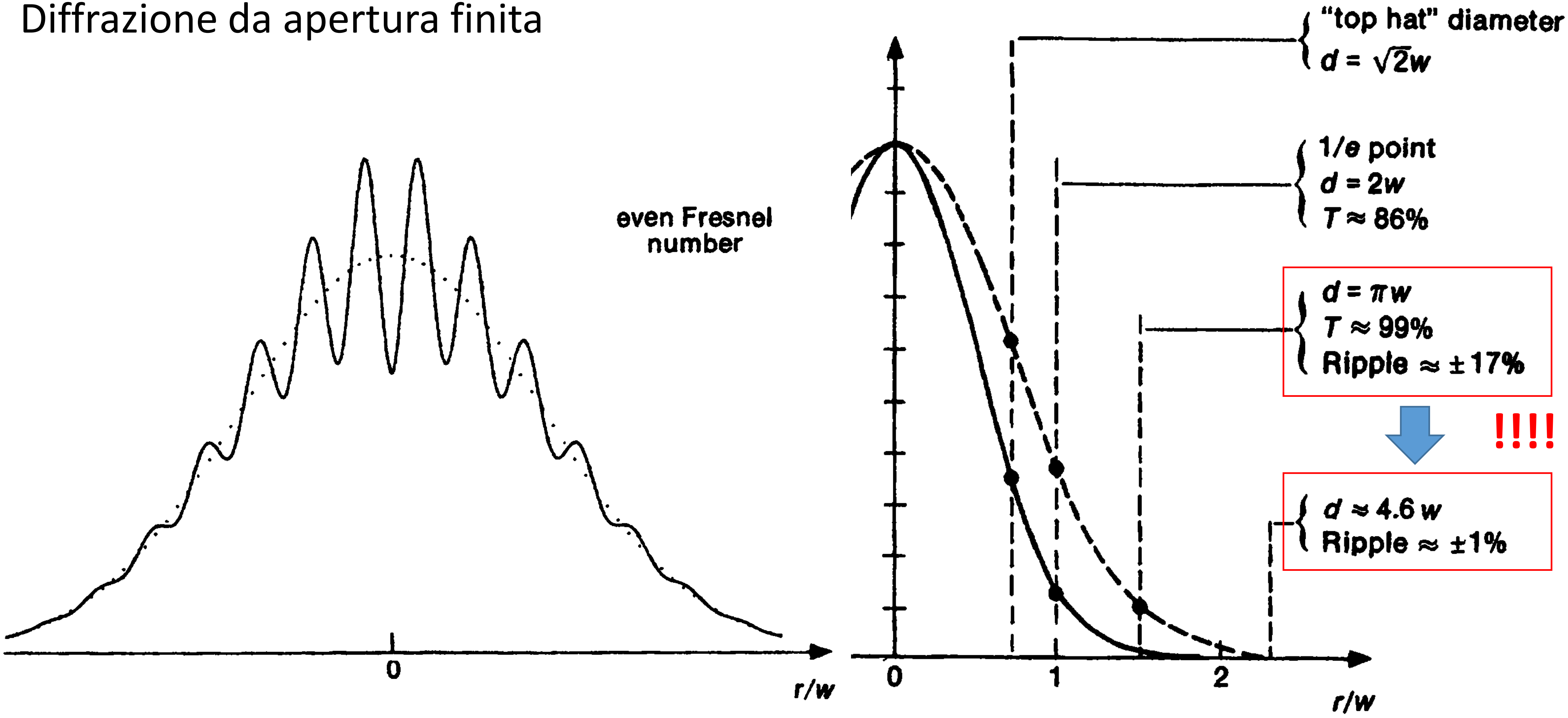
$I_0 = 2P_0/\pi w_0^2$

$$I(r) = \frac{2P}{\pi w^2}\, e^{-2r^2/w^2}$$



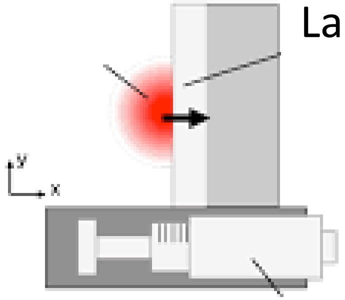
Fasci Gaussiani

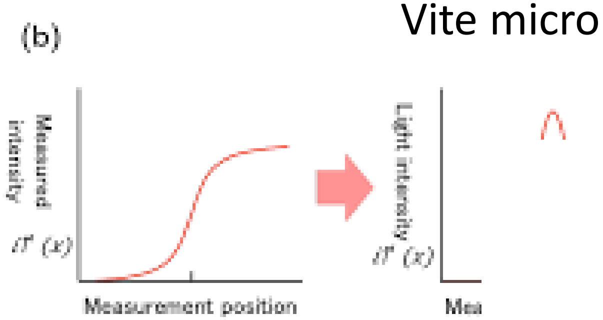
Diffrazione da apertura finita

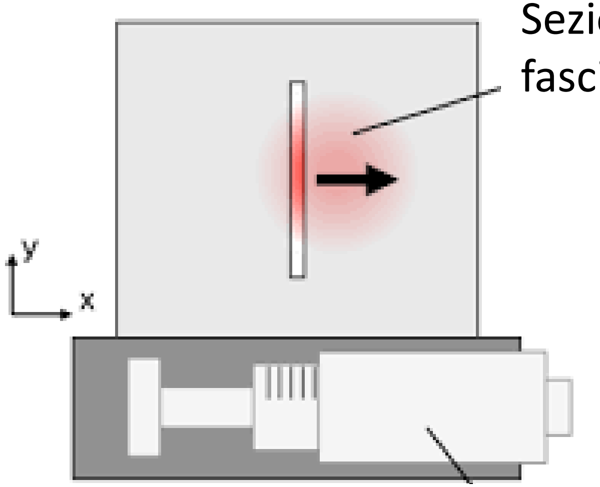


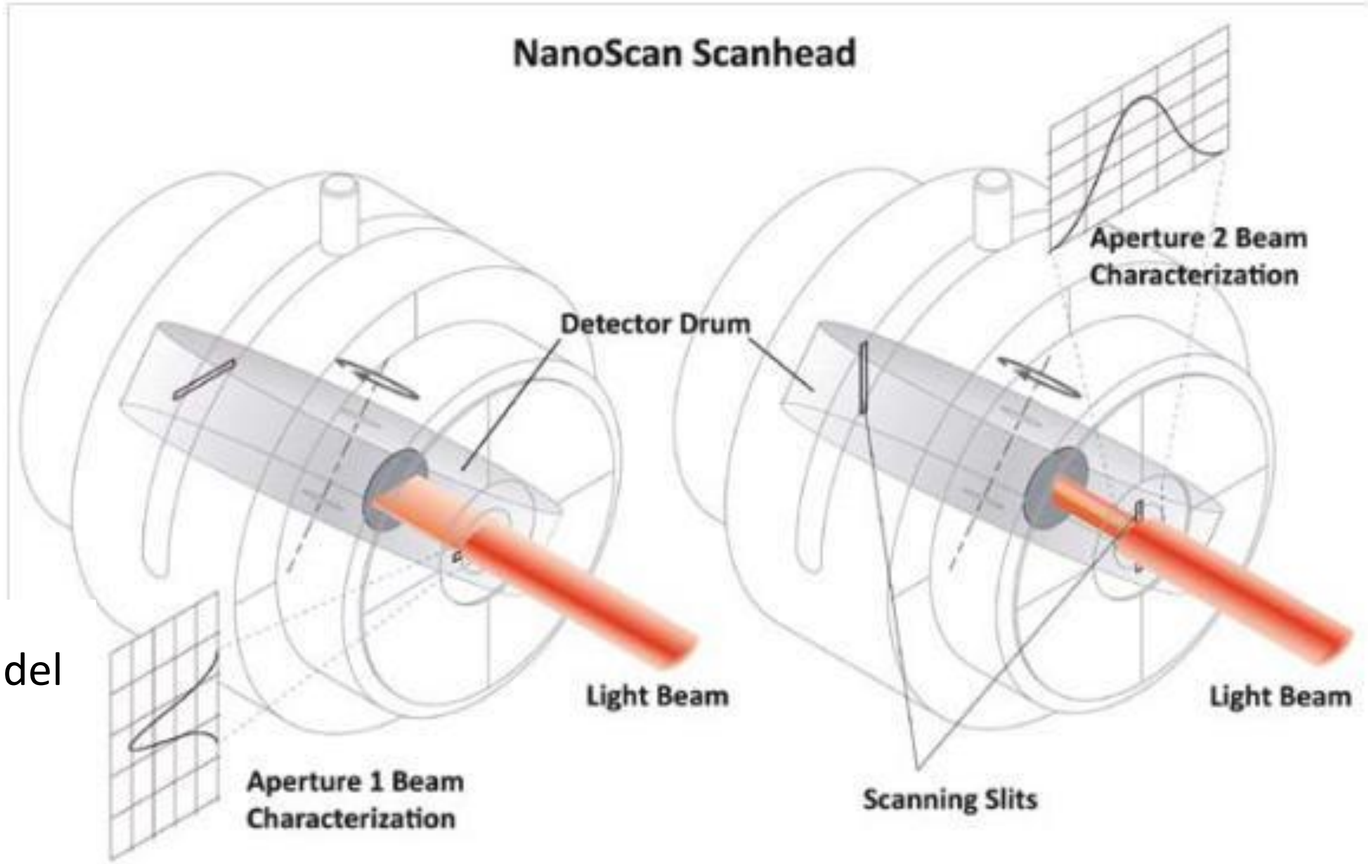
Fasci Gaussiani - definizioni

Misura dimensioni del fascio





