

Laboratorio di Fisica Atomica CdL Fisica e Astrofisica

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Programma Lezioni

- 4 lezioni

- Fasci Gaussiani

Soluzione eq. Maxwell parassiale, principali proprietà, propagazione, formulazione matriciale

Mer. 6 Novembre

- Polarizzazione onde e.m.

Stati di polarizzazione, rappresentazione con vettore di Jones, birifrangenza ed ottiche polarizzanti (lamine di ritardo, polarizzatori)

Mer. 13 Novembre

- Riflessione e rifrazione

Applicazioni eq. di Fresnel, riflessione totale, dispersione, trattamenti (coatings) AR, HR, cubi

Mer. 20 Novembre

- Propagazione guidata ed elementi elettro-ottici

Fibre ottiche, Acusto-ottici, elettro-ottici

Mer. 27 Novembre

Rifrazione – Riflessione - Polarizzazione

- Testi/articoli di riferimento:

- E. Hecht «Optics»

- G. R. Fowles “Introduction to modern optics”

- R.D. Guenther «Modern Optics»

Polarizzazione

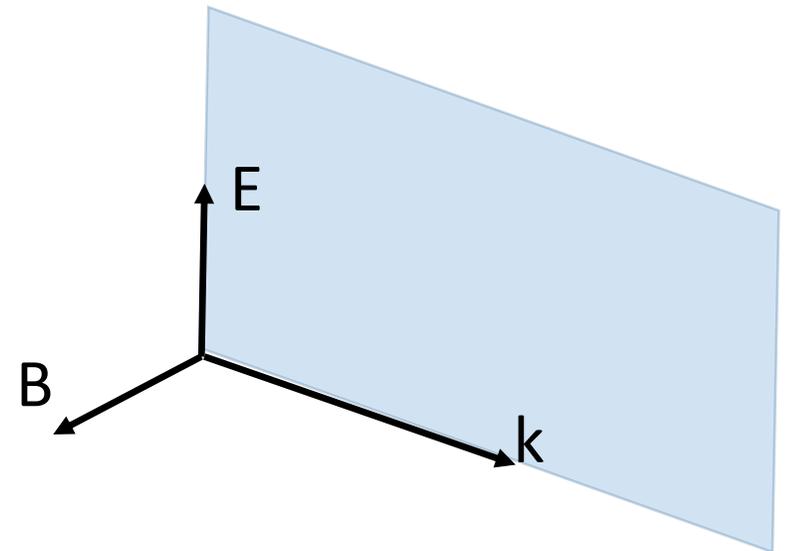
- Direzione del vettore campo elettrico

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \exp[i(\omega t - \vec{k} \cdot \vec{r})]$$

$$\frac{F_E}{F_H} = \frac{eE}{evB} \quad n|\mathbf{E}| = c|\mathbf{B}| \quad \frac{F_E}{F_H} = \frac{c}{nv} = 8.9 \times 10^5$$

Particella carica in aria alla velocità del suono $v = 335 \text{ m/sec}$

- Piano contenente vettore E e vettore k (piano di «polarizzazione» piano di «vibrazione»)



Polarizzazione lineare

(a)

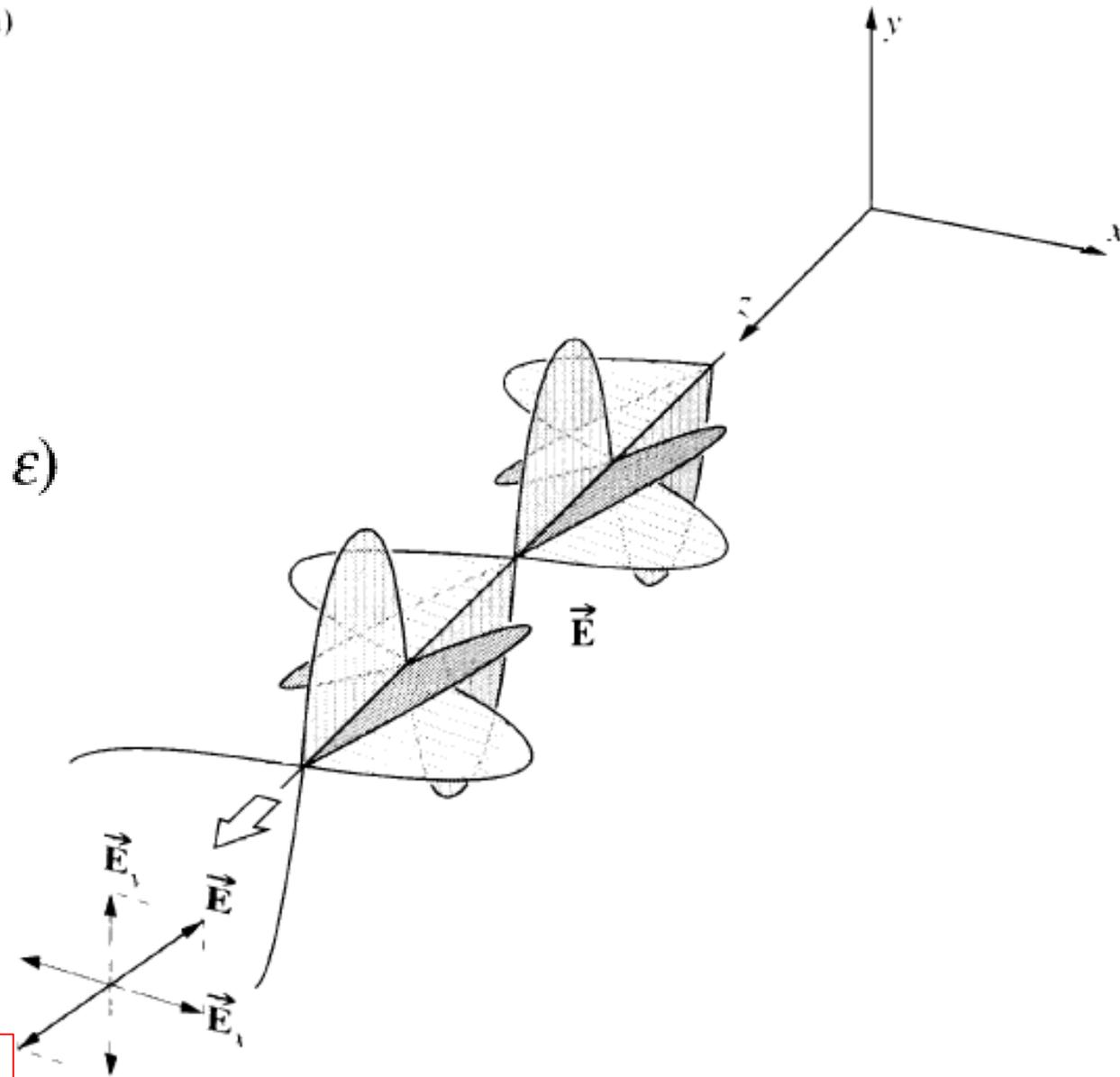
$$\vec{\mathbf{E}}(z, t) = \vec{\mathbf{E}}_x(z, t) + \vec{\mathbf{E}}_y(z, t)$$

$$\vec{\mathbf{E}}_x(z, t) = \hat{\mathbf{i}} E_{0x} \cos(kz - \omega t)$$

$$\vec{\mathbf{E}}_y(z, t) = \hat{\mathbf{j}} E_{0y} \cos(kz - \omega t + \varepsilon)$$

$$\varepsilon = 2m\pi$$

$$\vec{\mathbf{E}} = (\hat{\mathbf{i}} E_{0x} + \hat{\mathbf{j}} E_{0y}) \cos(kz - \omega t)$$



(linearly polarized - \mathcal{P} - state)

Polarizzazione lineare

$$\vec{E}(z, t) = \vec{E}_x(z, t) + \vec{E}_y(z, t)$$

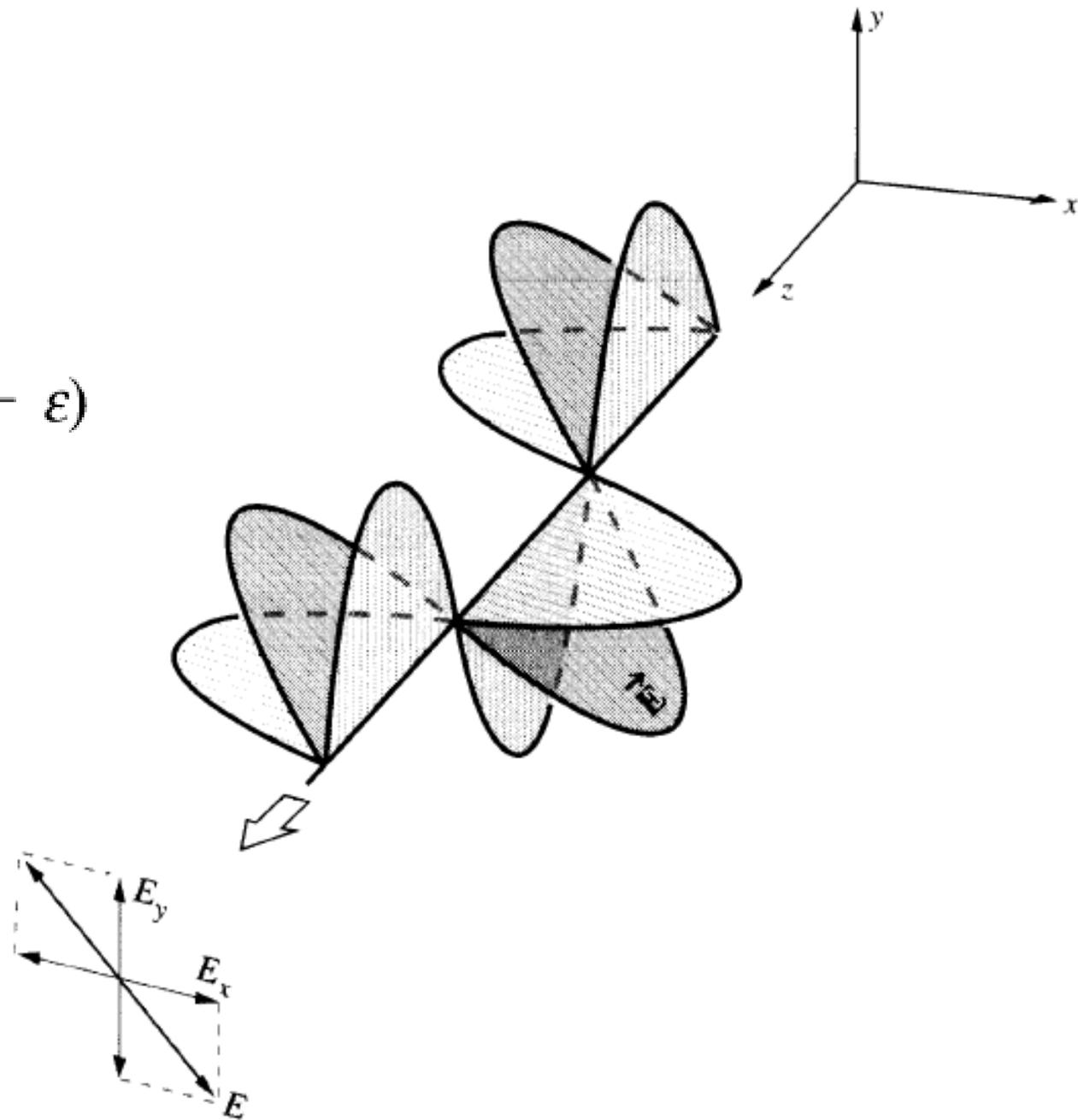
$$\vec{E}_x(z, t) = \hat{i} E_{0x} \cos(kz - \omega t)$$

$$\vec{E}_y(z, t) = \hat{j} E_{0y} \cos(kz - \omega t + \varepsilon)$$

$$\varepsilon = \pi + 2m\pi$$

$$\vec{E} = (\hat{i}E_{0x} - \hat{j}E_{0y}) \cos(kz - \omega t)$$

(linearly polarized - \mathcal{P} - state)



Polarizzazione circolare

$$\vec{\mathbf{E}}(z, t) = \vec{\mathbf{E}}_x(z, t) + \vec{\mathbf{E}}_y(z, t)$$

$$\vec{\mathbf{E}}_x(z, t) = \hat{\mathbf{i}} E_{0x} \cos(kz - \omega t)$$

$$\vec{\mathbf{E}}_y(z, t) = \hat{\mathbf{j}} E_{0y} \cos(kz - \omega t + \varepsilon)$$

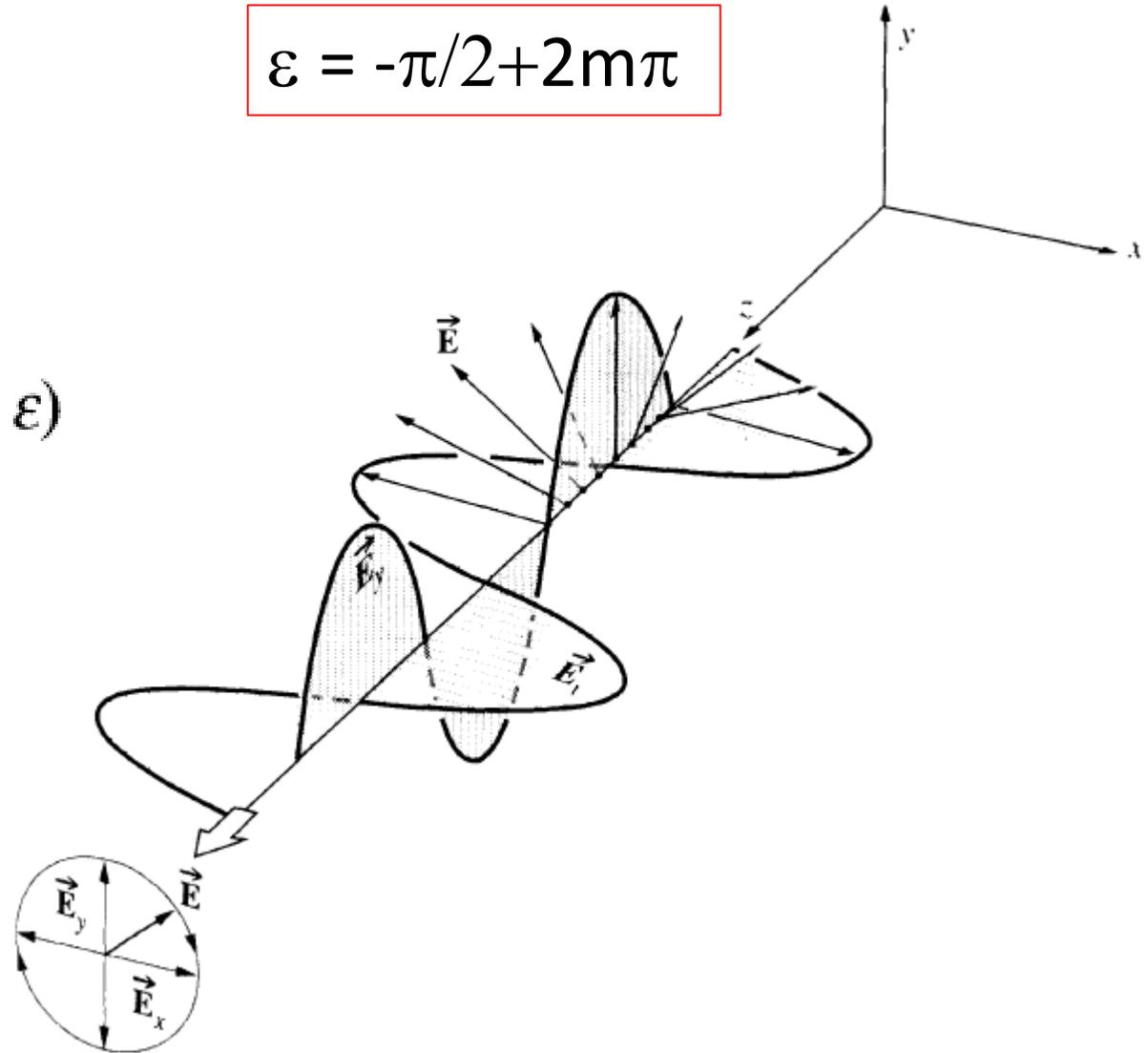
$$\varepsilon = -\pi/2 + 2m\pi$$

$$\vec{\mathbf{E}}_x(z, t) = \hat{\mathbf{i}} E_0 \cos(kz - \omega t)$$

$$\vec{\mathbf{E}}_y(z, t) = \hat{\mathbf{j}} E_0 \sin(kz - \omega t)$$

$$\vec{\mathbf{E}} = E_0[\hat{\mathbf{i}} \cos(kz - \omega t) + \hat{\mathbf{j}} \sin(kz - \omega t)]$$