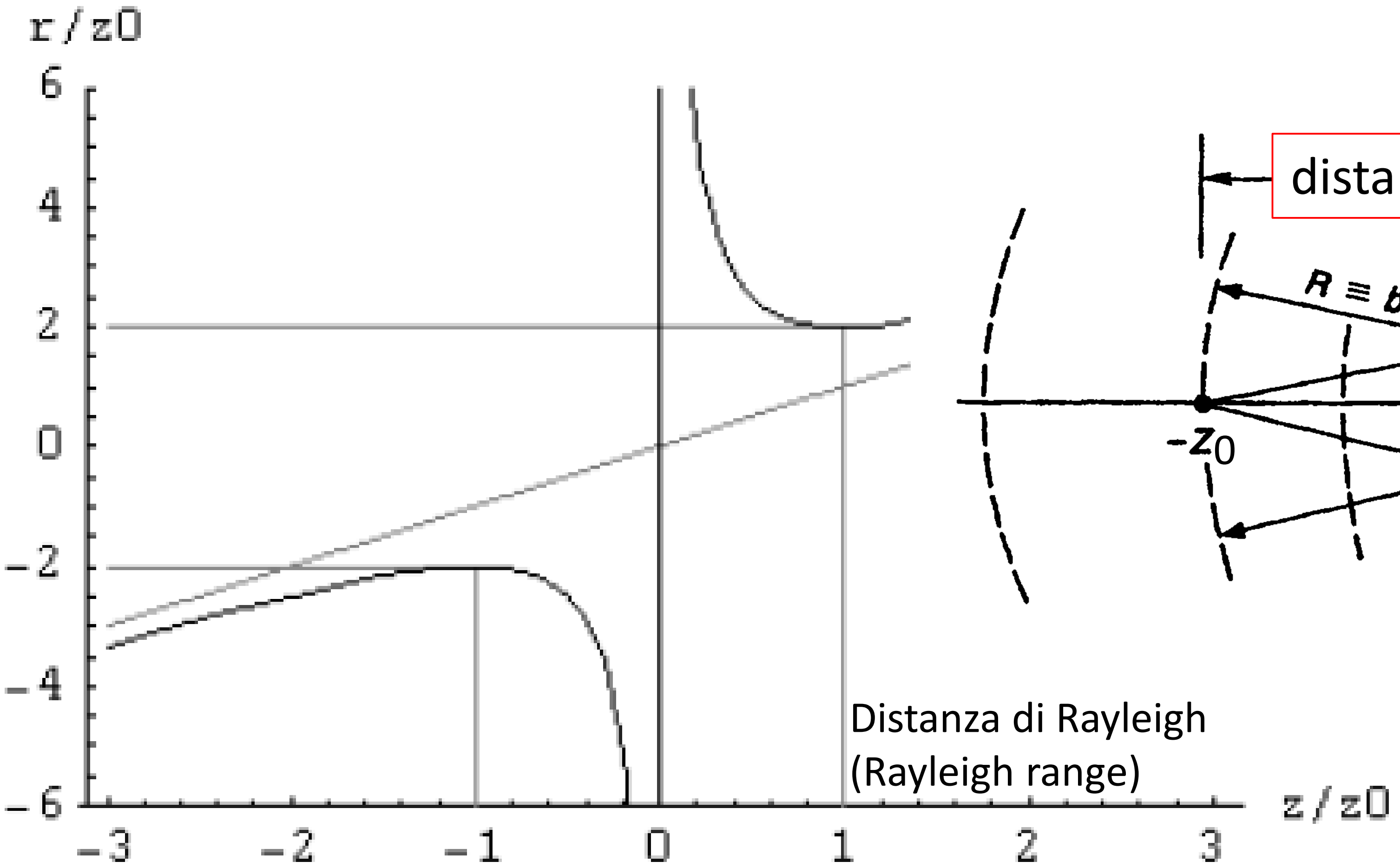




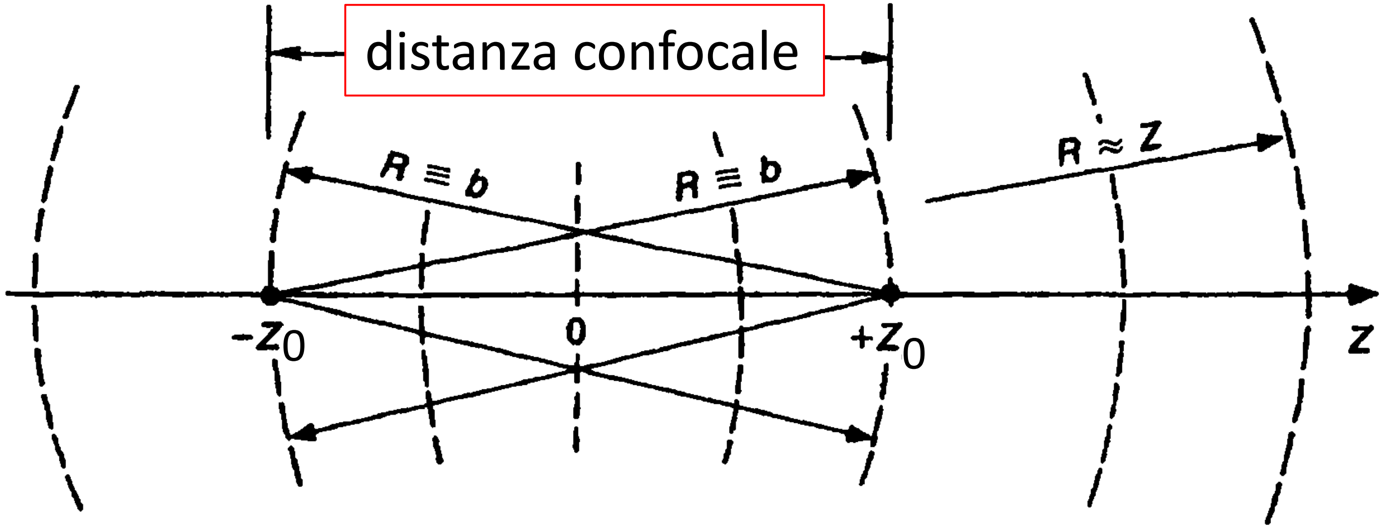
Fasci Gaussiani - definizioni

- Raggio di curvatura r(z)

$$r(z) = z\left(1+\left(\frac{z_0}{z}\right)^2\right) \approx \begin{cases} \infty & \text{for} \quad z \ll z_0 \\ 2z_0 & \text{for} \quad z = z_0 \\ z & \text{for} \quad z \gg z_0 \end{cases}$$



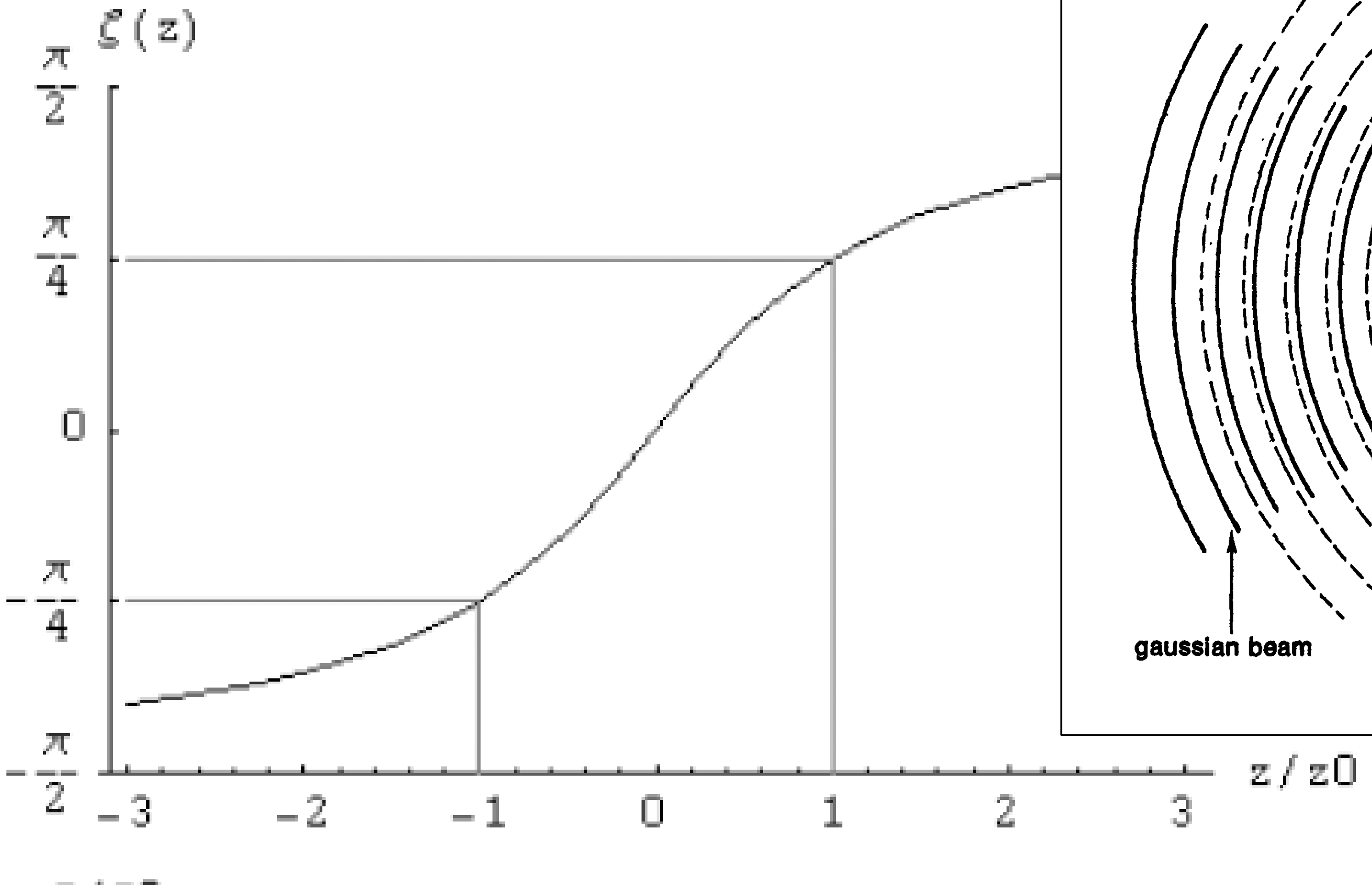
Distanza di Rayleigh
(Rayleigh range)

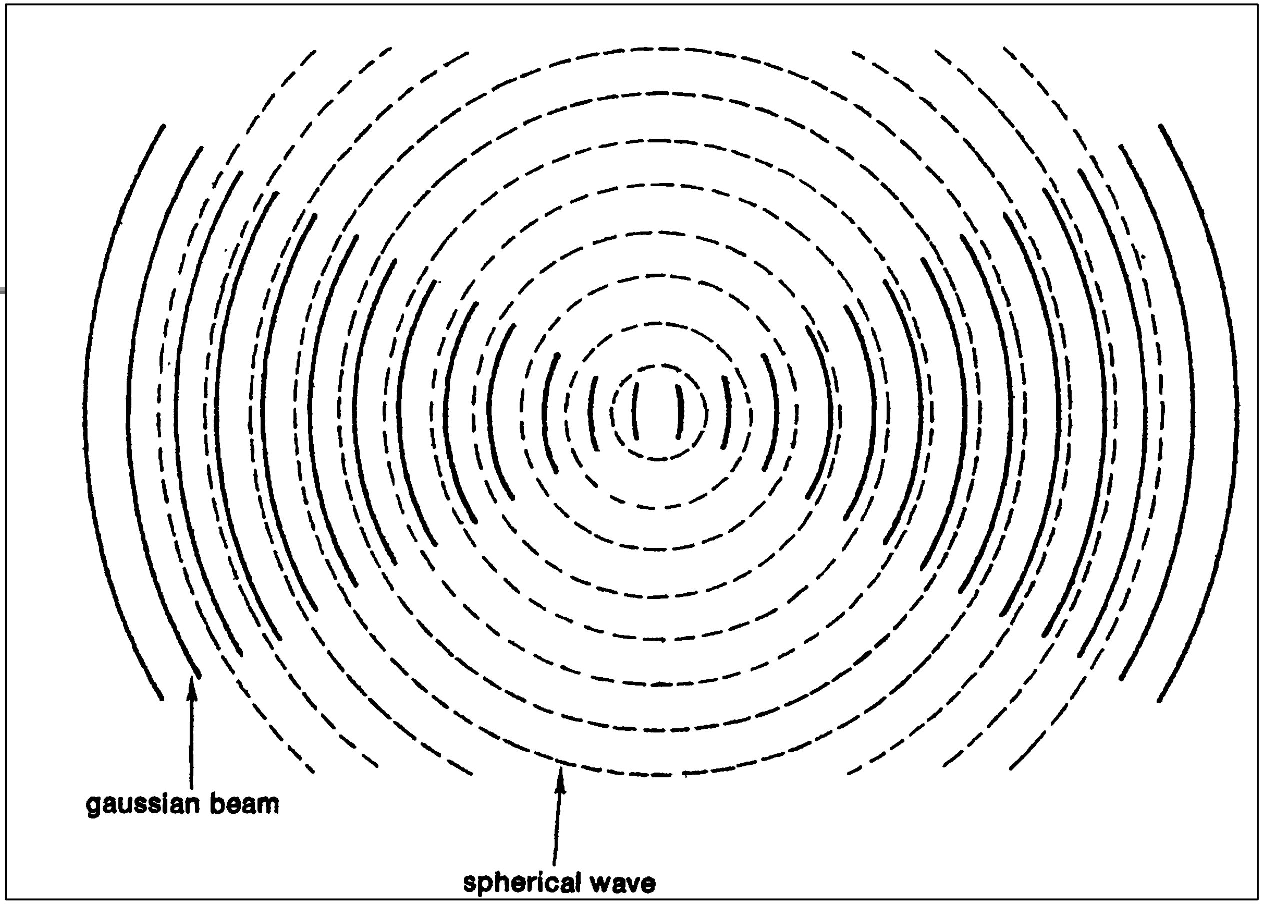


f=R/2 distanza focale

Fasci Gaussiani - definizioni

Fase di Guoy $\zeta(z) = \arctan\left[\frac{z}{z_0}\right]$





Fasci Gaussiani – ordini superiori

Separare dipendenza da x e y (soluzioni non simmetriche assialmente)

$$A(\vec{r}) \;=\; A_m(x,z)\,A_n(y,z)$$

$$\left(\frac{\partial^2}{\partial x^2} - 2\,i\,k\frac{\partial}{\partial z}\right)A_m(x,z) \approx 0$$

$$A_m(x,z) = A\Big[q(z)\Big]\,h_m\Big[\frac{x}{p(z)}\Big]\exp\Big[-i\,k\,\frac{x^2}{2q(z)}\Big]$$

$$H''_m - 2\frac{x}{p}H'_m + 2mH_m = 0$$

$$E(\vec{r}) = A_m(x,z)\,A_n(y,z)\;\exp\big[-i\,k\,z\big]$$
$$= E_{mn}\,\frac{w_0}{w(z)}\,H_m\Big[\frac{x\sqrt{2}}{w(z)}\Big]\,H_n\Big[\frac{y\sqrt{2}}{w(z)}\Big]\times$$
$$\exp\Big[i(m+n+1)\zeta(z) - i\,k\,\frac{\rho^2}{2\,q(z)} - i\,k\,z\Big]$$

Polinomi di Hermite

$$H_0(x) = 1$$
$$H_1(x) = x$$
$$H_2(x) = 4x^2 - 2$$
$$H_3(x) = 8x^3 - 12x.$$

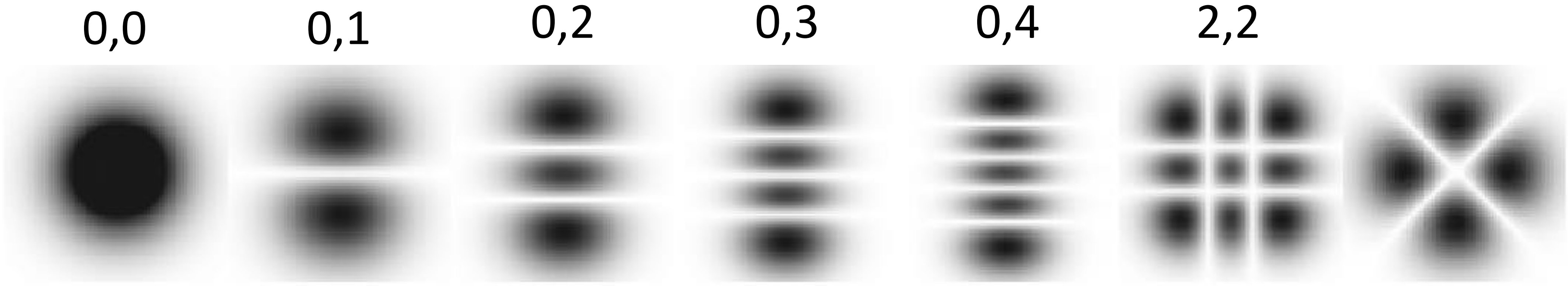
Fasci Gaussiani – ordini superiori

Separare dipendenza da x e y (soluzioni non simmetriche assialmente)

$$E(\vec{r}) = E_{mn}\,\frac{w_0}{w(z)}\,H_m\!\left[\frac{x\sqrt{2}}{w(z)}\right]\,H_n\!\left[\frac{y\sqrt{2}}{w(z)}\right]\exp\!\left[i\Phi(z) - \frac{\rho^2}{w(z)^2}\right]$$

$$\Phi(z) \equiv (m+n+1)\zeta(z) - k\left(\frac{\rho^2}{2\,r(z)} + z\right)$$

$$H_0(x) = 1$$
$$H_1(x) = x$$
$$H_2(x) = 4x^2 - 2$$
$$H_3(x) = 8x^3 - 12x.$$



Fasci Gaussiani – LaguerreGauss

In simmetria cilindrica

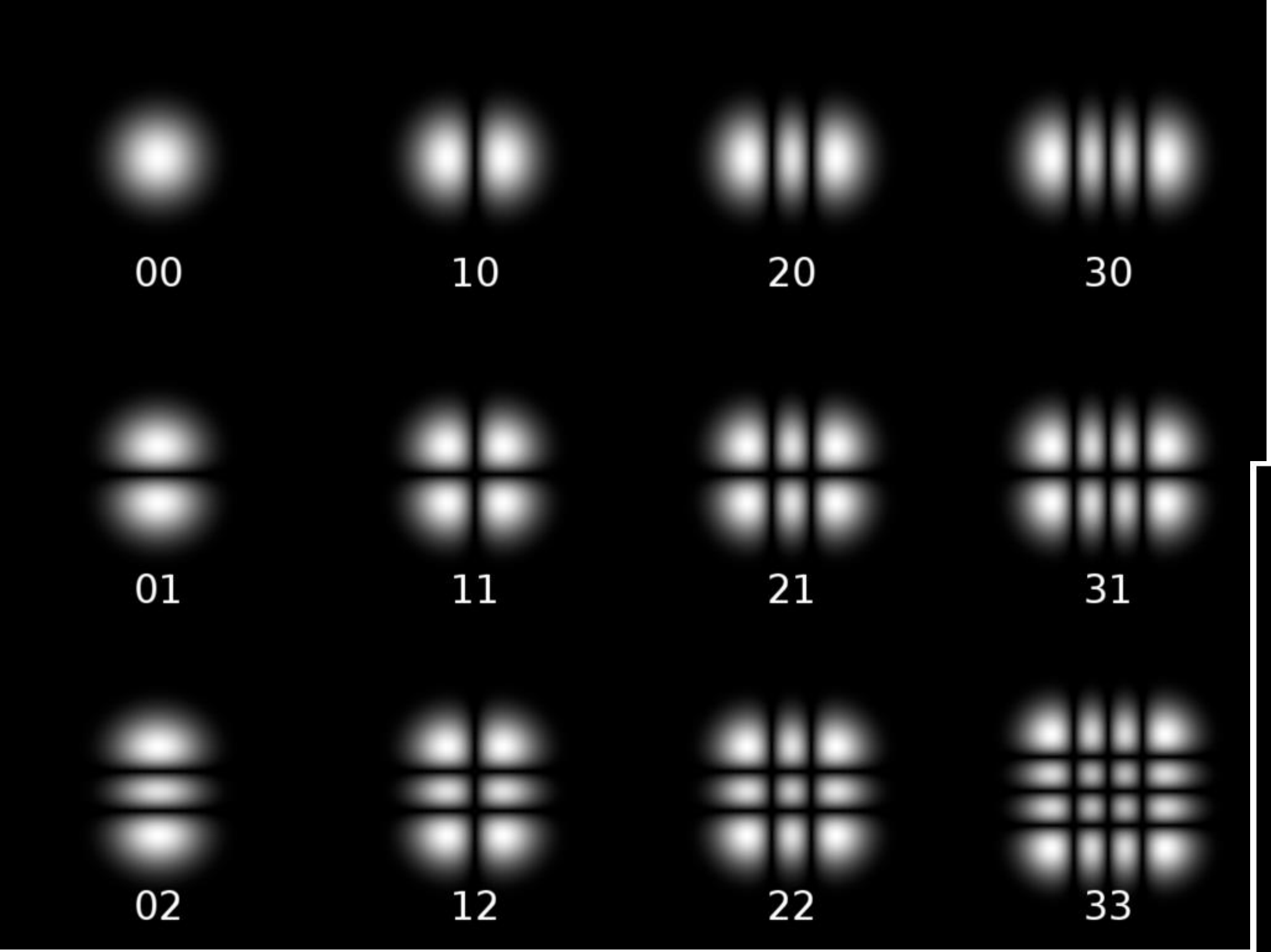
$$A_{pm}(r,\theta,z) = \sqrt{\frac{2p!}{(1+\delta_{0m})\pi(m+p)!}}\,\frac{\exp[i(2p+m+1)\zeta(z)]}{w(z)}$$
$$\times\left(\frac{r\sqrt{2}}{w(z)}\right)L_p^m\left[\frac{2r^2}{w(z)^2}\right]\exp\left[-ik\frac{r^2}{2q(z)}+im\theta\right]\,.$$