$$(\vec{\mathbf{E}} \cdot \vec{\mathbf{E}})^{1/2} = E_0$$



Polarizzazione circolare

$$\vec{E} = E_0[\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t)]$$

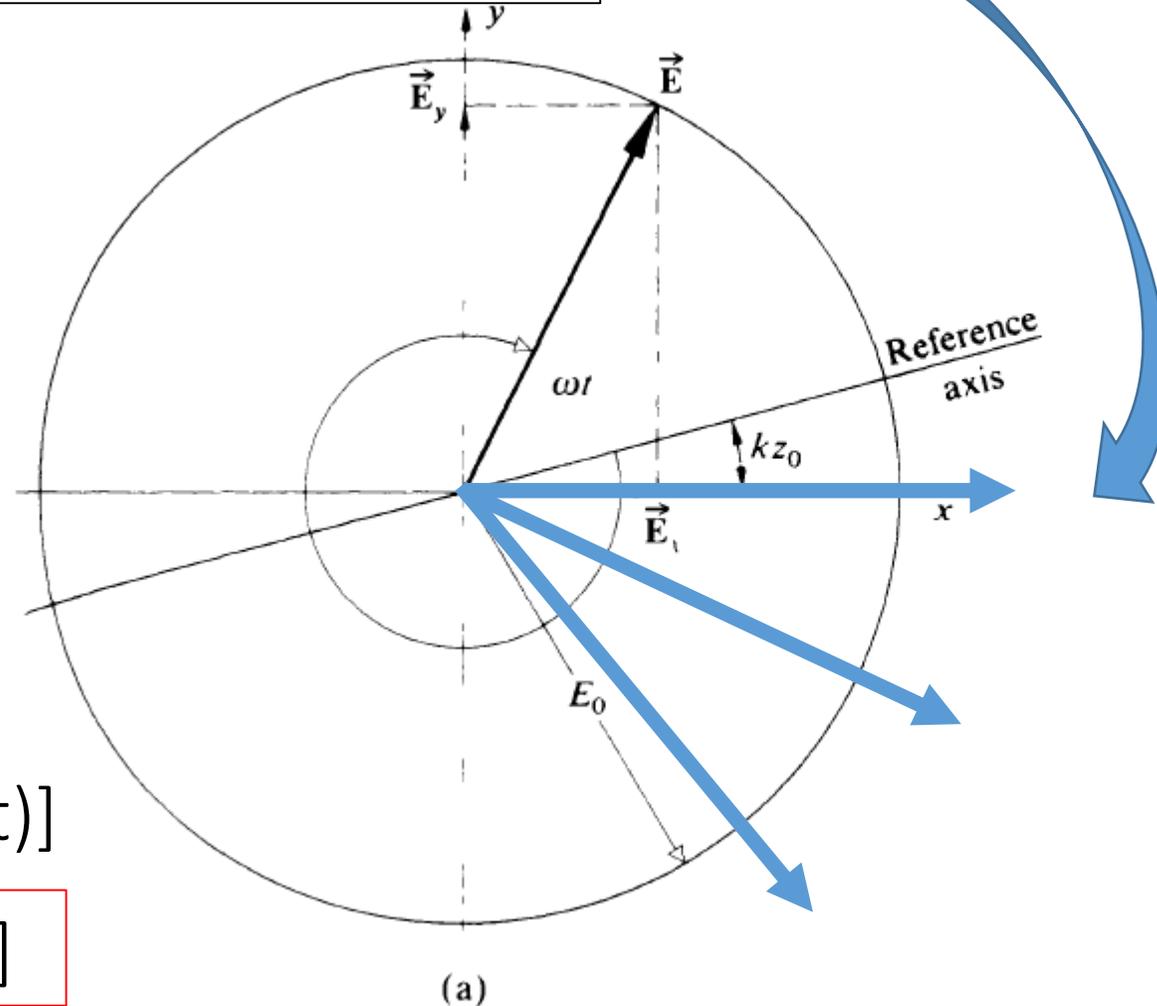
$$\varepsilon = -\pi/2 + 2m\pi$$

Onda polarizzata $\sigma+$

$$z = z_0 = 0$$

$$\mathbf{E} = E_0[\mathbf{i} \cos(-\omega t) + \mathbf{j} \sin(-\omega t)]$$

$$\mathbf{E} = E_0[\mathbf{i} \cos(\omega t) - \mathbf{j} \sin(\omega t)]$$



Senso orario

Polarizzazione circolare

$$\varepsilon = -\pi/2 + 2m\pi$$

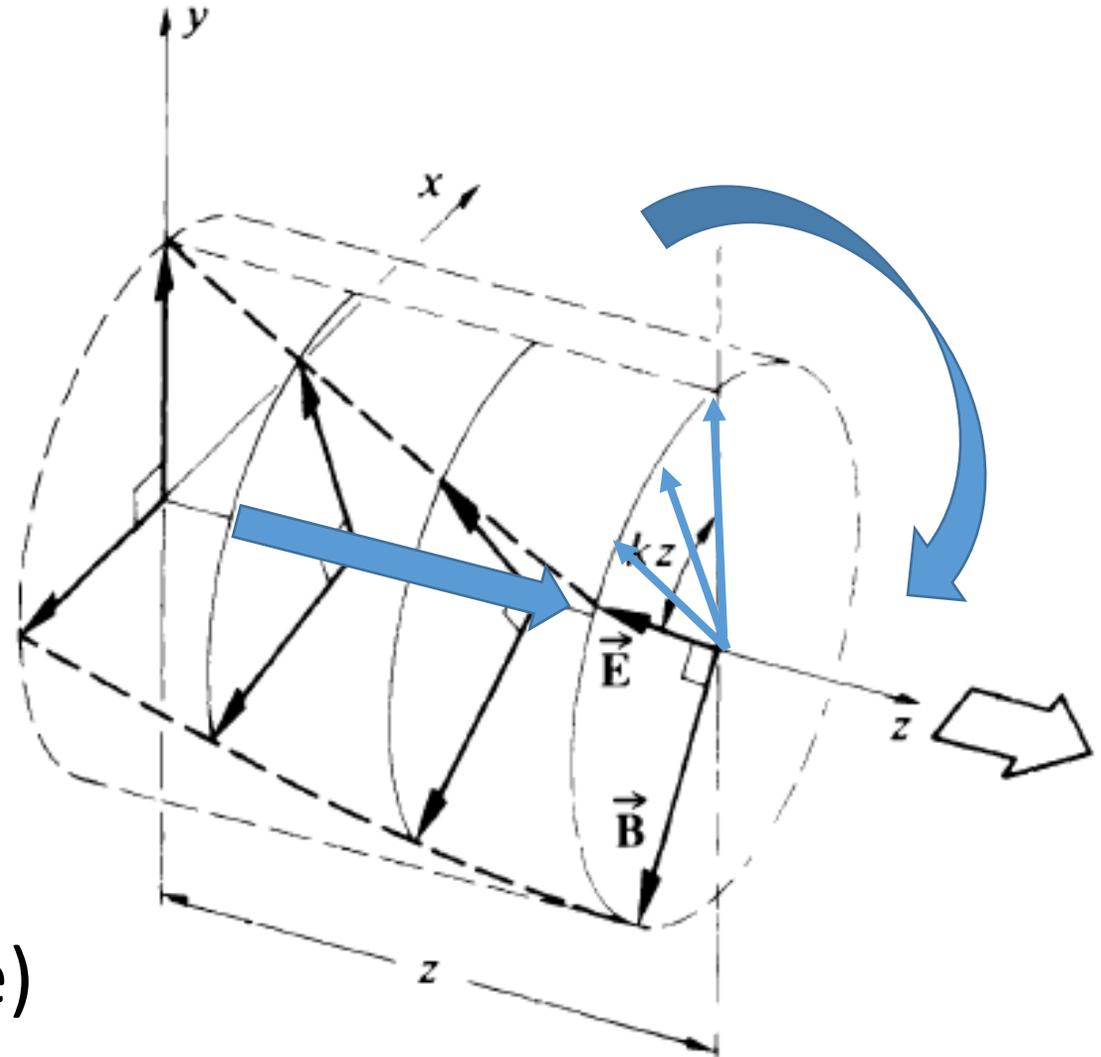
$$\vec{E} = E_0[\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t)]$$

$$t = t_0 = 0$$

$$\mathbf{E} = E_0[\mathbf{i} \cos(kz) + \mathbf{j} \sin(kz)]$$

Onda polarizzata $\sigma+$

(right-circularly polarized - \mathcal{R} -state)



Polarizzazione circolare

$$\varepsilon = \pi/2 + 2m\pi$$

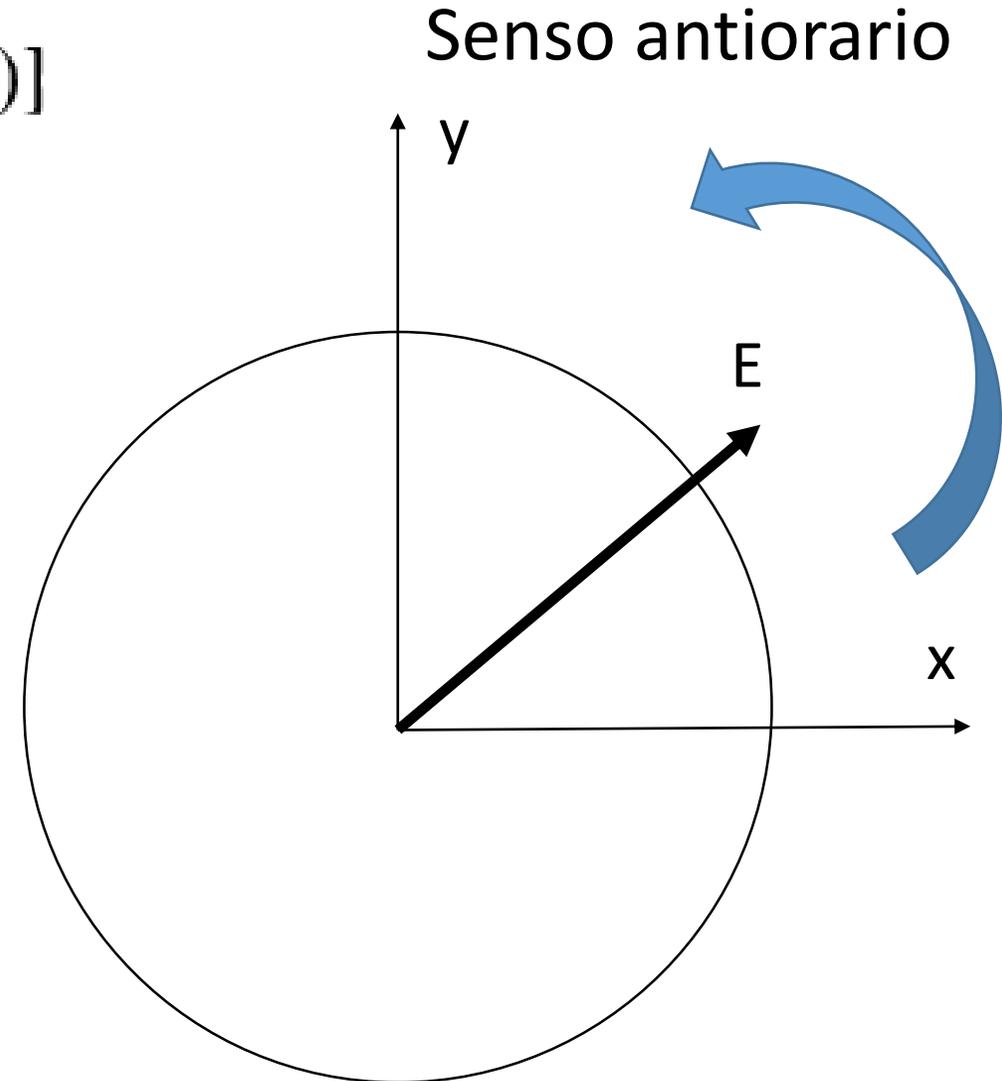
$$\vec{E} = E_0[\hat{i} \cos(kz - \omega t) - \hat{j} \sin(kz - \omega t)]$$

$$z = z_0 = 0$$

$$E = E_0[i \cos(\omega t) + j \sin(\omega t)]$$

Onda polarizzata σ^-

(Left-circularly polarized - \mathcal{L} - state)



Polarizzazione circolare

$$\vec{\mathbf{E}} = E_0[\hat{\mathbf{i}} \cos (kz - \omega t) + \hat{\mathbf{j}} \sin (kz - \omega t)] \quad \sigma+ (\mathcal{R}\text{-state})$$

+

+

$$\vec{\mathbf{E}} = E_0[\hat{\mathbf{i}} \cos (kz - \omega t) - \hat{\mathbf{j}} \sin (kz - \omega t)] \quad \sigma- (\mathcal{L}\text{-state})$$

=

=

$$\vec{\mathbf{E}} = 2E_0\hat{\mathbf{i}} \cos (kz - \omega t)$$

lin. (\mathcal{P} -state)

Onda polarizzata linearmente

Ellisse di polarizzazione

$$E_x = E_{0x} \cos(kz - \omega t) \quad \rightarrow \quad \sin(kz - \omega t) = [1 - (E_x/E_{0x})^2]^{1/2}$$

$$E_y = E_{0y} \cos(kz - \omega t + \varepsilon)$$

$$E_y/E_{0y} = \cos(kz - \omega t) \cos \varepsilon - \sin(kz - \omega t) \sin \varepsilon$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \varepsilon = -\sin(kz - \omega t) \sin \varepsilon$$

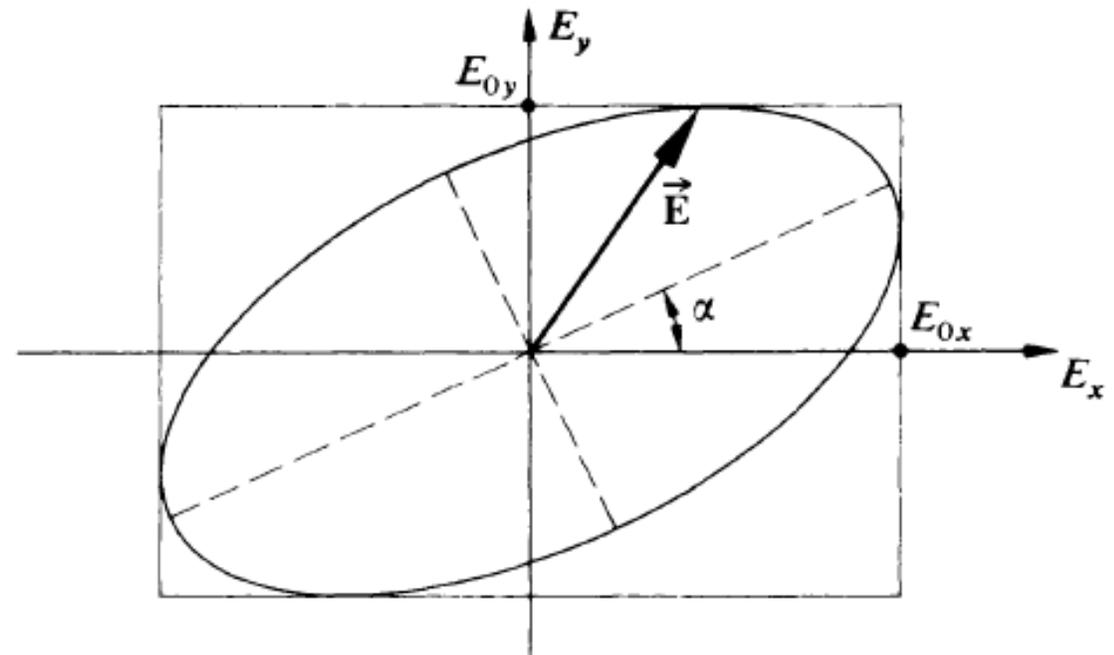
$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \varepsilon \right)^2 = \left[1 - \left(\frac{E_x}{E_{0x}} \right)^2 \right] \sin^2 \varepsilon$$

Ellisse di polarizzazione

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \varepsilon \right)^2 = \left[1 - \left(\frac{E_x}{E_{0x}} \right)^2 \right] \sin^2 \varepsilon$$

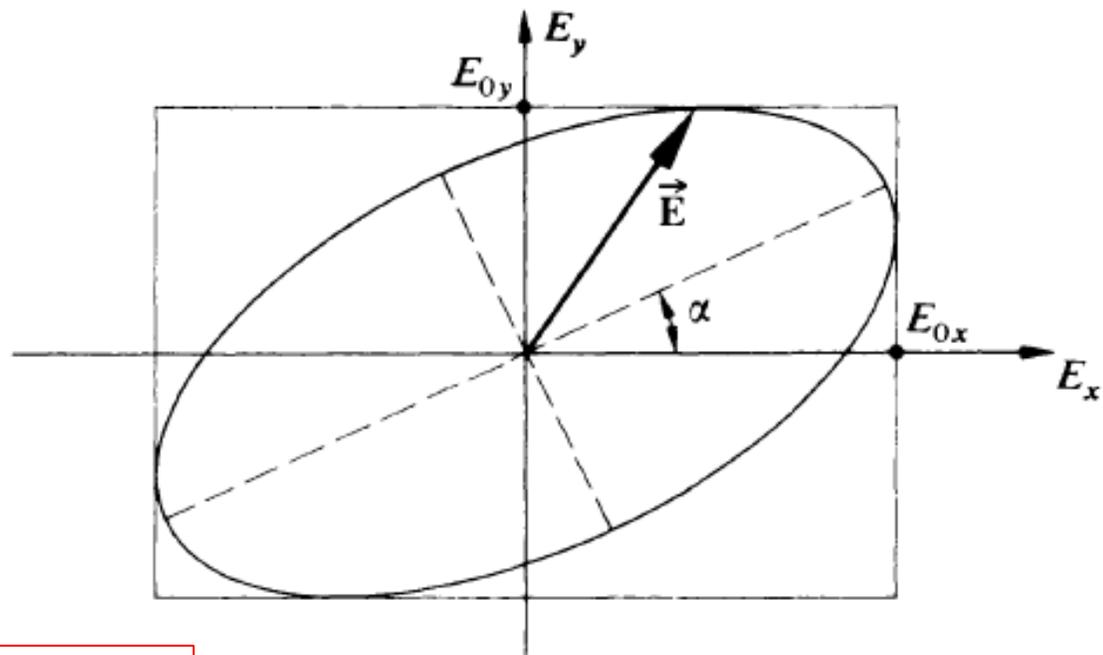
$$\left(\frac{E_y}{E_{0y}} \right)^2 + \left(\frac{E_x}{E_{0x}} \right)^2 - 2 \left(\frac{E_x}{E_{0x}} \right) \left(\frac{E_y}{E_{0y}} \right) \cos \varepsilon = \sin^2 \varepsilon$$

$$\tan 2\alpha = \frac{2E_{0x}E_{0y} \cos \varepsilon}{E_{0x}^2 - E_{0y}^2}$$

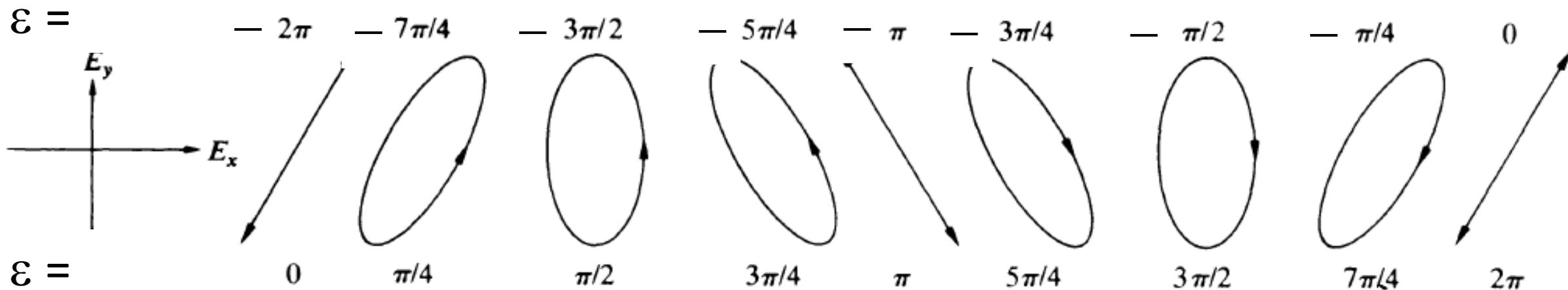


Ellisse di polarizzazione

$$\tan 2\alpha = \frac{2E_{0x}E_{0y} \cos \varepsilon}{E_{0x}^2 - E_{0y}^2}$$



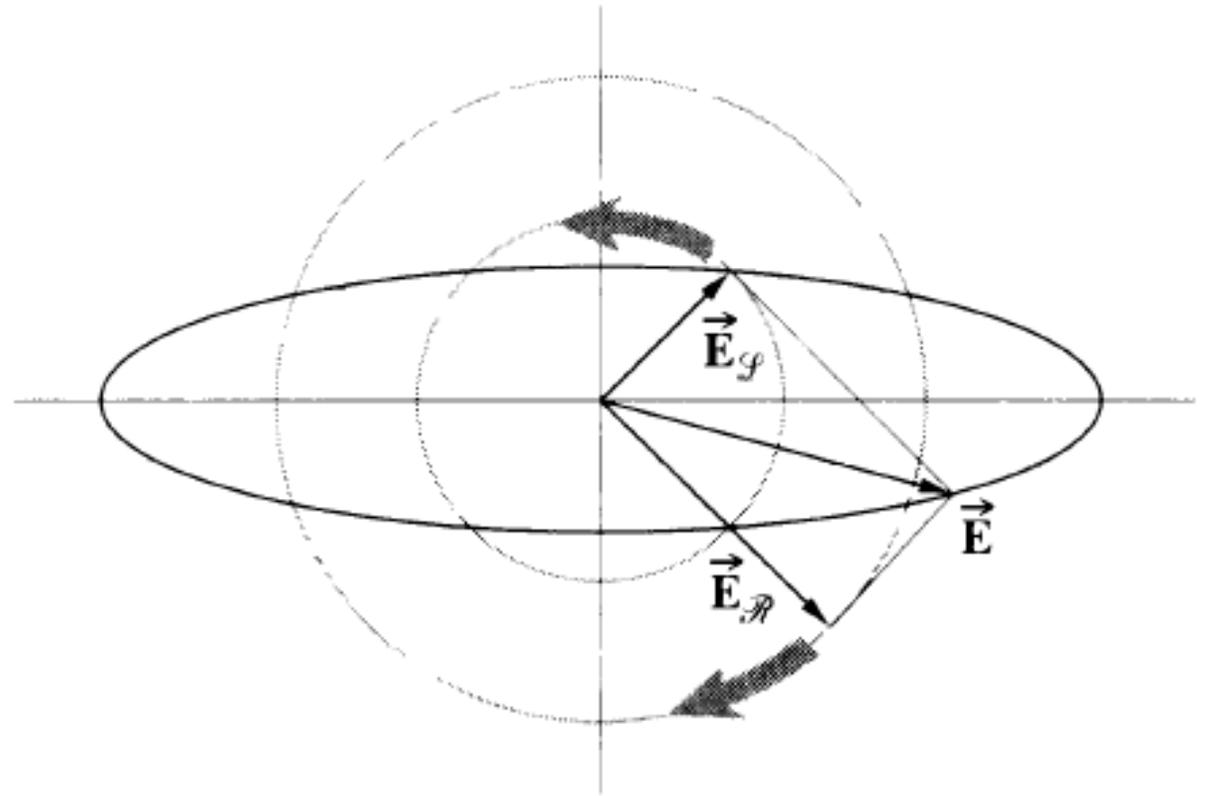
Polarizzazione Ellittica



\mathcal{E} -state

Ellisse di polarizzazione

Polarizzazione Ellittica



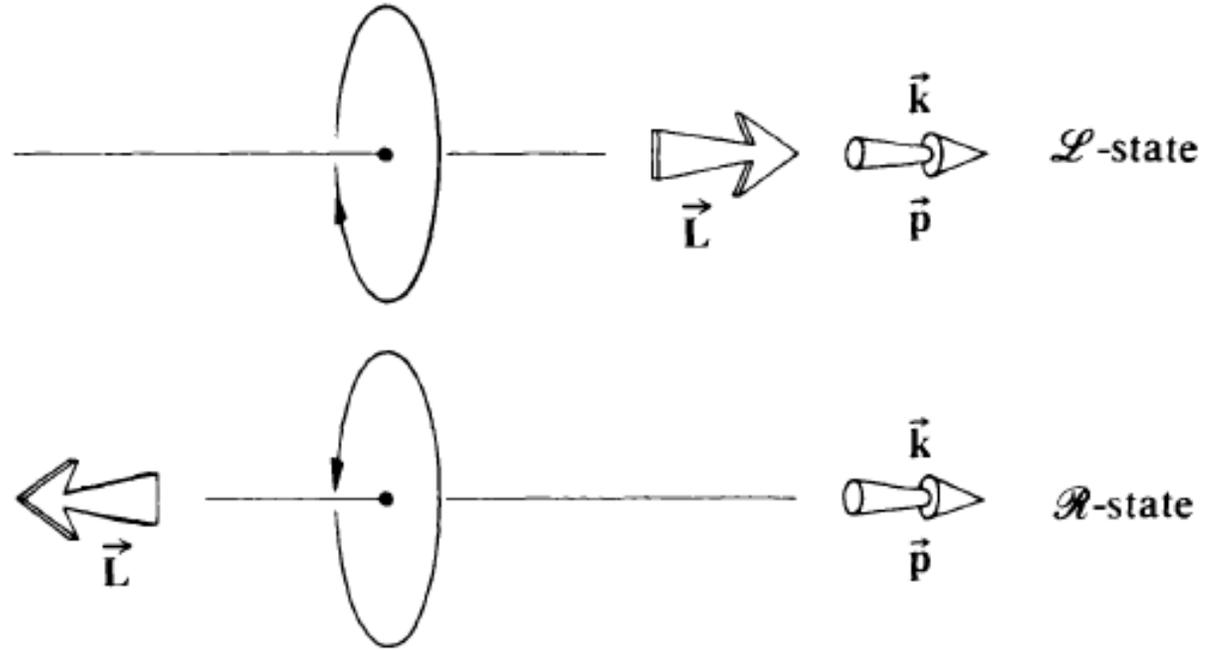
$$E\text{-state} = \mathcal{R}\text{-state} + \mathcal{L}\text{-state}$$