Polinomi di Laguerre

$$L_0{}^l(x) = 1$$

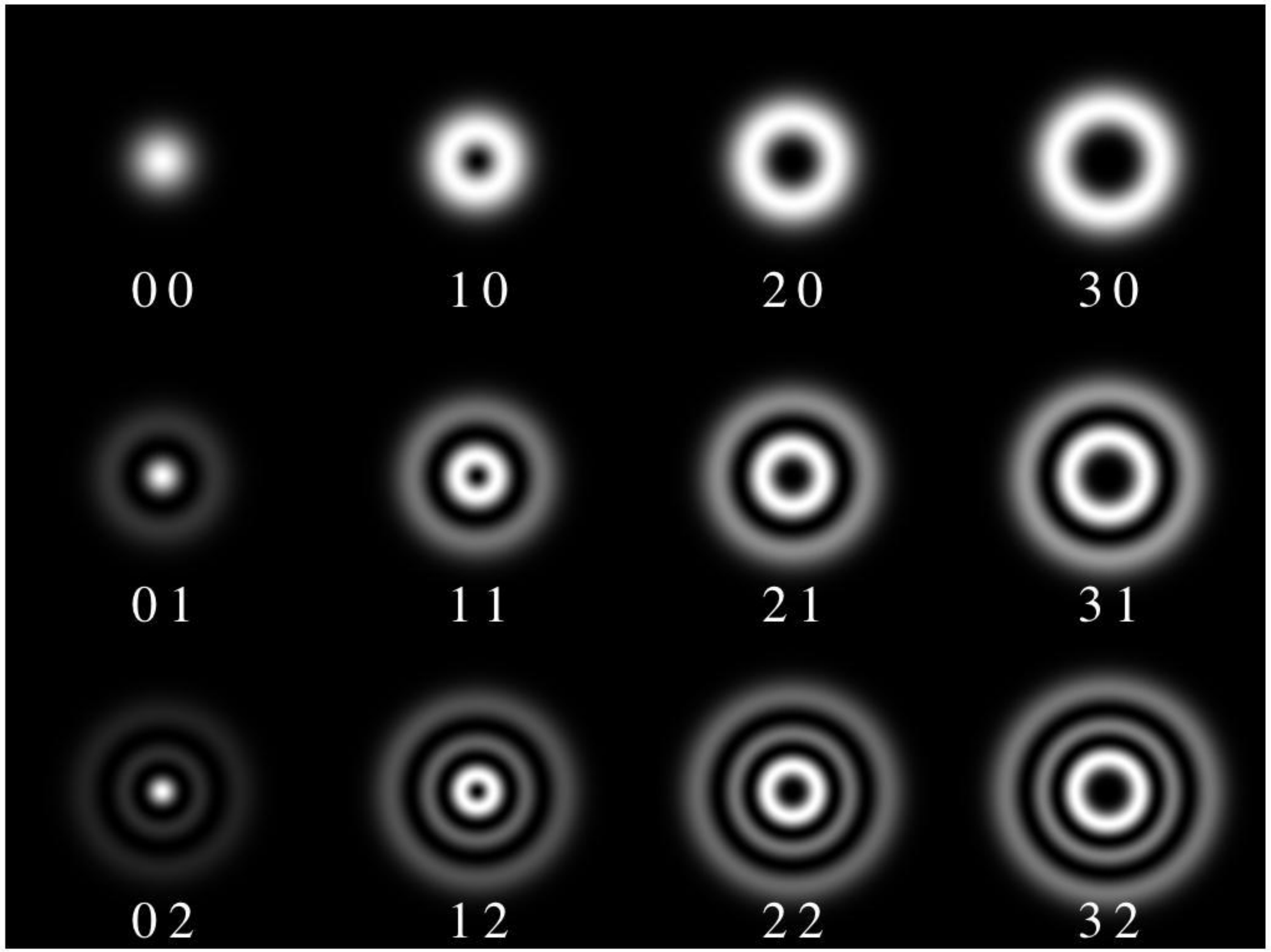
$$L_1{}^l(x) = l + 1 - x$$

$$L_2{}^l(x) = \tfrac{1}{2}(l+1)(l+2) - (l+2)x + \tfrac{1}{2}x^2.$$



Hermite-Gauss

Laguerre-Gauss



Fasci Gaussiani – M^2

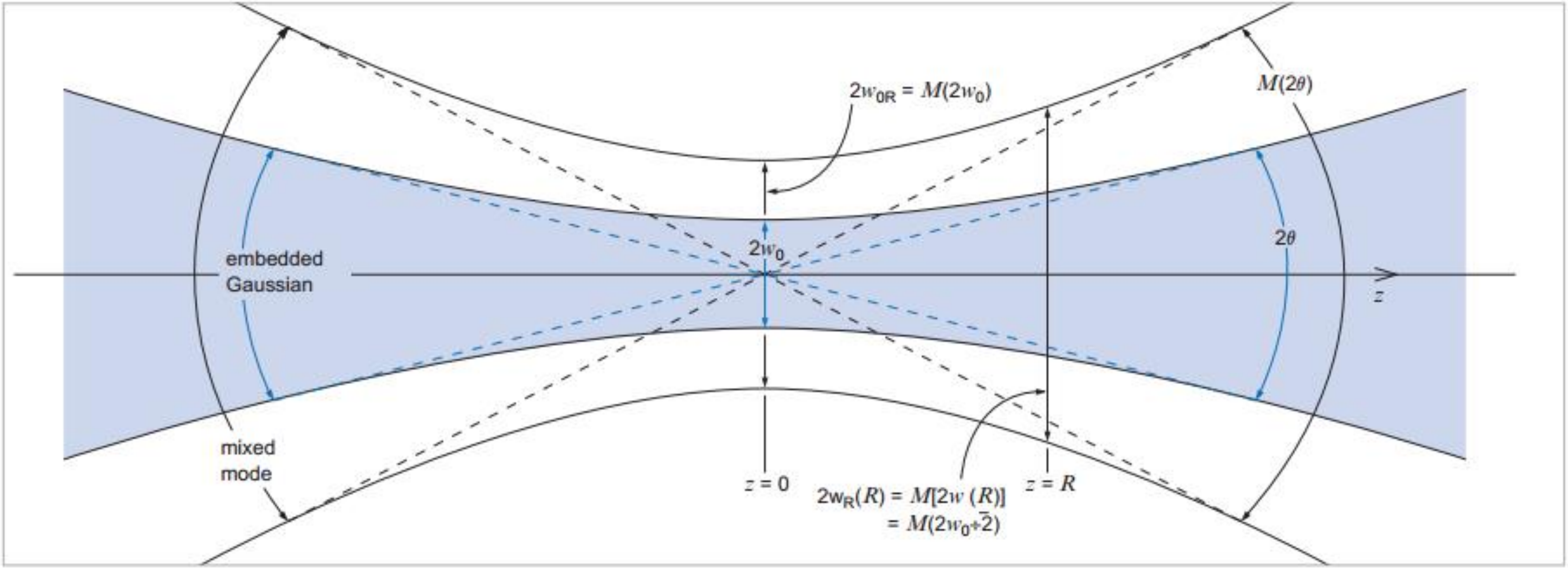
Misura della deviazione da un fascio gaussiano TEM00

$$w_0\,\Theta_{div} = w_0\,\frac{\lambda}{\pi w_0} = \frac{\lambda}{\pi}$$

Per ordini superiori il waist del fascio è generalmente un poco più grande ed anche la divergenza è maggiore.

$$w_m(z) = w_{00}(z)\sqrt{2\,m+1}$$

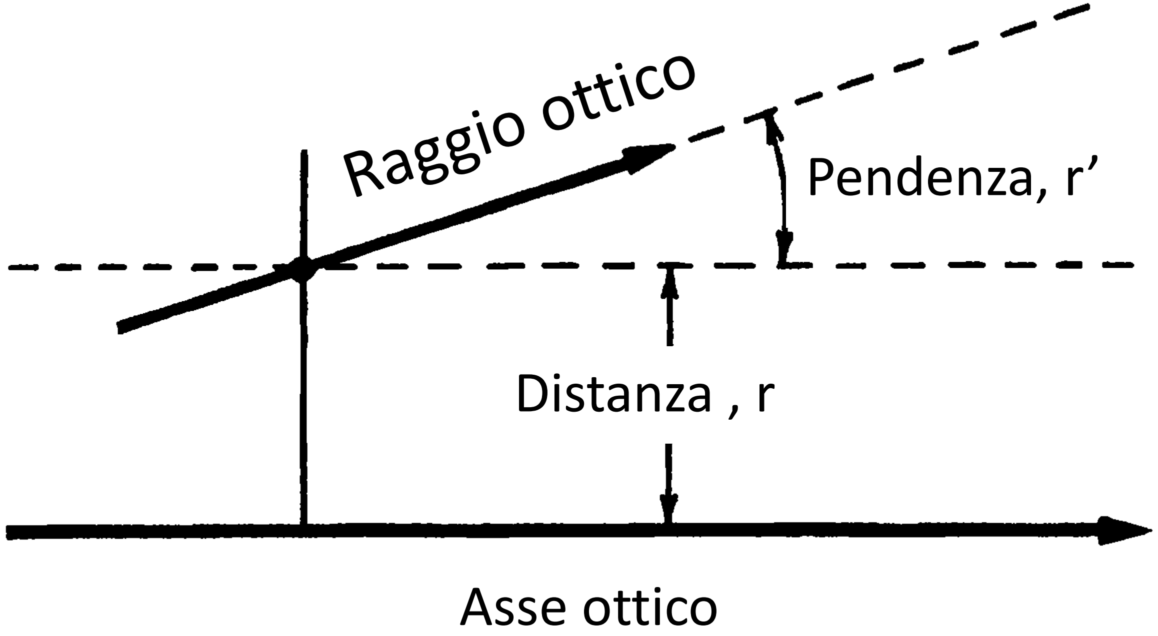
$$\mathrm{M}^2 \equiv \frac{\tilde{w}_0\,\tilde{\Theta}_{div}}{w_0\,\Theta_{div}} = \tilde{w}_0\,\tilde{\Theta}_{div}\,\frac{\pi}{\lambda} \geq 1$$



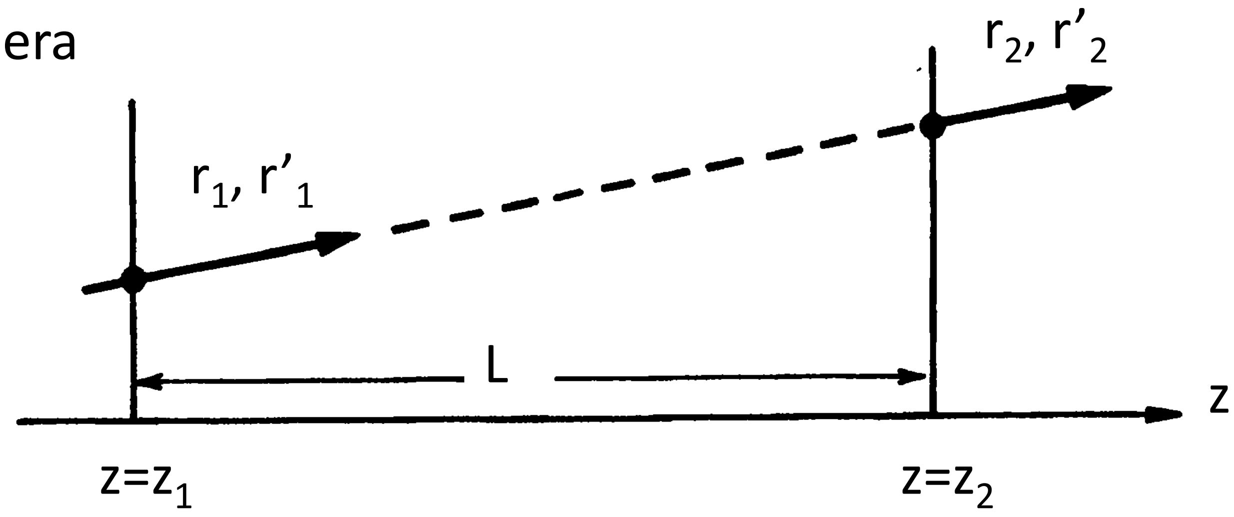
2w0R = M(2w0)
M(2θ)
2w0
2θ
embedded
Gaussian
z
mixed
mode
z = 0
z = R
2wR(R) = M[2w (R)]
= M(2w0+2)

Ottica Matriciale

- Raggi parassiali



- Propagazione libera

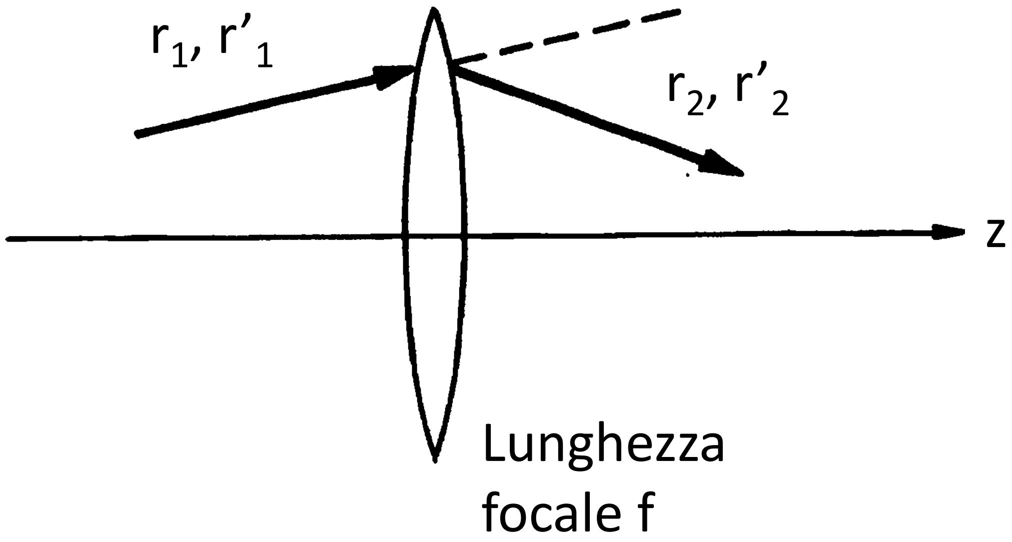


$$r_2 = r_1 + L\, dr_1/dz$$

$$dr_2/dz = dr_1/dz\,.$$

Ottica Matriciale

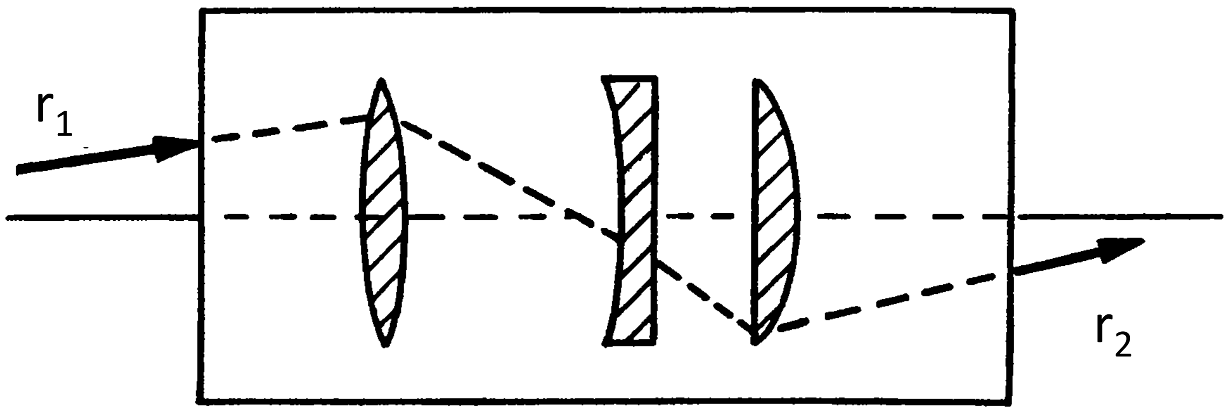
- Lente sottile



$$r_2 = r_1$$

$$dr_2/dz = -(1/f)\,r_1 + dr_1/dz\,.$$

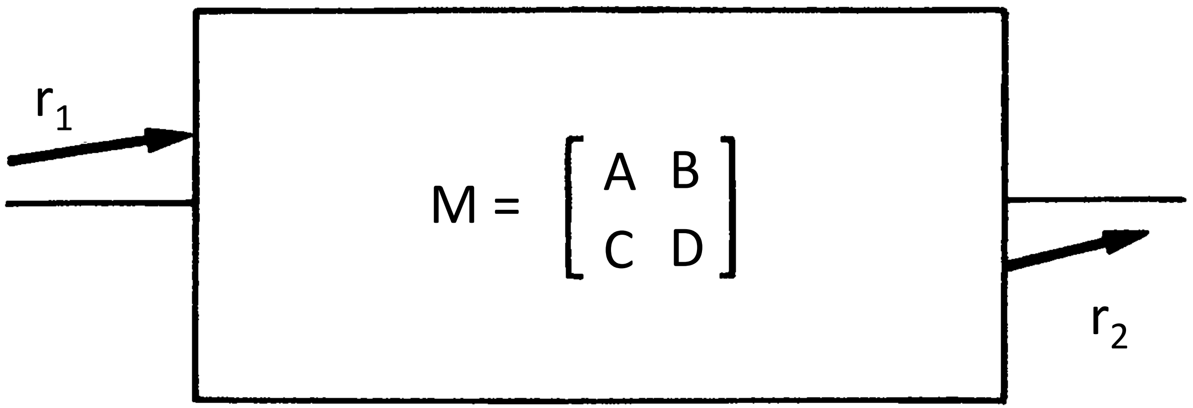
Ottica Matriciale



$$r'(z) \equiv n(z)\,\frac{dr(z)}{dz}$$

$$r_2 = Ar_1 + Br_1'$$

$$r_2' = Cr_1 + Dr_1'.$$

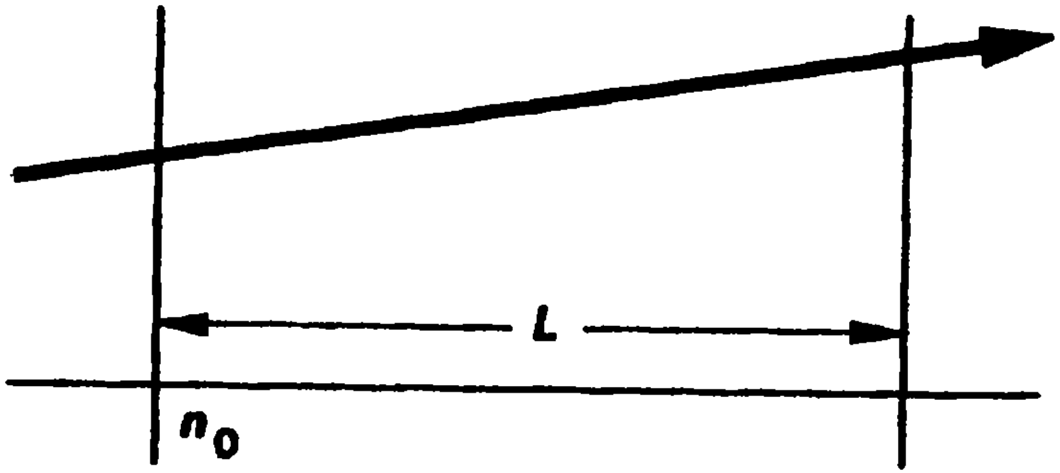


In forma «matriciale»

$$\boldsymbol{r}_2 \equiv \begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} r_1 \\ r_1' \end{bmatrix} \equiv \boldsymbol{M}\,\boldsymbol{r}_1$$