Polarizzazione circolare

Convenzione in fisica delle particelle

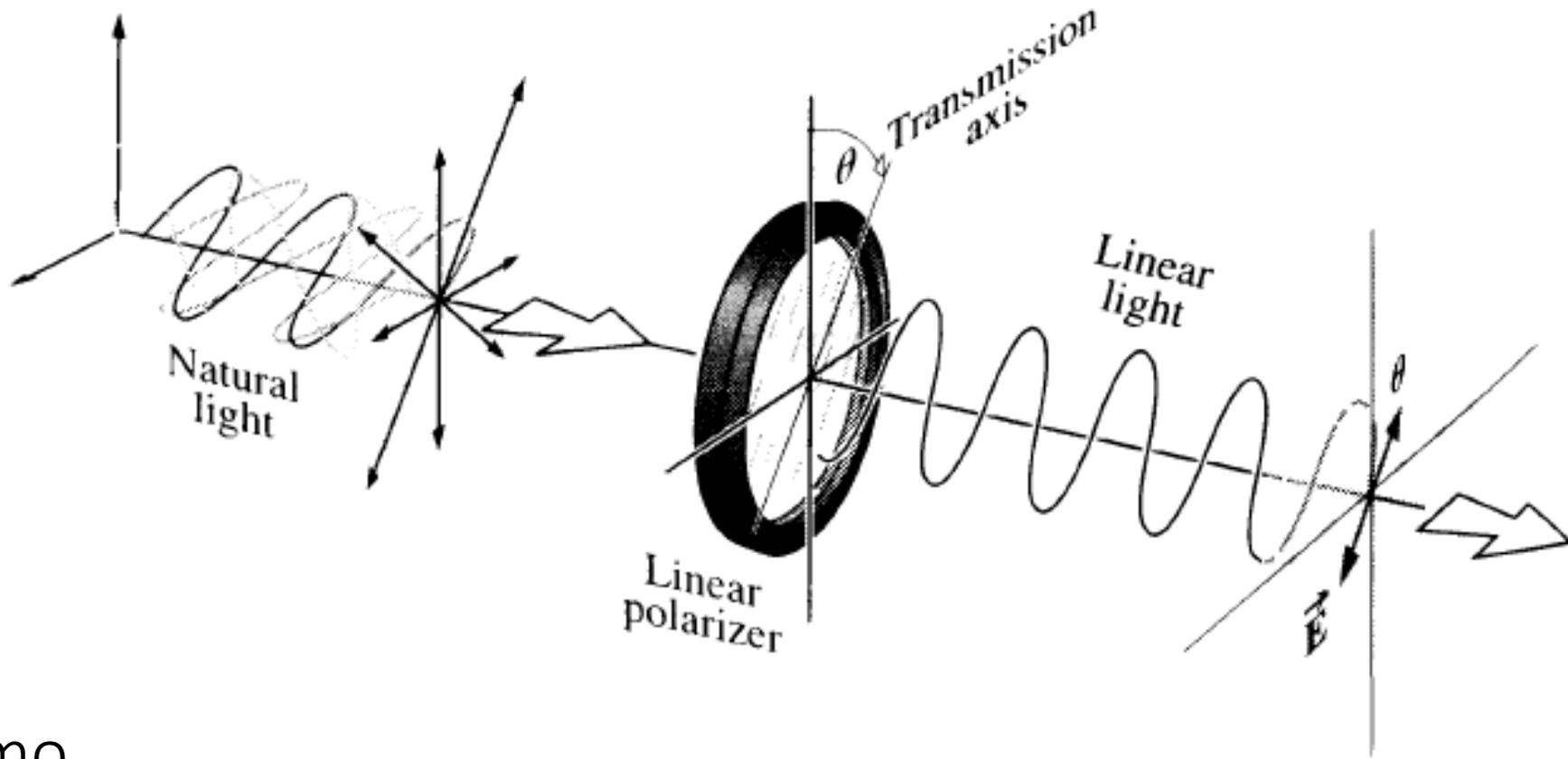
$$E = \hbar\omega$$

$$L = \pm \hbar$$



Opposta alla convenzione in ottica

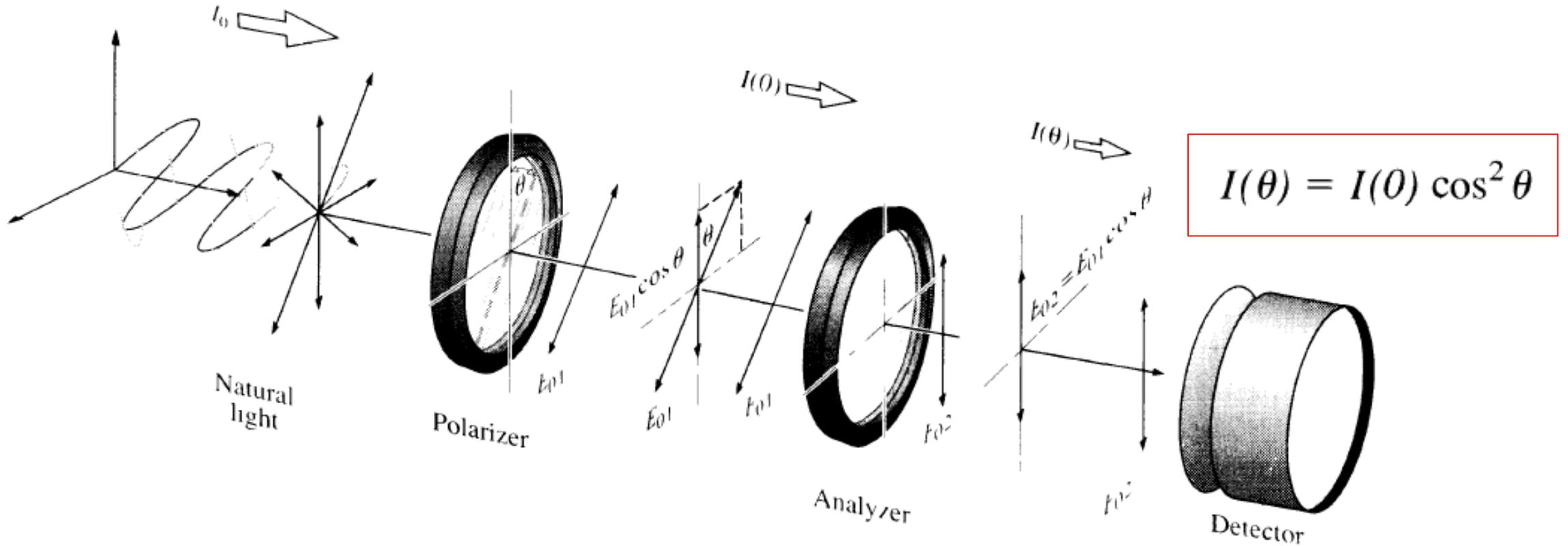
Polarizzatori



- Dicroismo
- Riflessione
- Scattering
- Birifrangenza

Materiali anisotropi

Polarizzatori



Legge di Malus

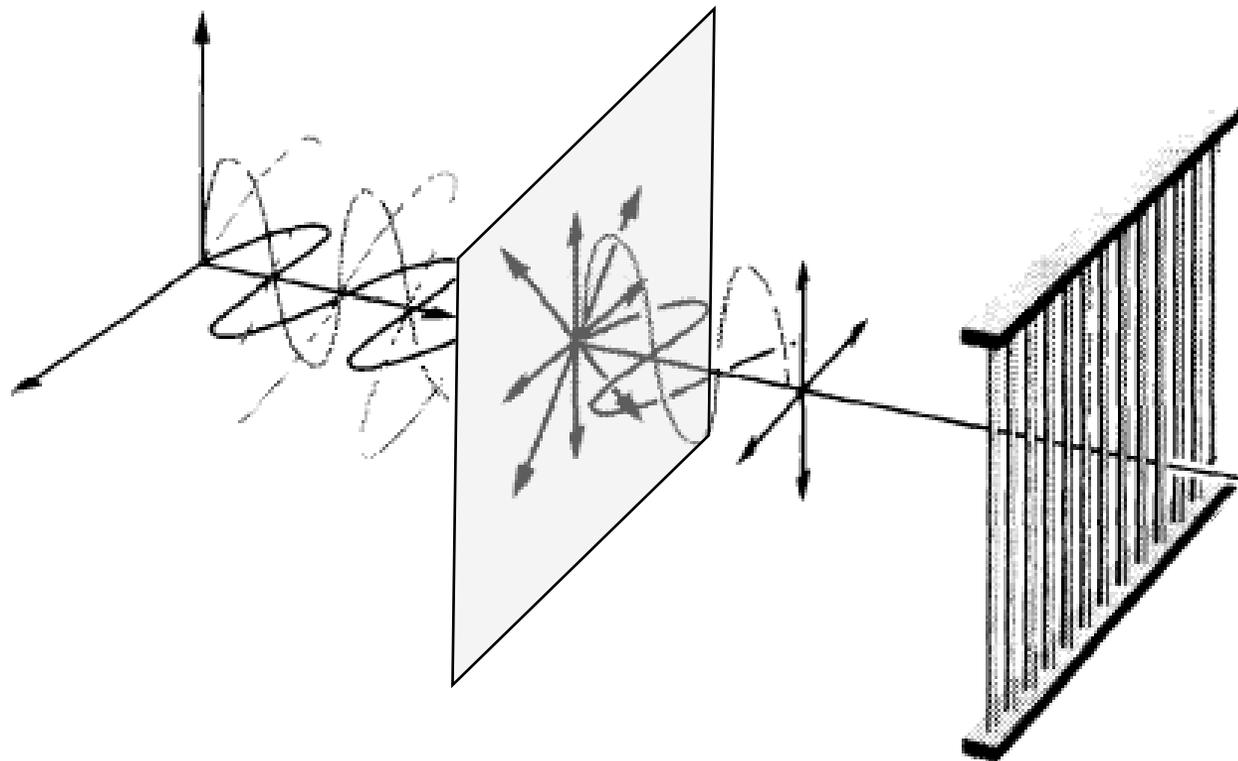
$$I = I_0 [\cos^2(\theta) + R \sin^2(\theta)]$$

R rapporto di estinzione

$$R = 10^{-2} - 10^{-3}$$

Dicroismo

- «Due colori»
- Assorbimento selettivo una delle due componenti \mathcal{P} ortogonali
 - Conducive «wire – grid» polarizer

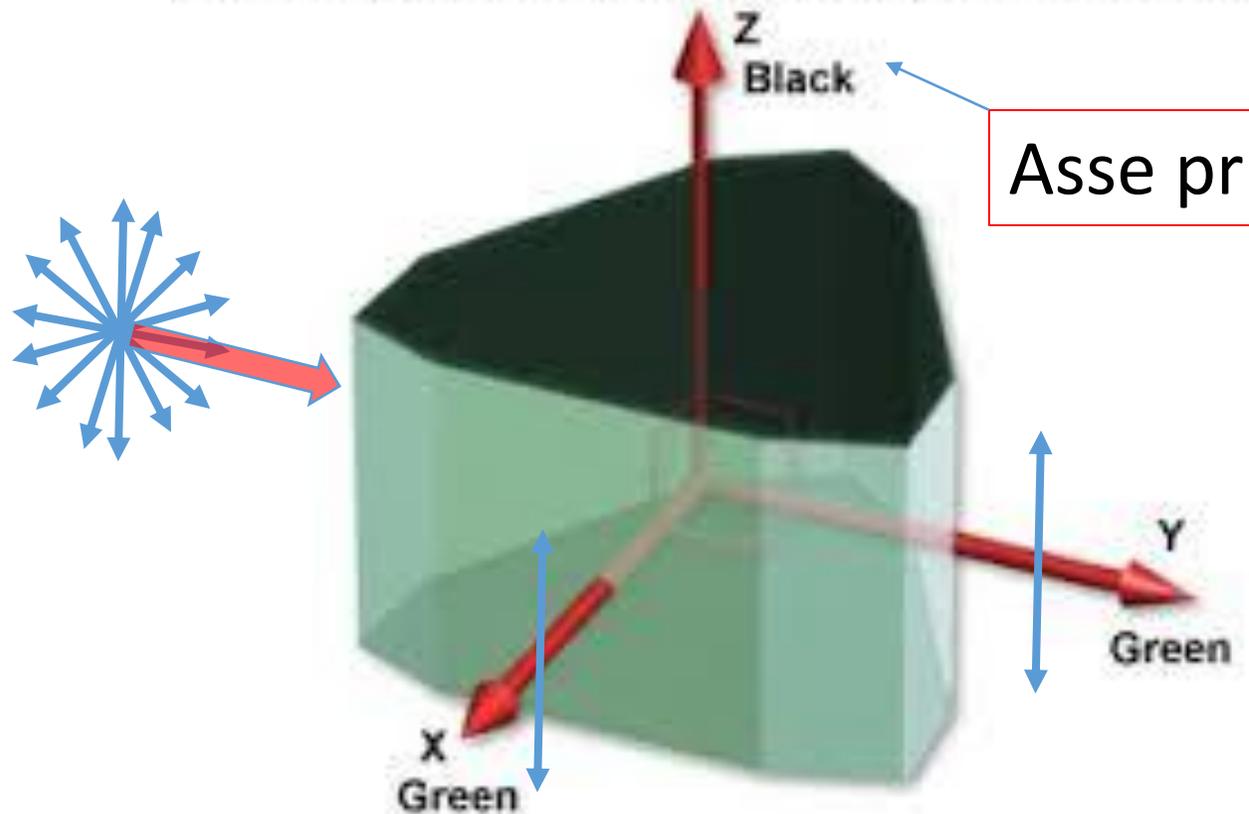


?

Cristalli dicroici

- Turmalina

Dichroic Tourmaline Crystal Light Transmission



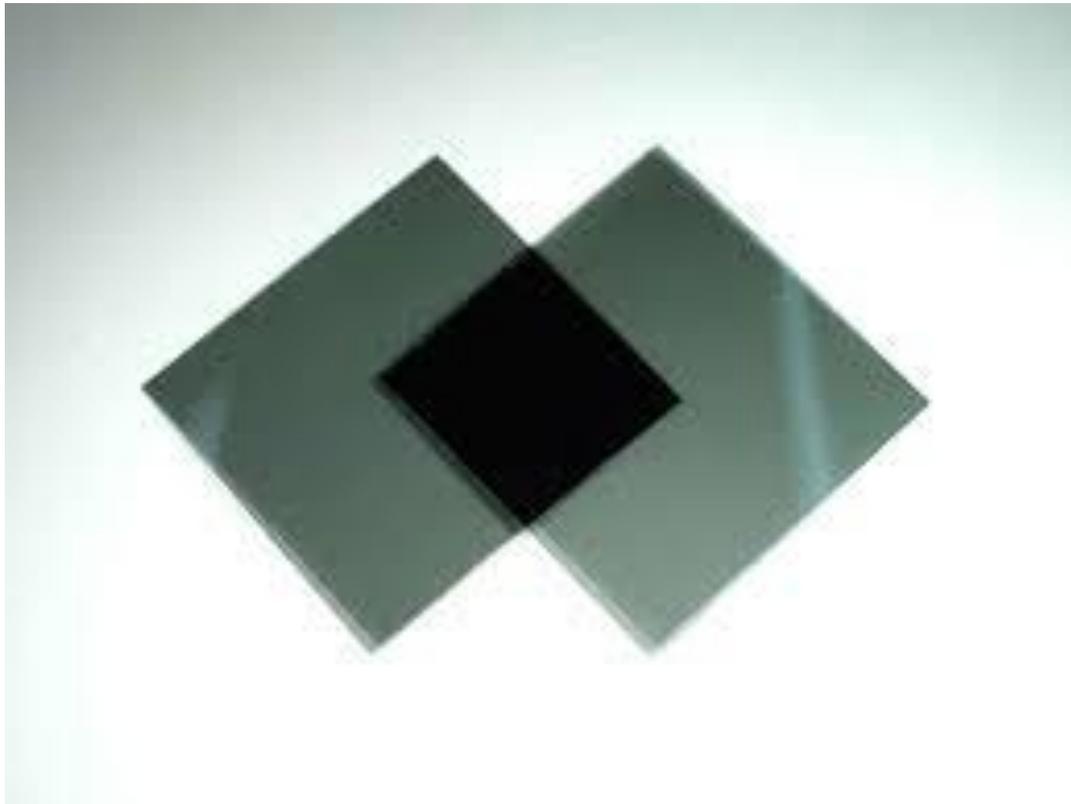
Asse principale o asse ottico

La componente del campo \mathbf{E} ortogonale a questo asse è \perp assorbita dal materiale

<https://www.youtube.com/watch?v=n0B2H8WcpwA>

Polaroid

- LPVISE2X2 Extinction Ratio:
 - >100:1 for 400 - 500 nm
 - >1000:1 for 500 - 700 nm
 - >5000:1 for 530 - 690 nm



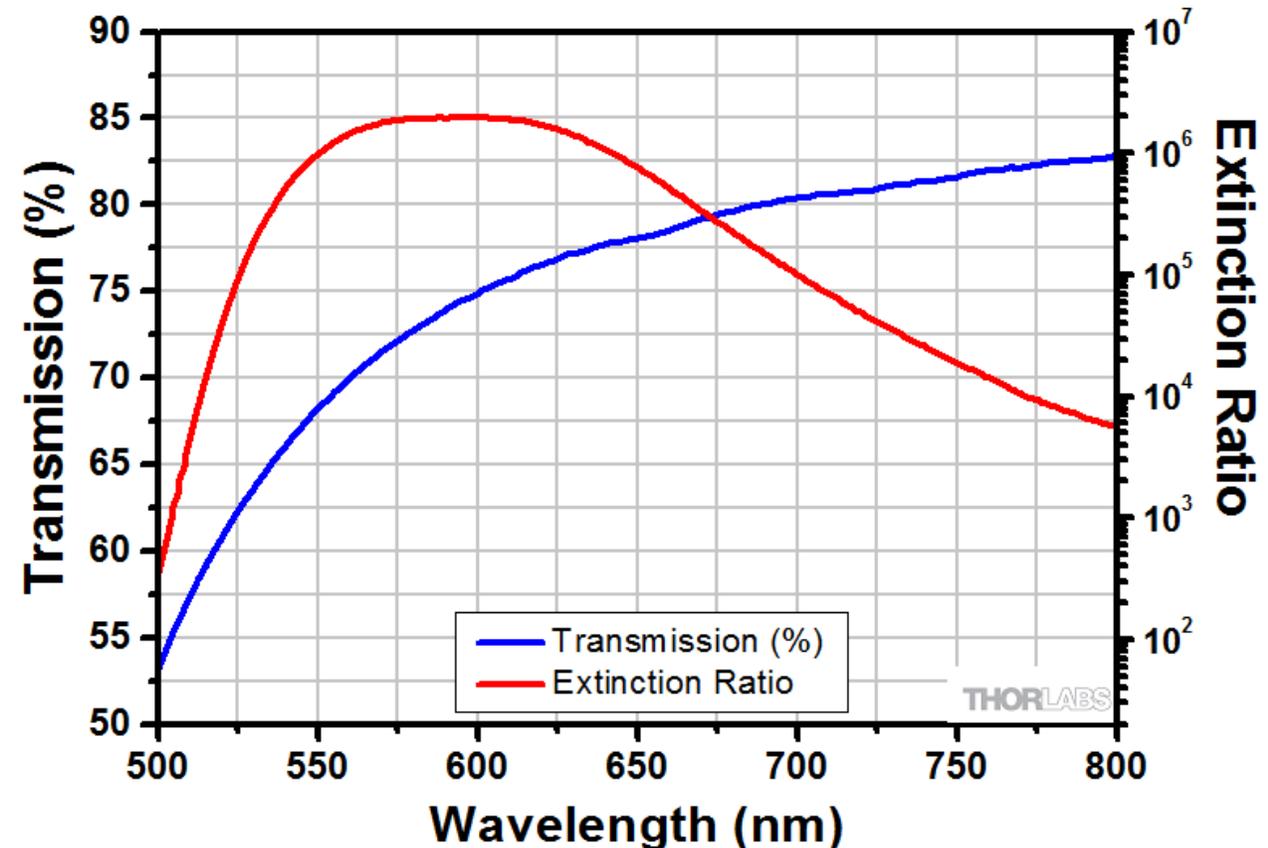
Nanoparticle Linear Film Polarizer

Spherical ellipsoid nanoparticles
embedded in sodium-silicate glass

Extinction Ratios up to 100 000:1

Laser Damage Thresholds up to 25 W/cm²

LPVISC Series



Polarizer Selection Guide

Parametri

- Intervallo di lunghezza d'onda
- Rapporto di estinzione
- Trasmissione
- Dimensioni
- Soglia di danneggiamento
- Prezzo €€€

https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=752

Wire Grid Polarizers

Film Polarizers

Beamsplitting Polarizers

alpha-BBO Polarizers

Calcite Polarizers

Quartz Polarizers

Magnesium Fluoride Polarizers

Yttrium Orthovanadate (YVO₄) Polarizers

Rutile Polarizers

Birifrangenza

- Anisotropia ottica

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

~~$$\mathbf{P} = \epsilon_0 \chi \mathbf{E}$$~~

- In generale \mathbf{D} e \mathbf{P} non paralleli ad \mathbf{E}

$$(D_1 \quad D_2 \quad D_3) = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$

Crystal Class	Tensor	Number of Coefficients	Optical Type
Cubic, three 4-fold axes	$\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix}$	1	Isotropic
Trigonal, one 3-fold symmetry axis	$\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_1 & 0 \\ 0 & 0 & S_2 \end{bmatrix}$	2	Uniaxial
Triclinic, no symmetry (at most, a center of symmetry)	$\begin{bmatrix} S_{11} & S_{12} & S_{31} \\ S_{12} & S_{22} & S_{23} \\ S_{31} & S_{23} & S_{33} \end{bmatrix}$	6	Biaxial
Monoclinic one 2-fold symmetry axis	$\begin{bmatrix} S_1 & 0 & S_{31} \\ 0 & S_2 & 0 \\ S_{31} & 0 & S_3 \end{bmatrix}$	4	
Orthorombic, three perpendicular 2-fold axes of	$\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$	3	

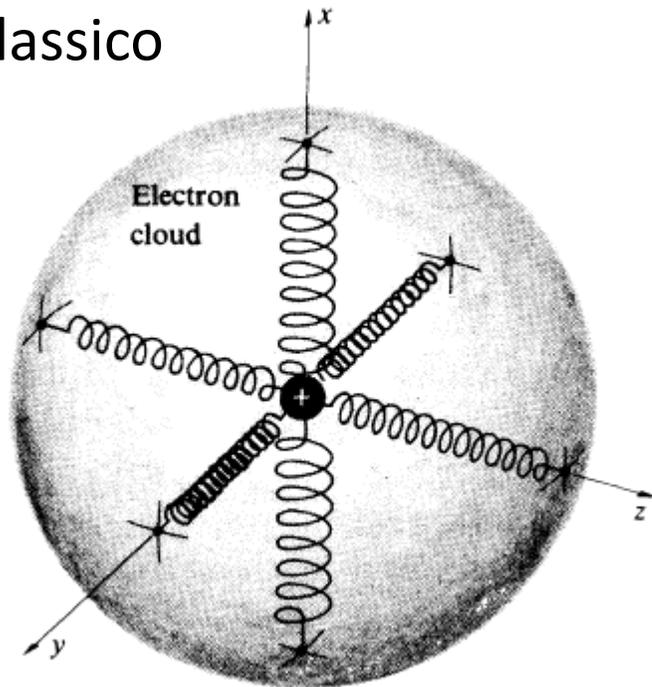
Birifrangenza

- Anisotropia ottica

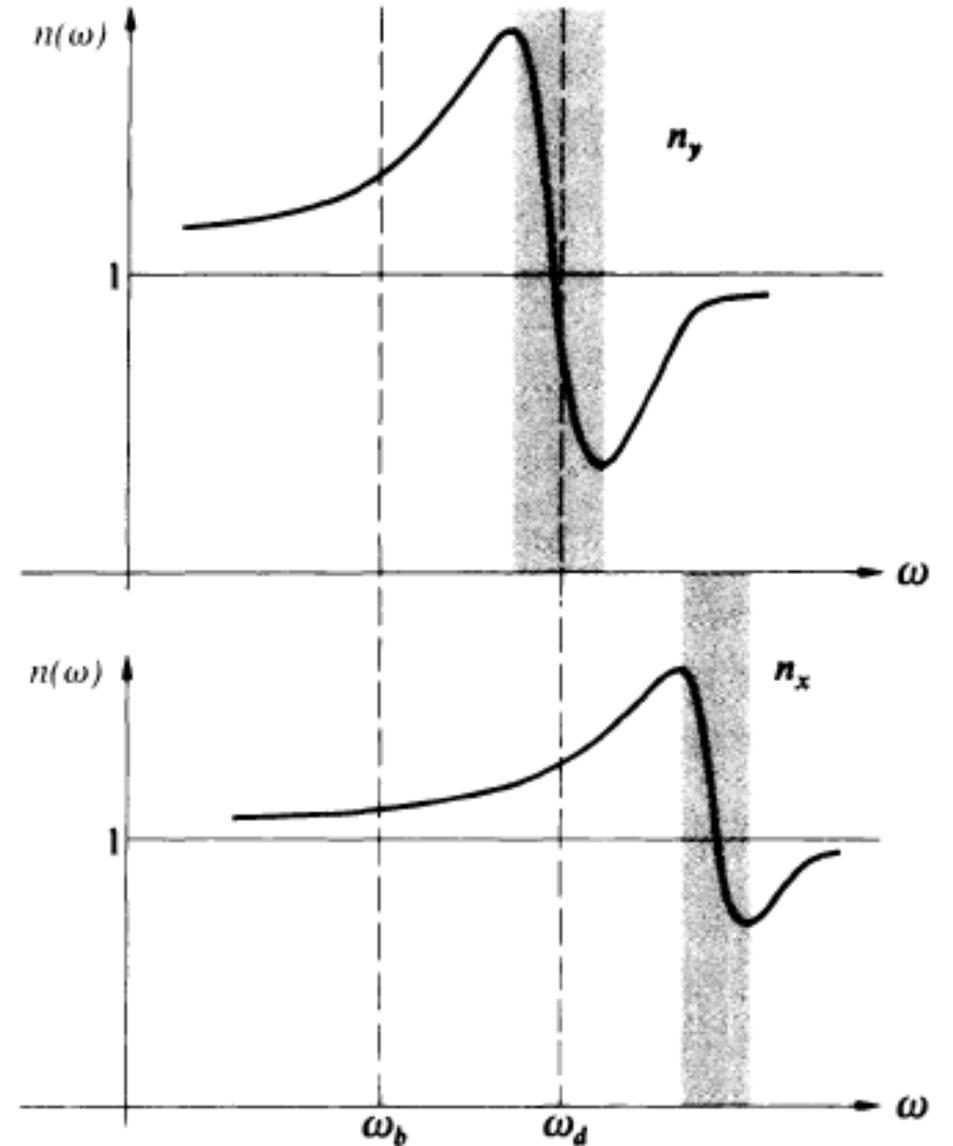
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$P = \epsilon_0 \chi E$$