Ottica Matriciale

(a) "Free space" region, index n_0, length L

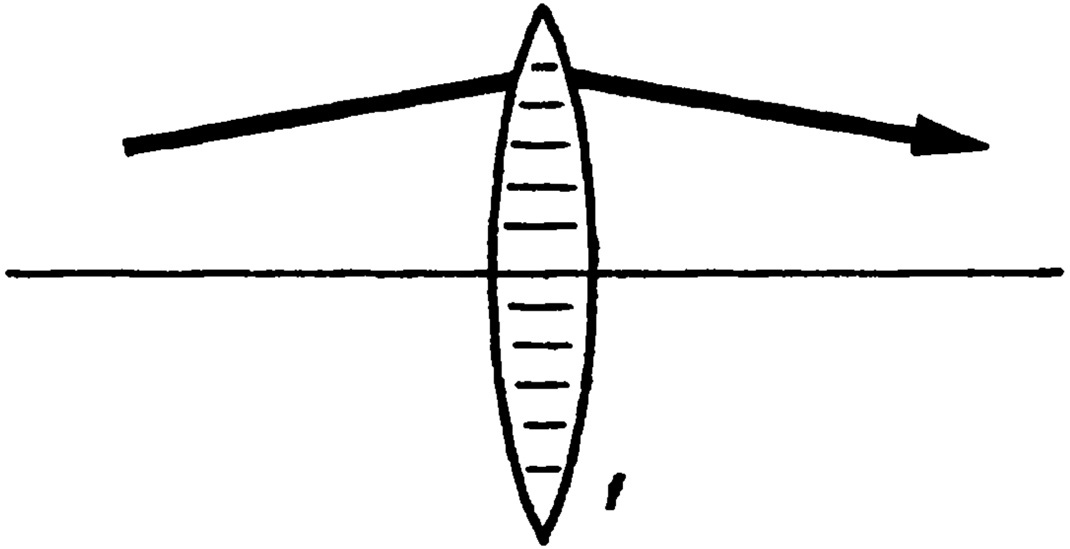


$$\begin{bmatrix} 1 & L/n_o \\ 0 & 1 \end{bmatrix}$$

Ottica Matriciale

(b) Thin lens, focal length f

$f > 0$ for converging lens

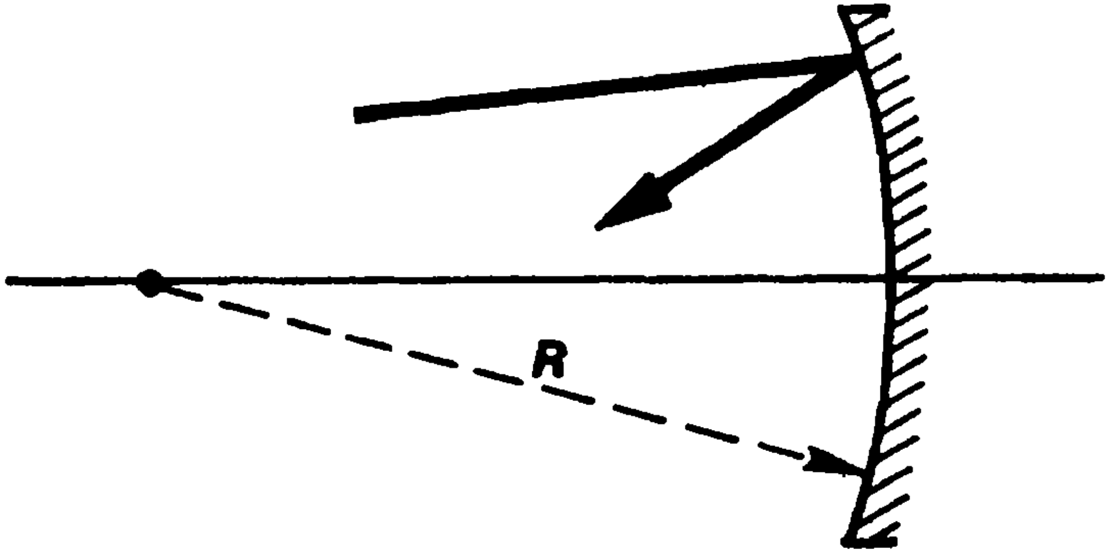


$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

Ottica Matriciale

(c) Curved mirror, radius R, normal incidence

$R > 0$ for concave mirror



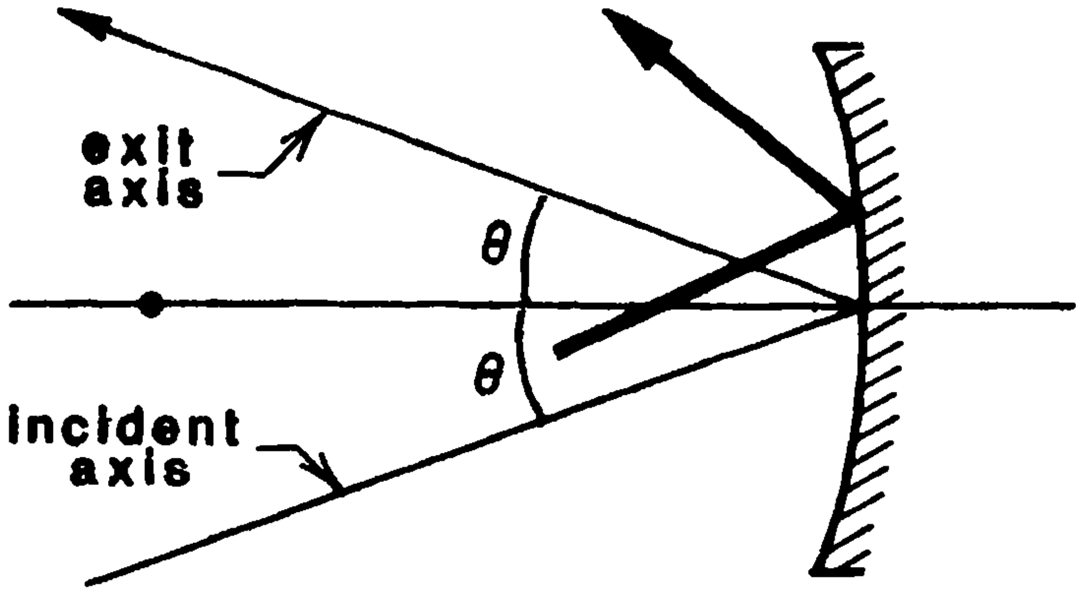
$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$$

Ottica Matriciale

(d) Curved mirror, arbitrary incidence

$R_e = R\cos\theta$ in the plane of incidence ("tangential")

$R_e = R/\cos\theta$ $\perp$ to plane of incidence ("sagittal")



$$\begin{bmatrix} 1 & 0 \\ -2/R_e & 1 \end{bmatrix}$$

Ottica Matriciale

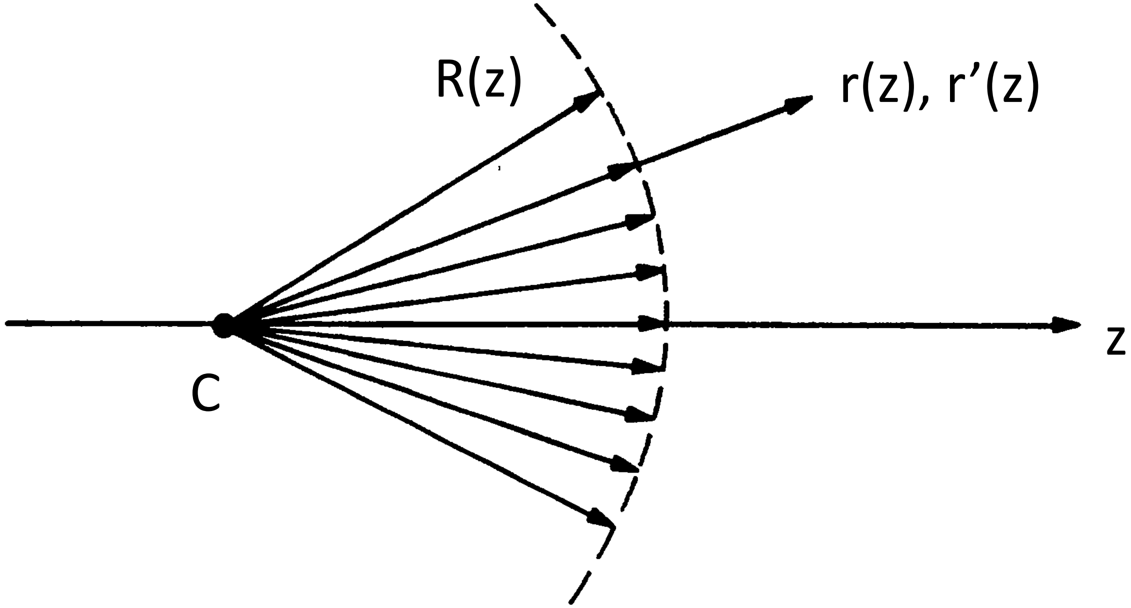
$$r_1 = M_1 \, r_0$$
$$r_2 = M_2 \, r_1 = M_2 \, M_1 \, r_0$$
$$r_3 = M_3 \, r_2 = M_3 \, M_2 \, M_1 \, r_0,$$

$$r_n = [M_n M_{n-1} \cdots M_2 M_1] \, r_0 = M_{\text{tot}} \, r_0.$$

$$M_{\text{tot}} \equiv M_n M_{n-1} \cdots M_2 M_1.$$

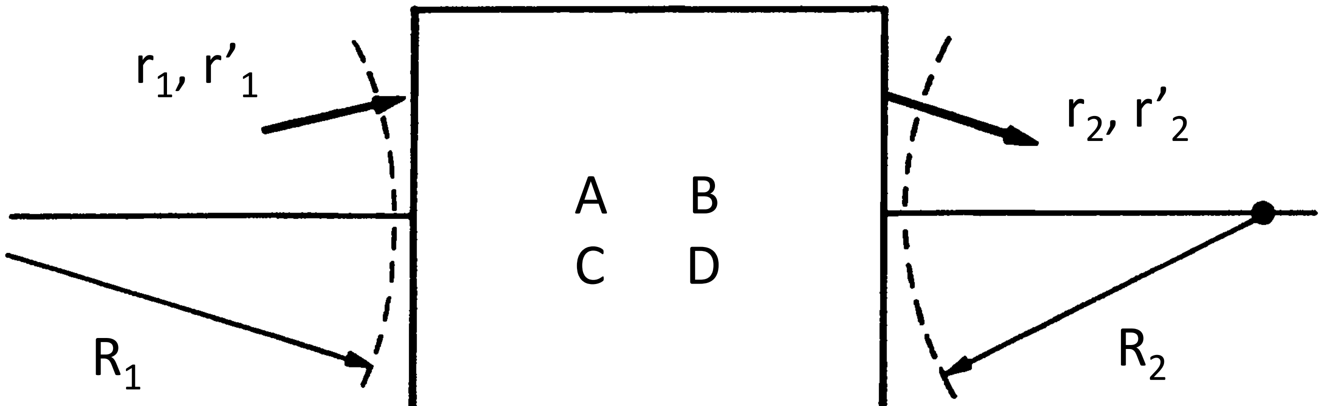
Ottica Matriciale

- Onde sferiche



$$r'(z) = n(z)\frac{dr(z)}{dz} = \frac{n(z)r(z)}{R(z)}$$

$$R(z) \equiv \frac{n(z)\,r(z)}{r'(z)}$$



$$\frac{R_2}{n_2} \equiv \frac{r_2}{r_2'} = \frac{Ar_1 + Br_1'}{Cr_1 + Dr_1'} = \frac{A(R_1/n_1) + B}{C(R_1/n_1) + D}.$$

$$\hat{R}(z) \equiv R(z)/n(z)$$

$$\hat{R}_2 = \frac{A\hat{R}_1 + B}{C\hat{R}_1 + D}.$$

Ottica Matriciale

Fasci gaussani

Parametro q ridotto

$$\frac{1}{\hat{q}} \equiv \frac{n}{\tilde{q}} \equiv \frac{n}{R} - j\frac{n\lambda}{\pi w^2} = \frac{1}{\hat{R}} - j\frac{\lambda_0}{\pi w^2}$$

$$\hat{R}_2 = \frac{A\hat{R}_1 + B}{C\hat{R}_1 + D}.$$

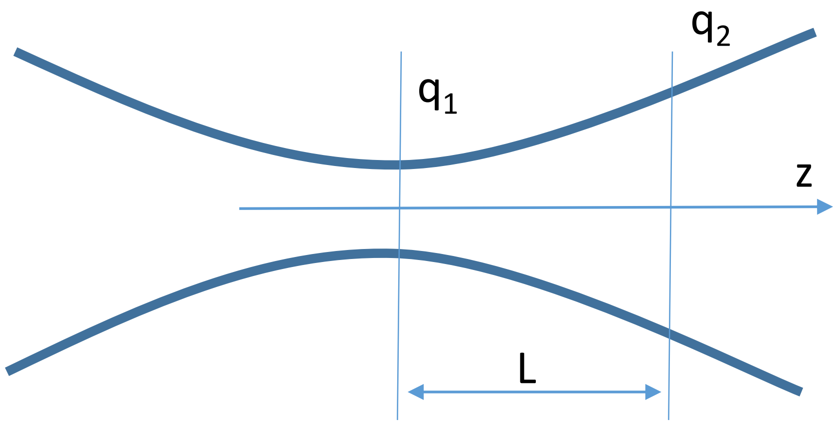
$$\Rightarrow$$

$$\hat{q}_2 = \frac{A\hat{q}_1 + B}{C\hat{q}_1 + D}$$

Ottica Matriciale - Esempi

$$\hat{q}_2 = \frac{A\hat{q}_1 + B}{C\hat{q}_1 + D}$$

– propagazione libera $\quad q(z=0) = i\,z_0 = i\,\frac{\pi\omega_1^2}{\lambda} = q_1$



$$\mathrm{M} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$q_2 = q_1 + L = i\,z_0 + L = \left(\frac{1}{r_2} - \frac{i\,\lambda}{\pi\omega_2^2}\right)^{-1}$$

$$\left(\frac{1}{r_2} - \frac{i\,\lambda}{\pi\omega_2^2}\right) = \frac{1}{L + iz_0} = \frac{L - iz_0}{L^2 + z_0^2}$$

– parte reale $\quad \frac{1}{r_2} = \frac{L}{L^2 + z_0^2}$

– parte immaginaria $\quad \frac{\lambda}{\pi\omega_2^2} = \frac{z_0}{L^2 + z_0^2}$

$$r_2(L) = L\left(1 + \left(\frac{z_0}{L}\right)^2\right)$$

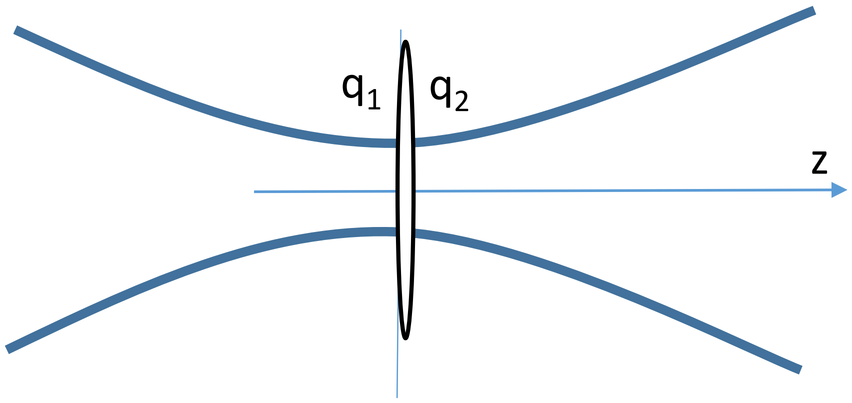
$$\omega_2(L) = z_0\sqrt{1 + \left(\frac{L}{z_0}\right)^2}$$

Ottica Matriciale -Esempi

$$\hat{q}_2 = \frac{A\hat{q}_1 + B}{C\hat{q}_1 + D}$$

Lente sottile $\quad q(z=0) = i\,z_0 = i\,\frac{\pi\omega_1^2}{\lambda} = q_1$

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \qquad q_2 = \frac{q_1 + 0}{-q_1/f + 1}$$



$$\frac{1}{q_2} = \frac{1 - q_1/f}{q_1} = -\frac{1}{q_1} - \frac{1}{f}$$

$$\frac{1}{r_2} - i\,\frac{\lambda}{\pi\omega_2^2} = \frac{1}{i\,\pi\omega_1^2/\lambda} - \frac{1}{f}$$

– parte reale $\qquad \frac{1}{r_2} = -\frac{1}{f}$

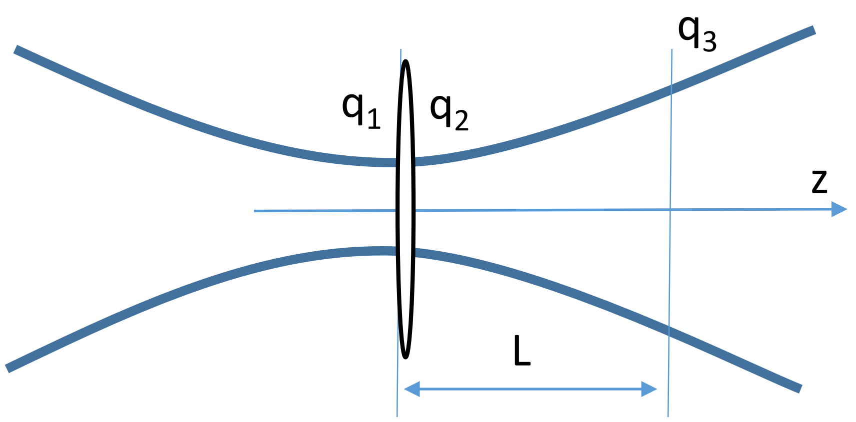
– parte immaginaria $\qquad \omega_2 = \omega_1$

Ottica Matriciale -Esempi

$$\hat{q}_2 = \frac{A\hat{q}_1 + B}{C\hat{q}_1 + D}$$

Lente sottile+
Propagazione
libera

$$q(z=0) = i\,z_0 = i\,\frac{\pi\omega_1^2}{\lambda} = q_1$$



$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{L}{f} & L \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$q_3 = \frac{(1-L/f)q_1 + 0}{-q_1/f + 1}$$

In alternativa:

$$q_3 = q_2 + L$$

(da soluzione precedente)

$$\frac{1}{q_3} = \frac{1}{r_3} - i\,\frac{\lambda}{\pi\omega_3^2}$$

$$\frac{1}{q_2} = -\frac{1}{f} - i\,\frac{\lambda}{\pi\omega_1^2}$$

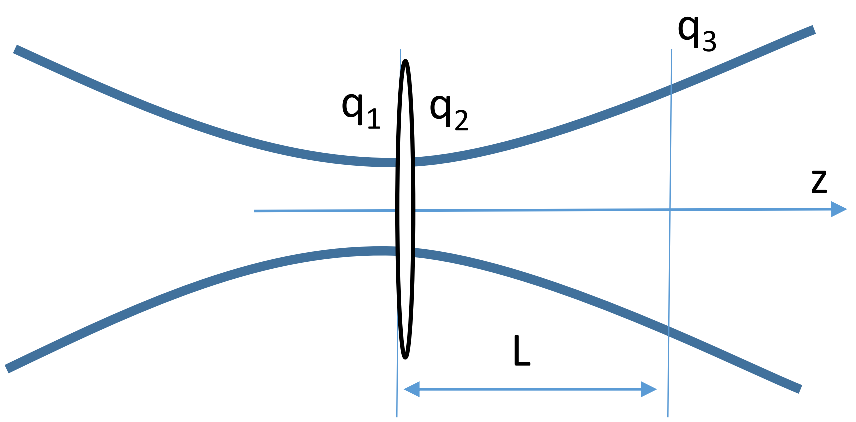
$$\left(\frac{1}{r_3} - i\,\frac{\lambda}{\pi\omega_3^2}\right)^{-1} = \left(-\frac{1}{f} - i\,\frac{\lambda}{\pi\omega_1^2}\right)^{-1} + L$$