- modello semiclassico

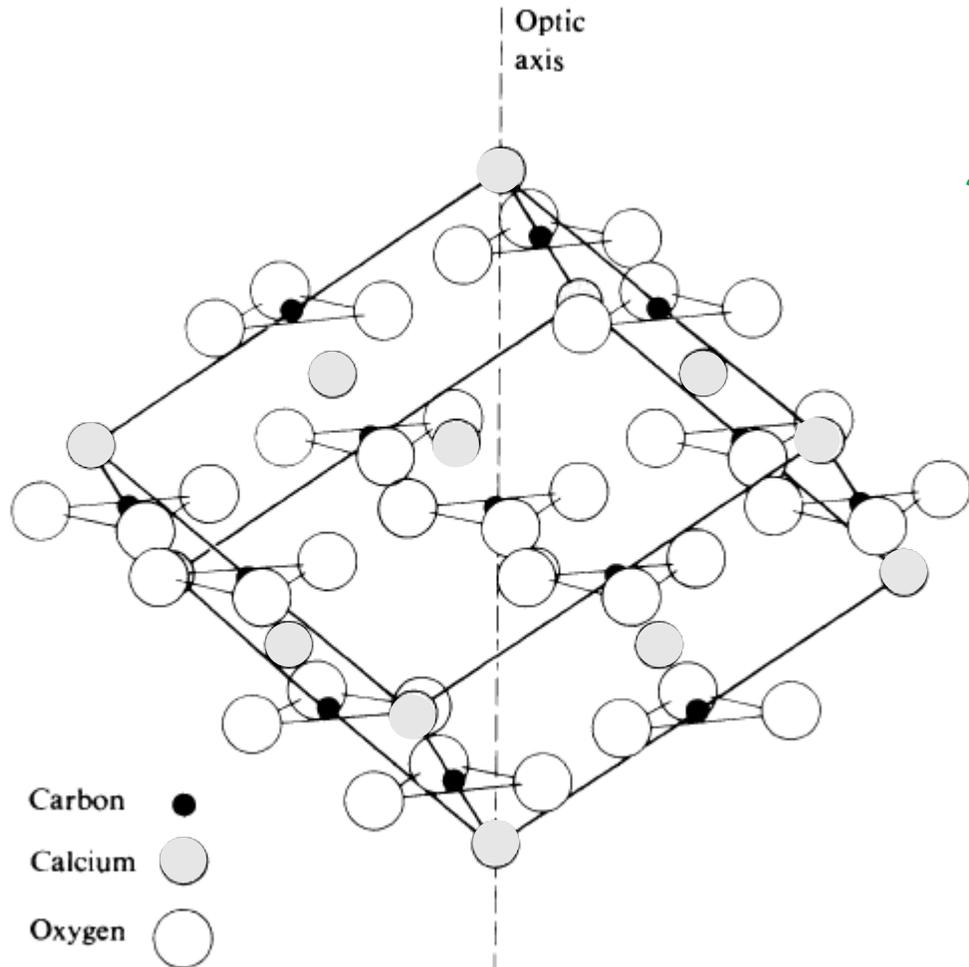


- anisotropia negli indici di rifrazione

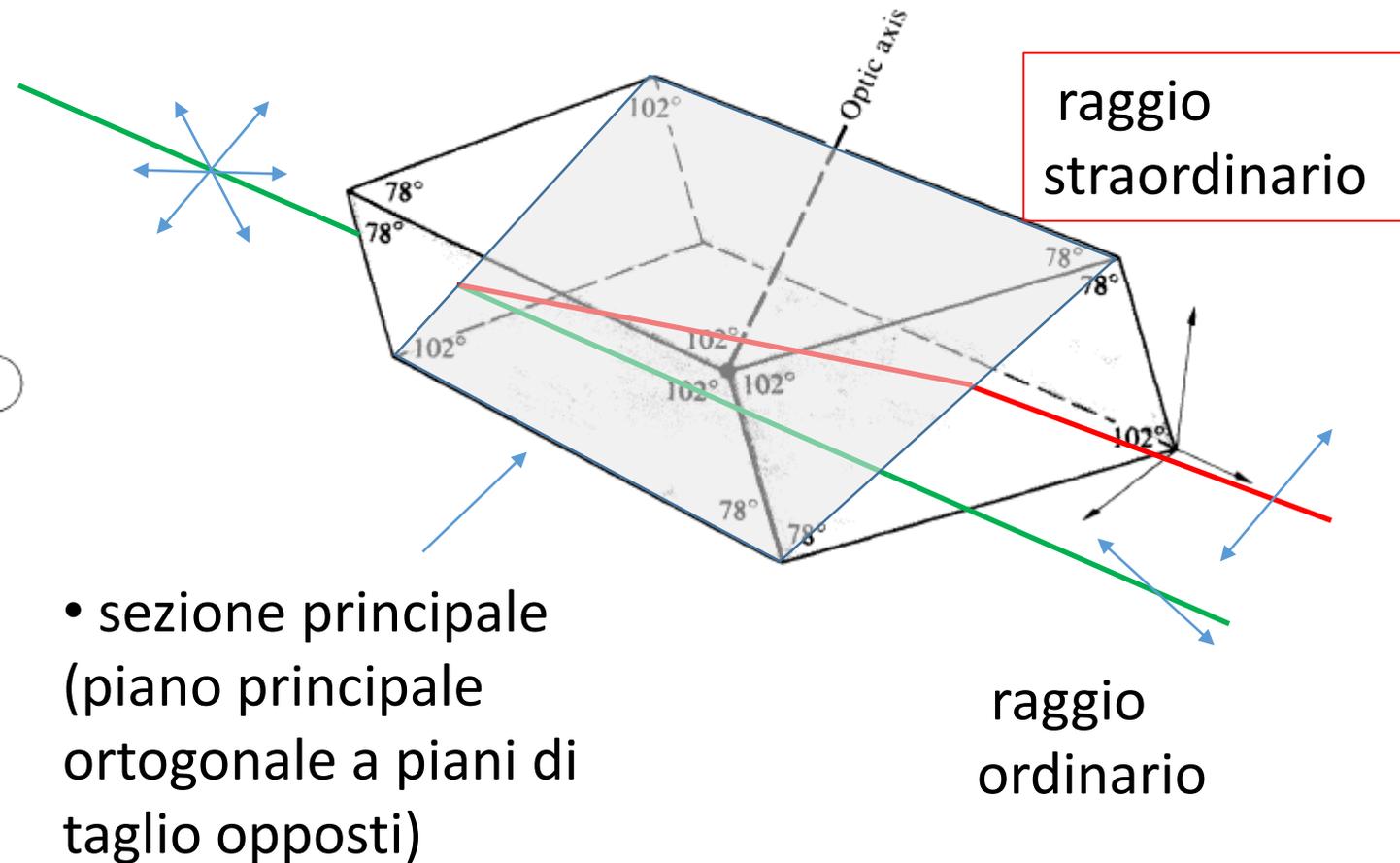


Birifrangenza

- Calcite/Carbonato di Calcio(CaCO_3)

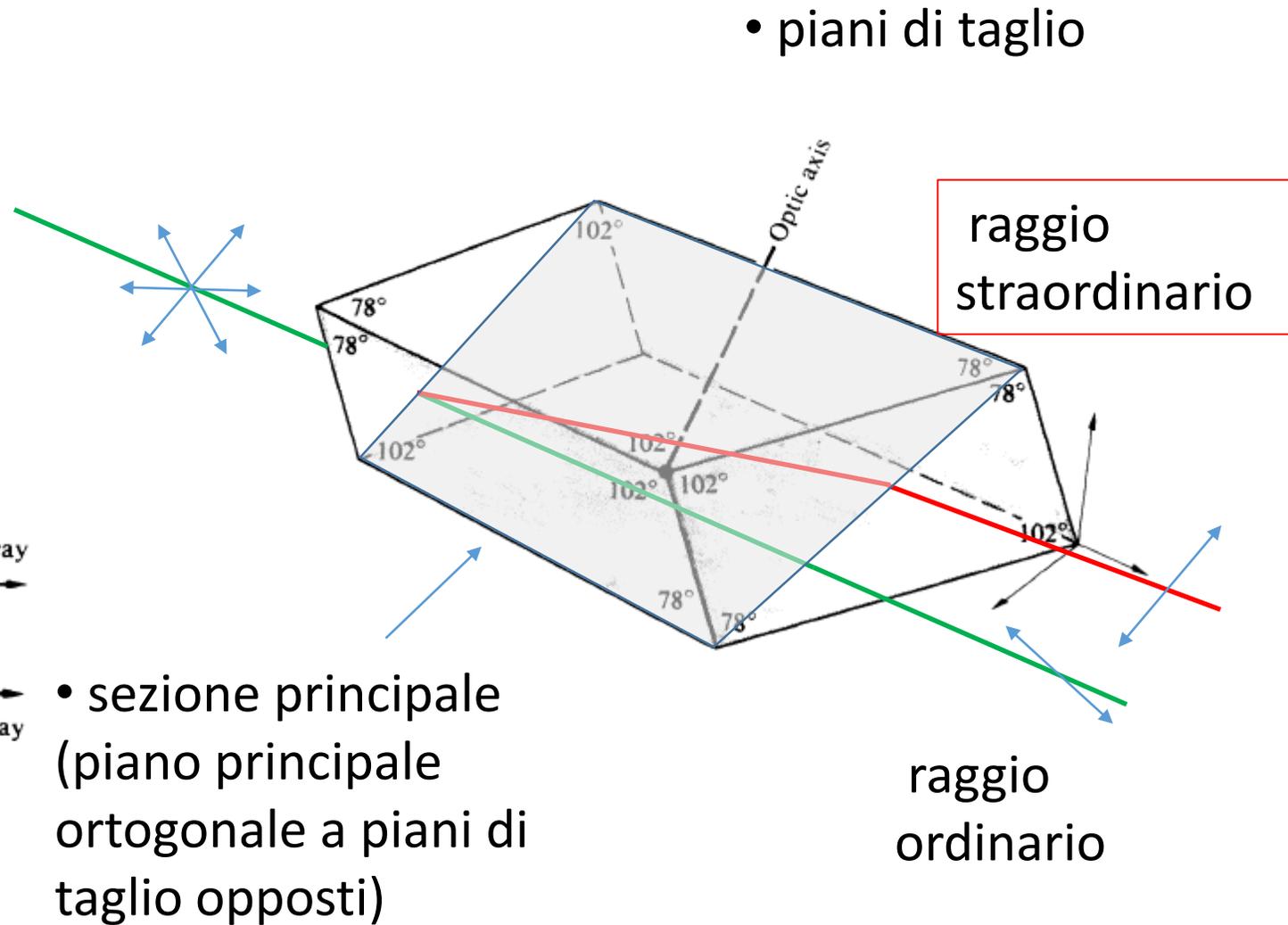
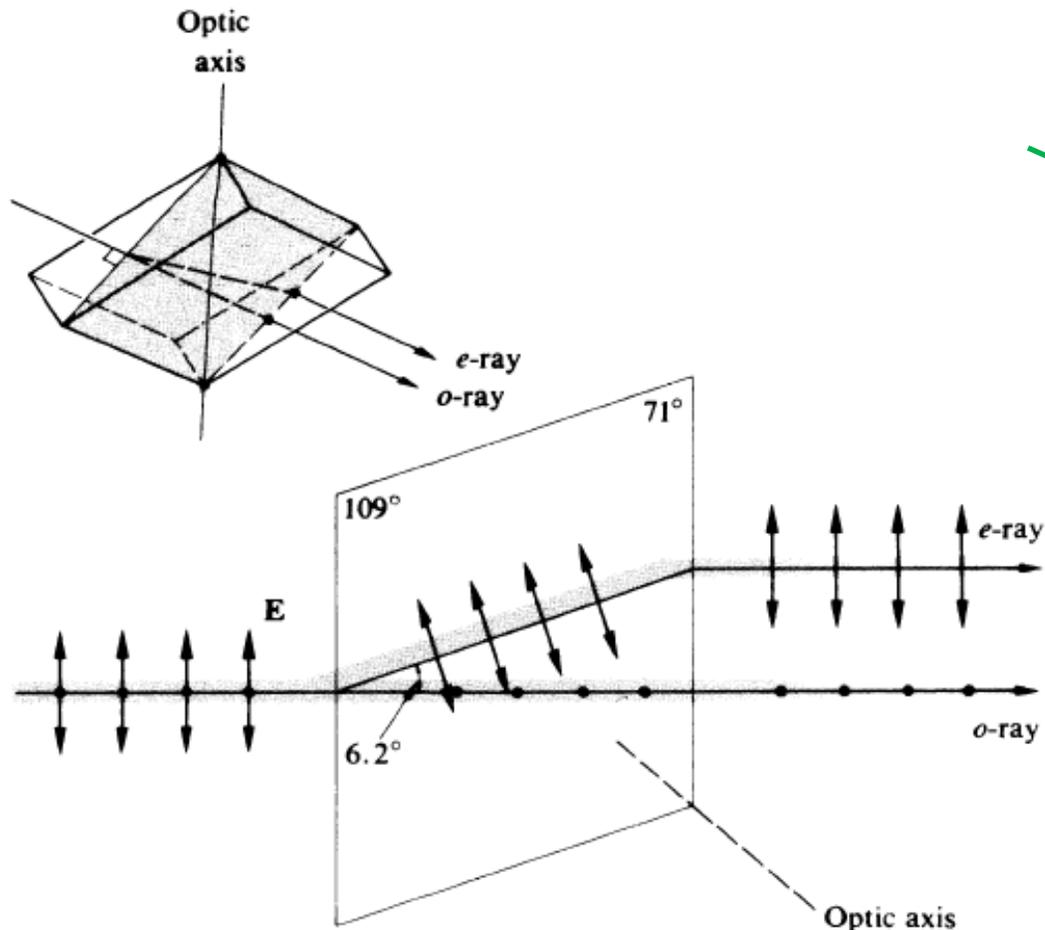


- piani di taglio



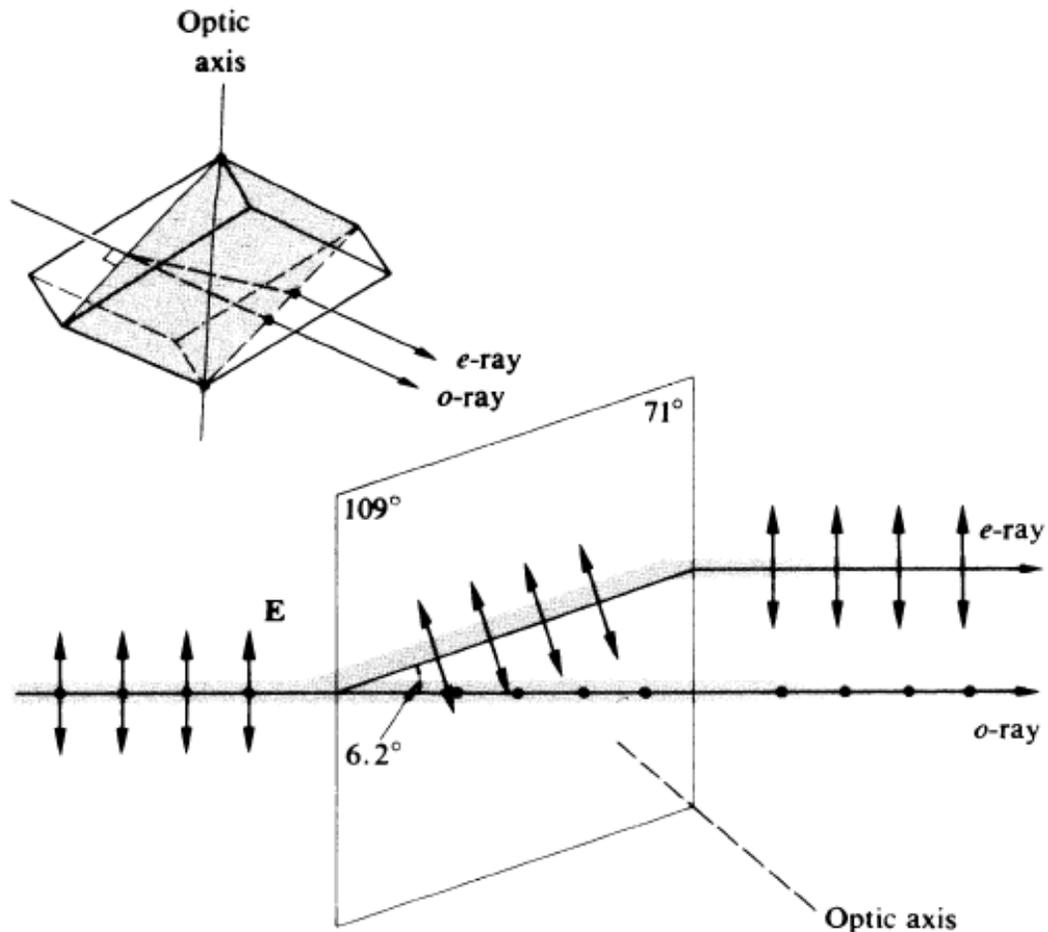
Birifrangenza

- Calcite/Carbonato di Calcio(CaCO_3)

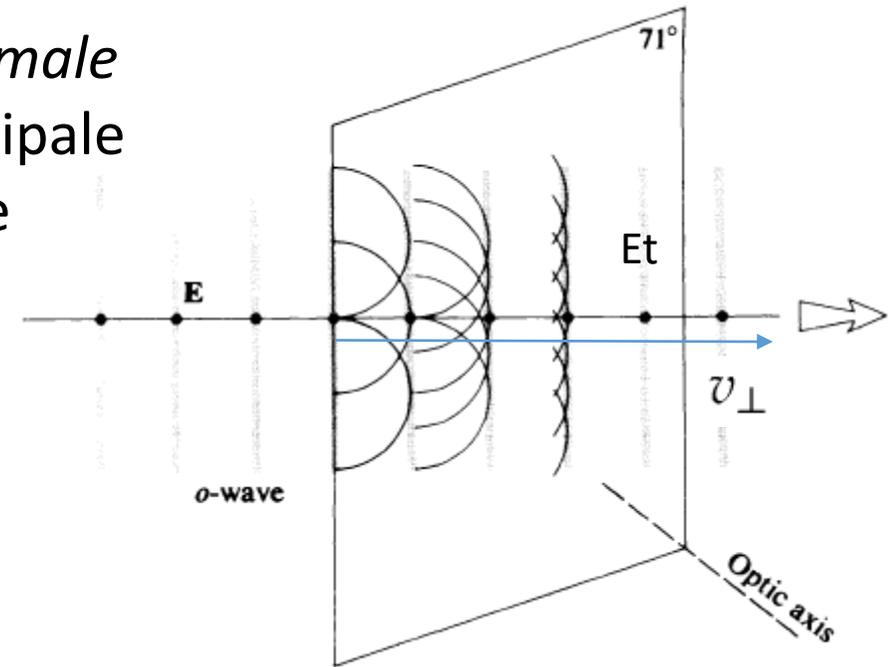


Birifrangenza

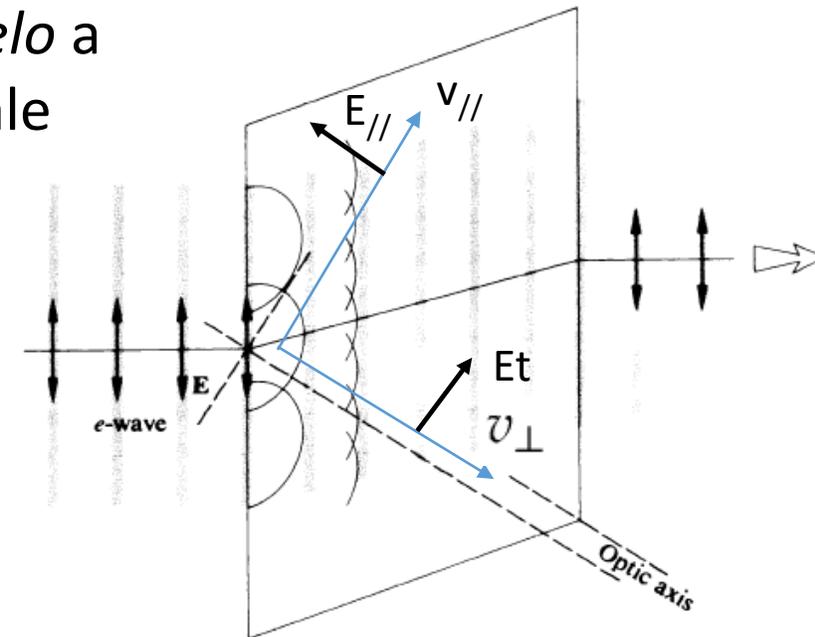
- Calcite/Carbonato di Calcio(CaCO_3)



- Campo E *normale* a sezione principale
propagazione isotropa

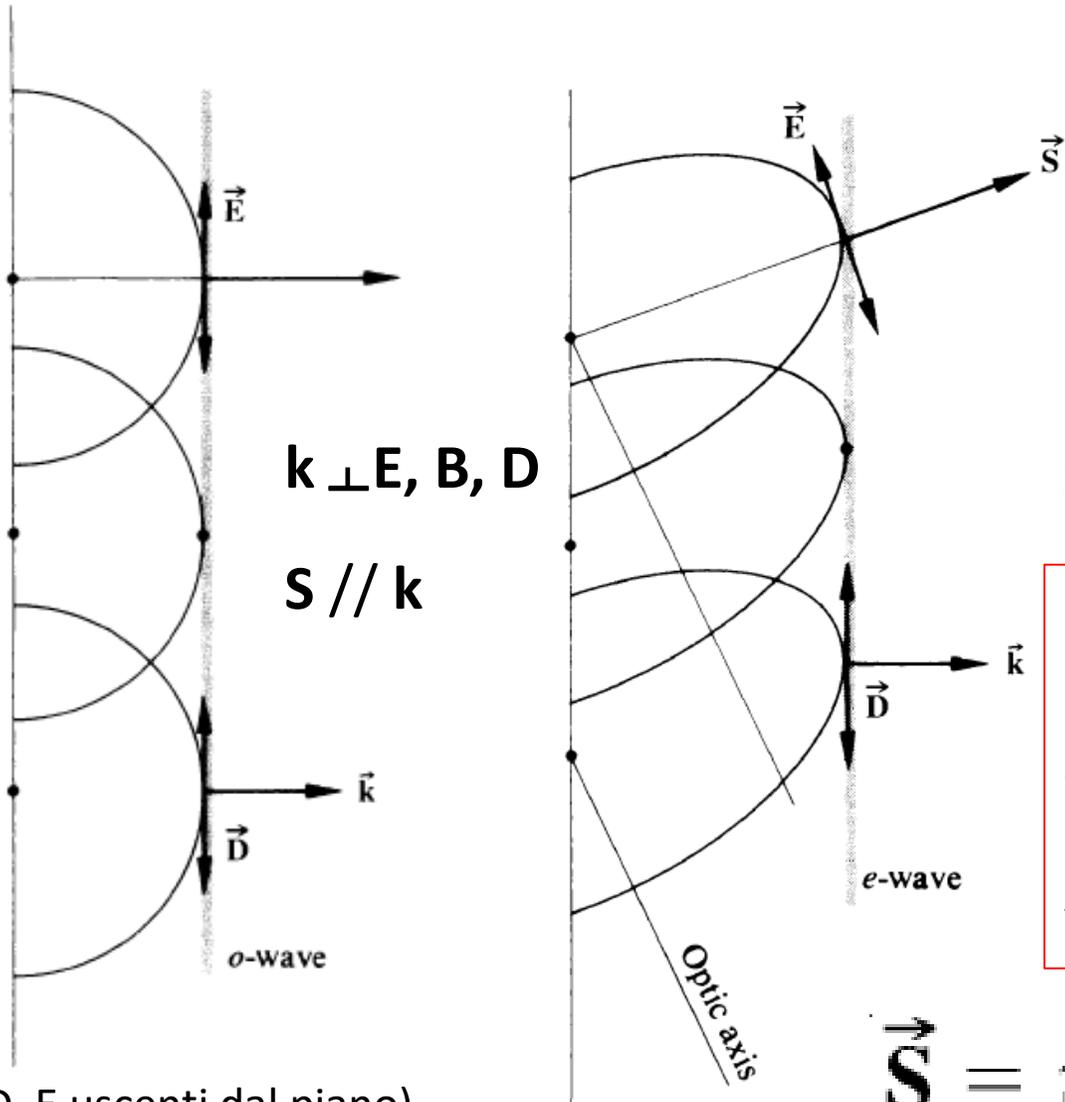


- Campo E *parallelo* a sezione principale
propagazione **anisotropa**



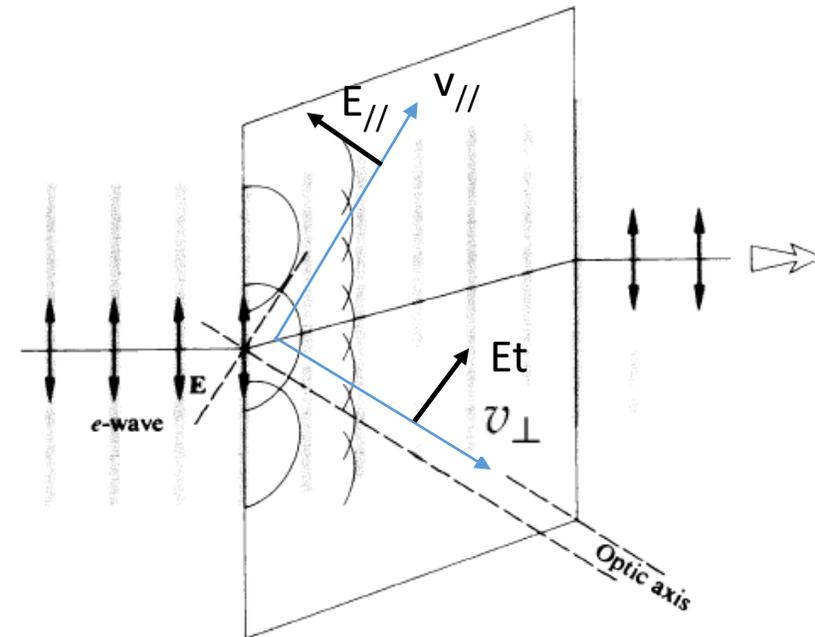
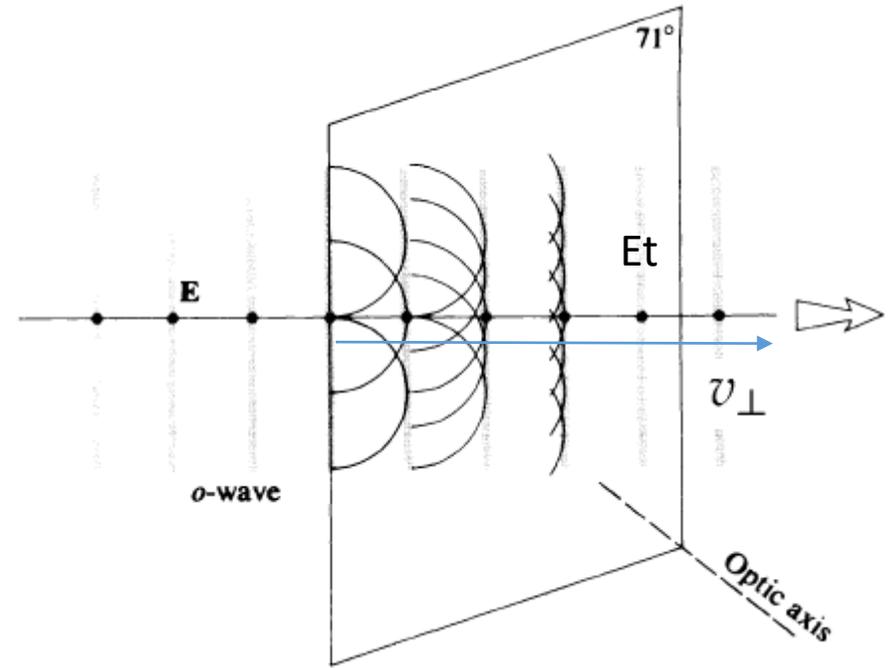
Birifrangenza

- Calcite/Carbonato di Calcio(CaCO₃)



(D, E uscenti dal piano)

$$\vec{S} = v^2 \epsilon \vec{E} \times \vec{B}$$



Birifrangenza

- Calcite/Carbonato di Calcio (CaCO_3)

$$n_o \equiv c/v_{\perp}$$

$$n_e \equiv c/v_{\parallel}$$

$$v_{\parallel} > v_{\perp}$$

$$\Delta n = (n_e - n_o) = -0.172$$

Uniassiale *negativa*

- Uniassiale *positivo*

Quarzo

Ghiaccio

...

- Campo E *parallelo* a sezione principale propagazione **anisotropa**

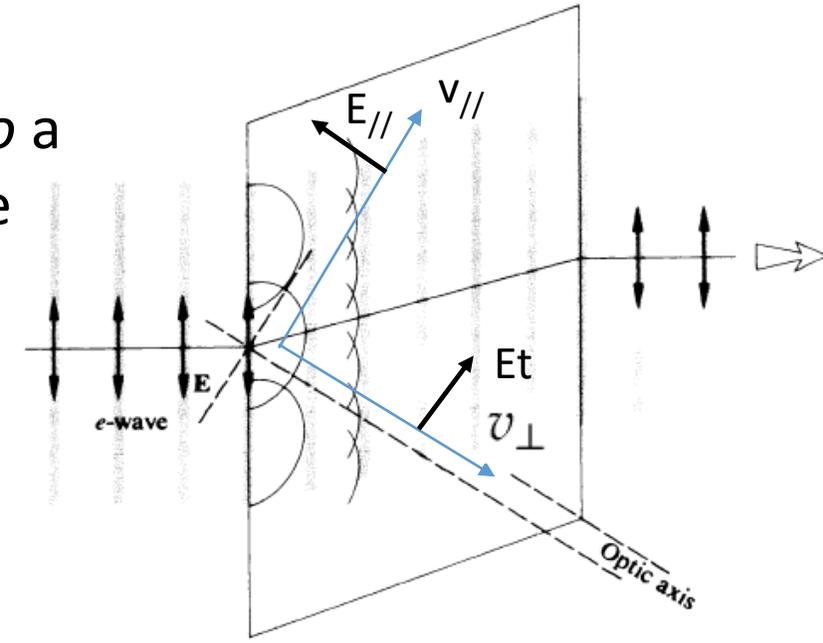


TABLE 8.1 Refractive Indices of Some Uniaxial Birefringent Crystals ($\lambda_0 = 589.3$ nm)

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium nitrate	1.5854	1.3369
Ice	1.309	1.313
Rutile (TiO_2)	2.616	2.903

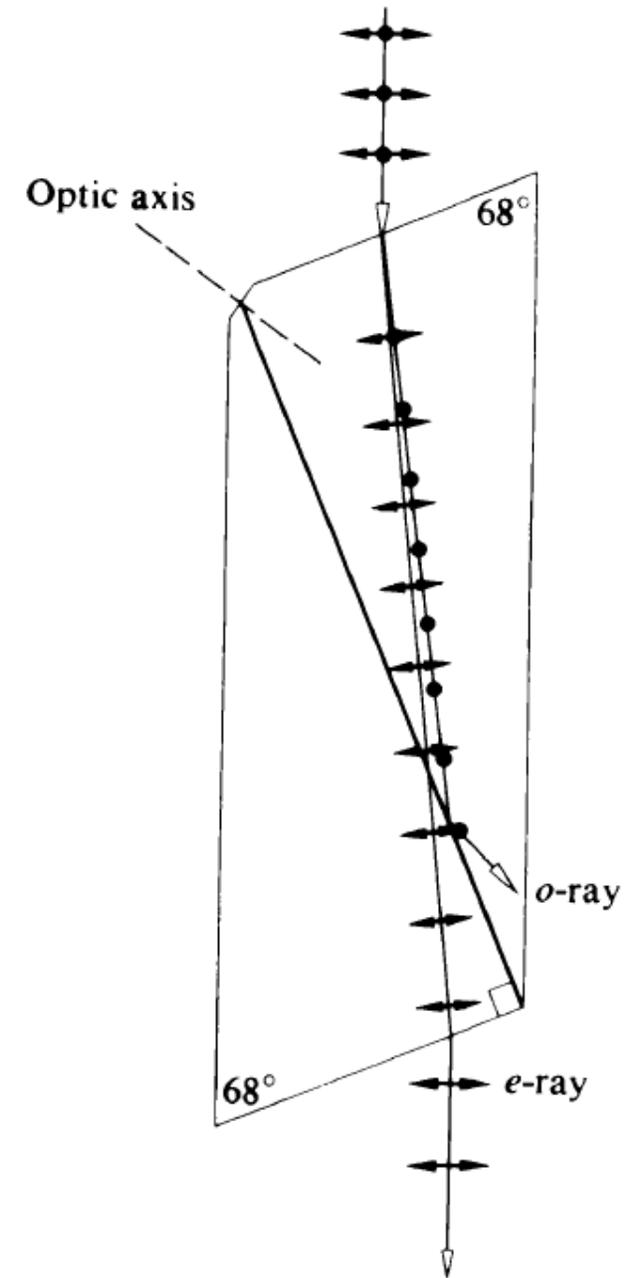
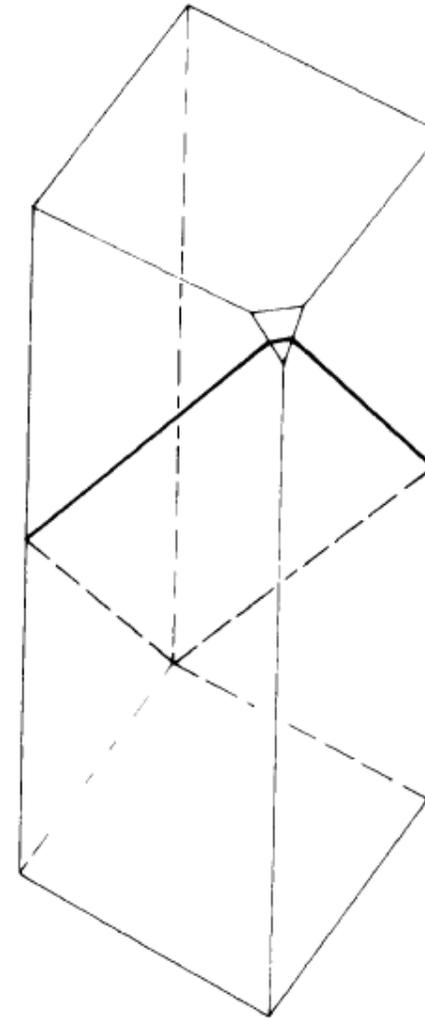
Polarizzatori Birifrangenti