Ottica Matriciale -Esempi

$$\hat{q}_2 = \frac{A\hat{q}_1 + B}{C\hat{q}_1 + D}$$

Lente sottile+
Propagazione
libera

$$q(z=0) = i\,z_0 = i\,\frac{\pi\omega_1^2}{\lambda} = q_1$$



$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{L}{f} & L \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$q_3 = q_2 + L$$

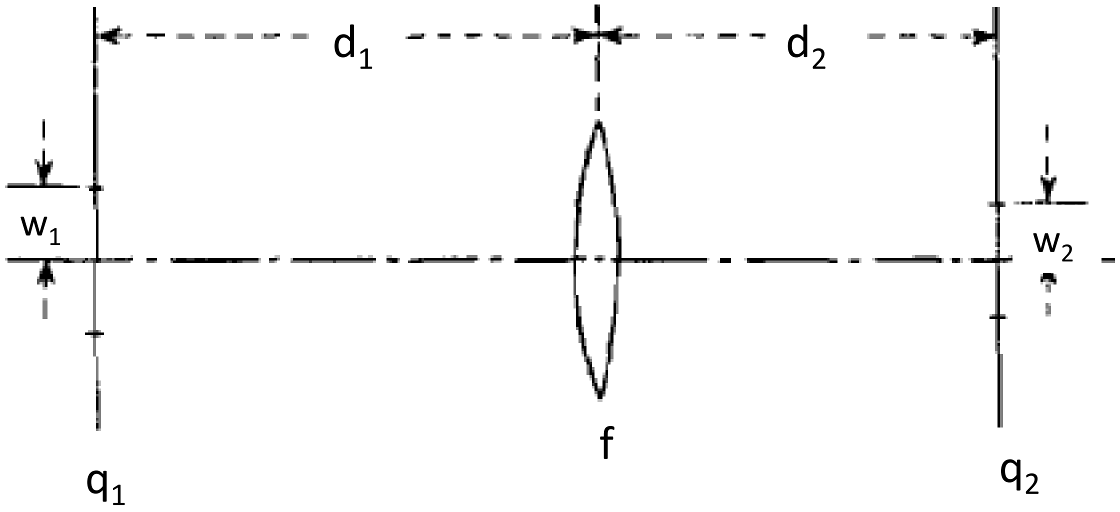
$$\left(\frac{1}{r_3} - i\,\frac{\lambda}{\pi\omega_3^2}\right)^{-1} = \left(-\frac{1}{f} - i\,\frac{\lambda}{\pi\omega_1^2}\right)^{-1} + L$$

$$r_3(L) = \frac{((L-f)\pi\omega_1^2)^2 + (f\lambda L)^2}{(L-f)(\pi\omega_1^2)^2 + f^2\lambda^2 L}$$

$$\omega_3^2(L) = \frac{((L-f)\pi\omega_1^2)^2 + (f\lambda L)^2}{(\pi\omega_1 f)^2}$$

Ottica Matriciale -Esempi

- Mode matching



$$M = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - d_2/f & d_1 + d_2 - d_1 d_2/f \\ -1/f & 1 - d_1/f \end{pmatrix}$$

$$q(z=0) = i\, z_0 = i\, \frac{\pi \omega_1^2}{\lambda} = q_1$$

Impongo $q_2 = i\, \frac{\pi \omega_2^2}{\lambda}$

$$q_2 = \frac{(1 - d_2/f) q_1 + (d_1 + d_2 - d_1 d_2/f)}{1 - d_1/f - q_1/f}$$

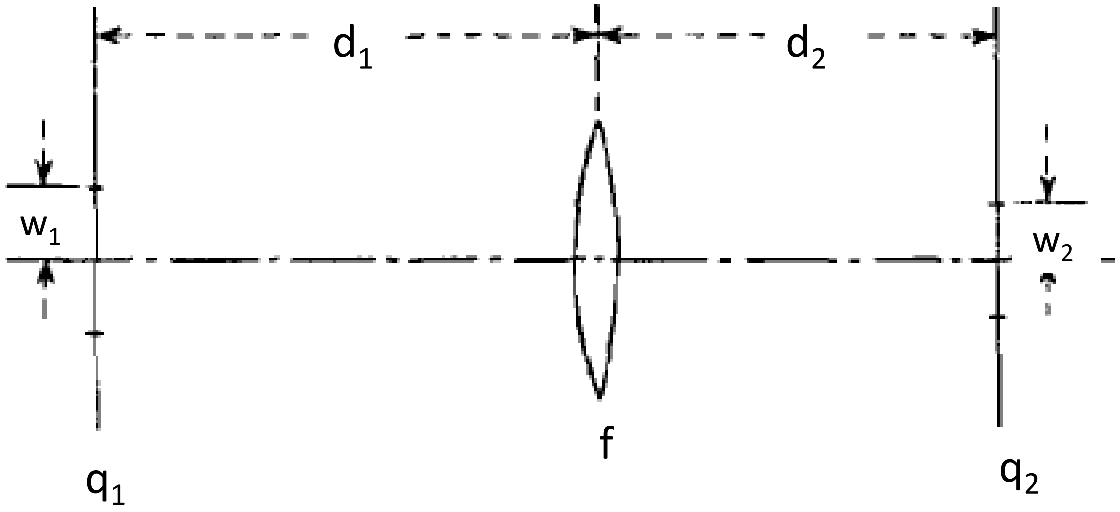
$$i\, \frac{\pi \omega_2^2}{\lambda} = d_2 - f - \frac{f^2 \lambda}{(d_1 - f)\lambda + i\, \pi \omega_1^2}$$

$$\frac{\omega_2^2}{\omega_1^2} = \frac{d_2 - f}{d_1 - f}$$

$$(d_1 - f)(d_2 - f) = f^2 - f_0^2$$

Ottica Matriciale -Esempi

Mode matching



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - d_2/f & d_1 + d_2 - d_1 d_2/f \\ -1/f & 1 - d_1/f \end{pmatrix}$$

$$i\,\frac{\pi\omega_2^2}{\lambda} = d_2 - f - \frac{f^2\lambda}{(d_1 - f)\lambda + i\,\pi\omega_1^2}$$

$$\frac{\omega_2^2}{\omega_1^2} = \frac{d_2 - f}{d_1 - f}$$

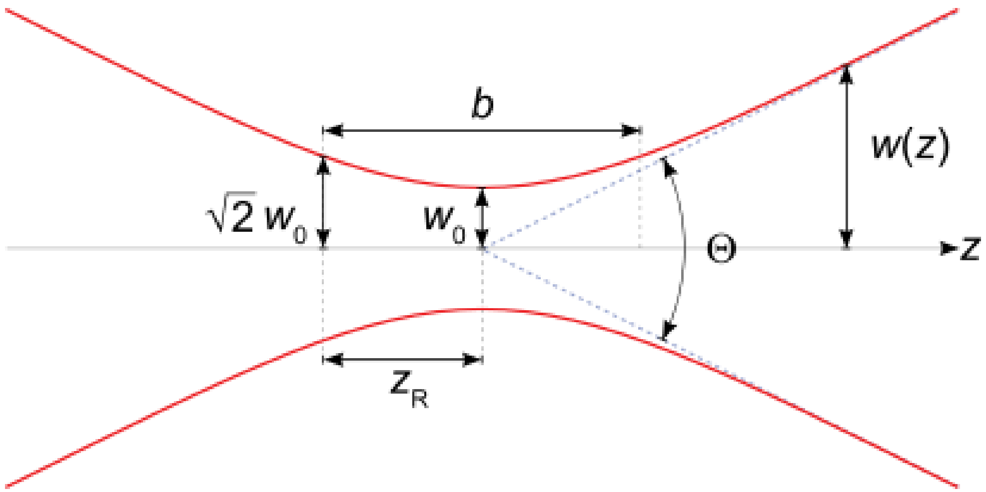
$$(d_1 - f)(d_2 - f) = f^2 - f_0^2$$

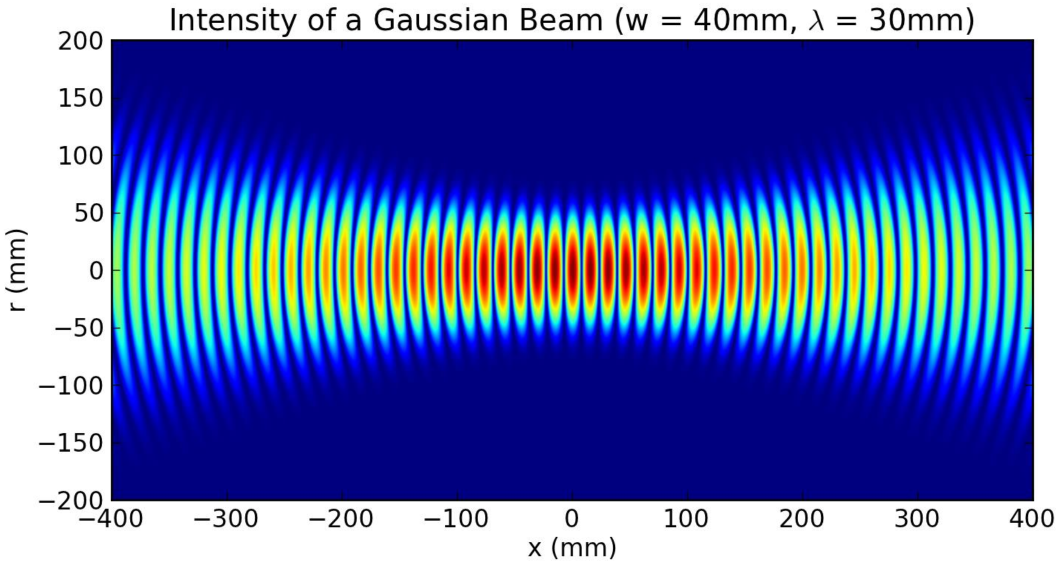
$$d_1 = f \pm \frac{w_1}{w_2}\sqrt{f^2 - f_0^2},$$

$$d_2 = f \pm \frac{w_2}{w_1}\sqrt{f^2 - f_0^2}.$$

$f > f_0$

con $f_0 = \pi w_1 w_2/\lambda$


b
w(z)
√2 w0
w0
Θ
z
zR


Intensity of a Gaussian Beam (w = 40mm, λ = 30mm)
r (mm)
x (mm)
200
150
100
50
0
−50
−100
−150
−200
−400
−300
−200
−100
0
100
200
300
400