- Prisma di Nicol

- Due parti di calcite unite con balsamo del Canada $n_{CB}=1.55$

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864

angolo critico per riflessione
totale interna $\theta_c=69^\circ$



- raggio ordinario deflesso e assorbito

- raggio straordinario parallelo al fascio in ingresso

Polarizzatori Birifrangenti

- Polarizzatore di Glan – Focault
Glan -Taylor

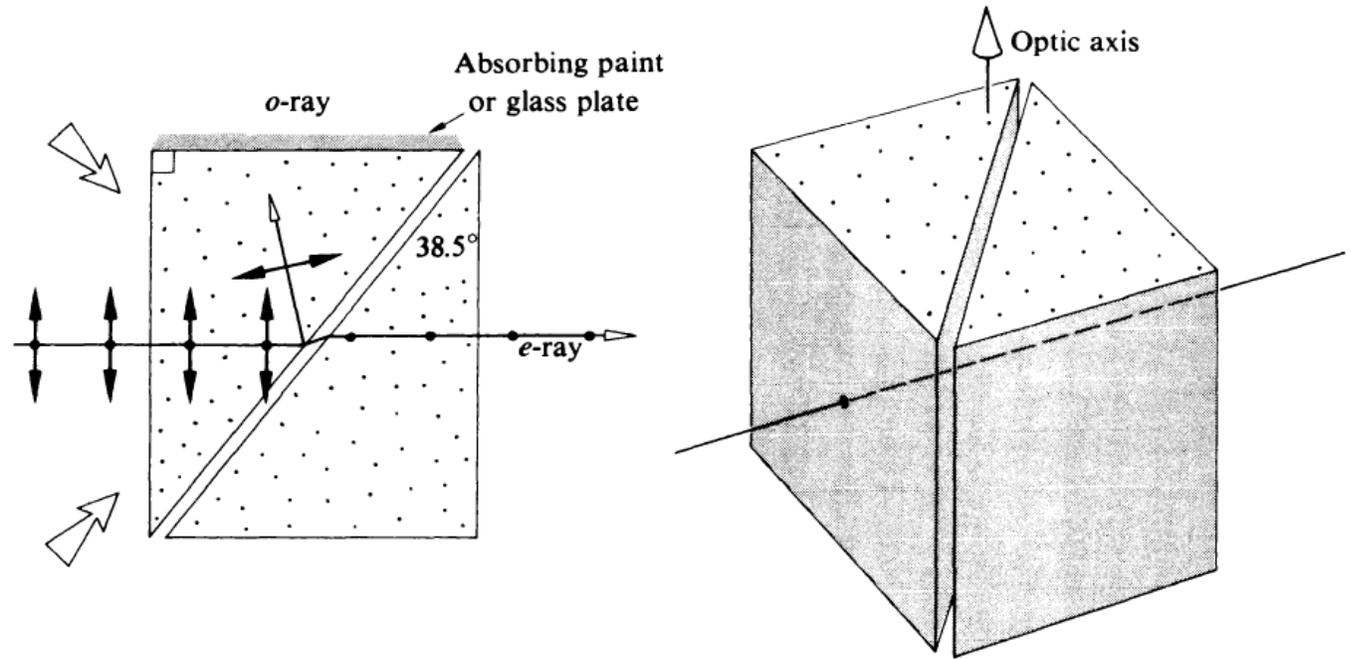
- Due parti di calcite separate da aria

angolo critico per riflessione
totale interna (o-ray)

$$n_e < 1/\sin \theta < n_o$$

$$\Delta\theta = 10^\circ$$

- raggio ordinario deflesso e
assorbito
- raggio straordinario
parallelo al fascio in ingresso



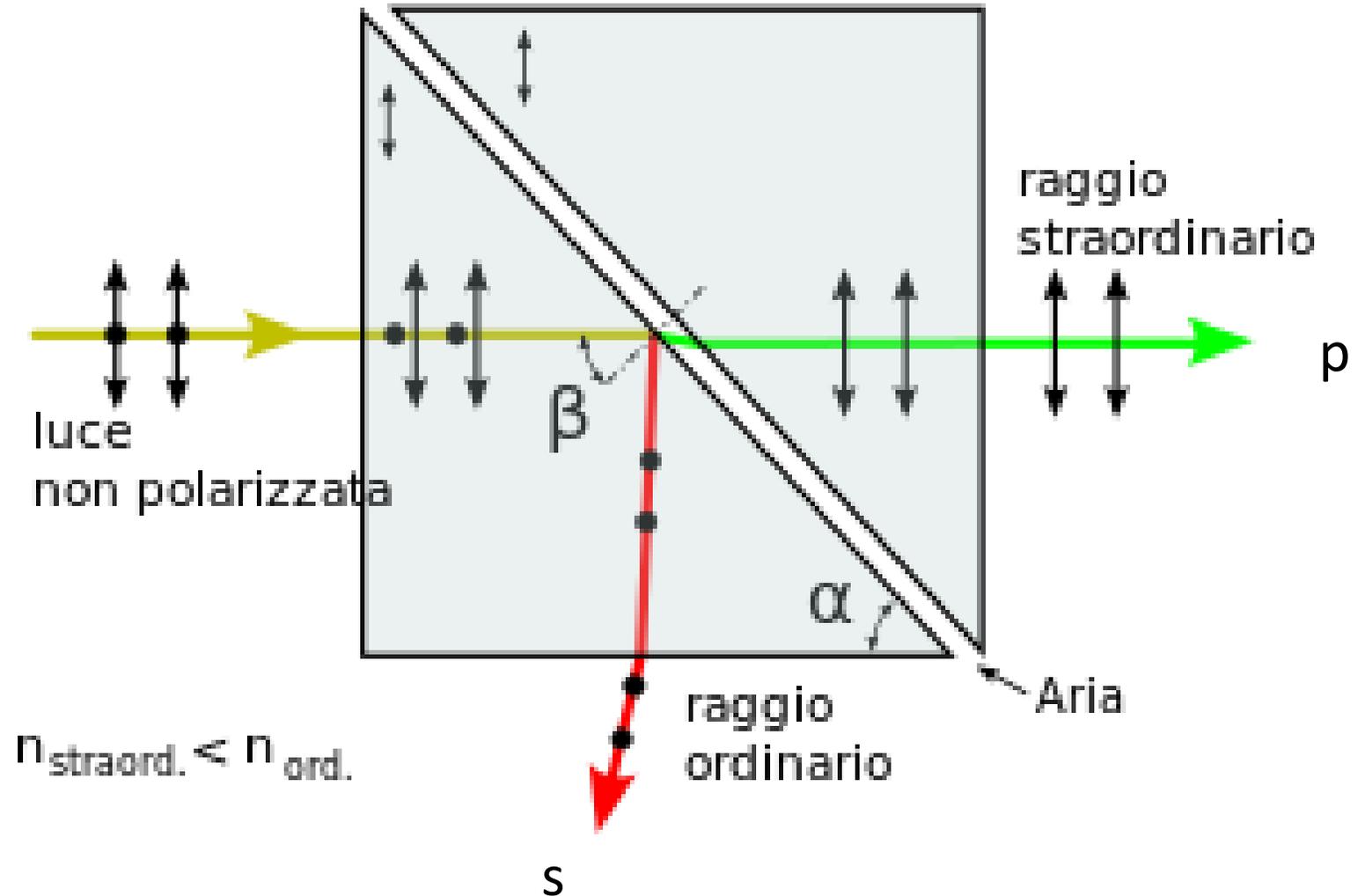
Prismi polarizzatori

- Glan-Taylor (calcite)

$n_o=1.66$

$n_e=1.49$

$$\beta \geq \arcsin \left(\frac{n_{aria}}{n_{ord}} \right)$$



Polarizzatori Birifrangenti

- Polarizzatore di Glan - Thompson

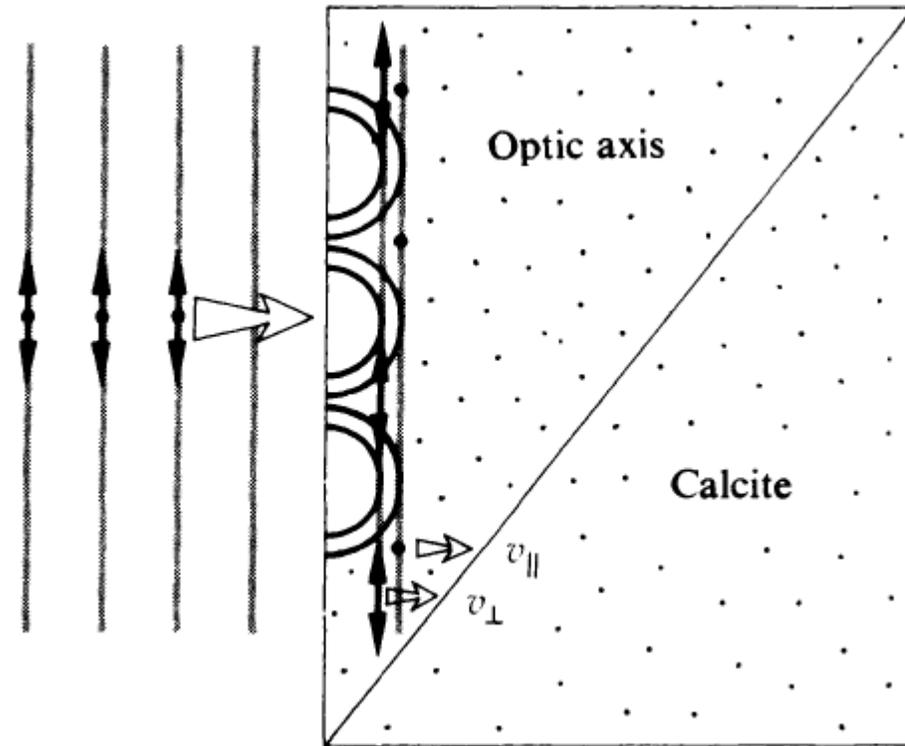
- Due parti di calcite a contatto

angolo critico per riflessione
totale interna (o-ray)

$$n_e < 1/\sin \theta < n_o$$

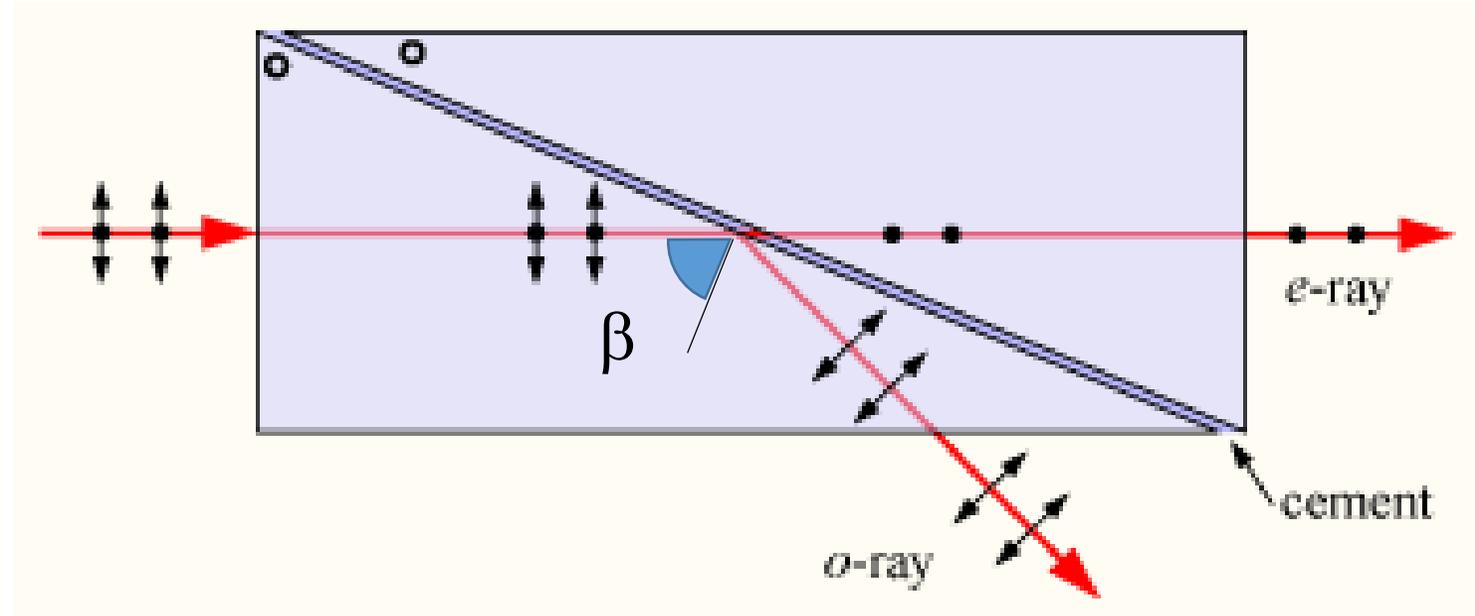
$$\Delta\theta = 30^\circ$$

- raggio ordinario deflesso e
assorbito
- raggio straordinario
parallelo al fascio in ingresso



Prismi polarizzatori

- Glan-Thompson (calcite) – (basse intensità)



$n_o=1.66$

$n_e=1.49$

$n_b=1.53$ (balsamo del Canada, da abete...)

Raggio O : $\beta > \arcsin\left(\frac{n_b}{n_o}\right)$ circa 67°

Raggio E : β' vicino a $\arctg(n_e/n_b)$

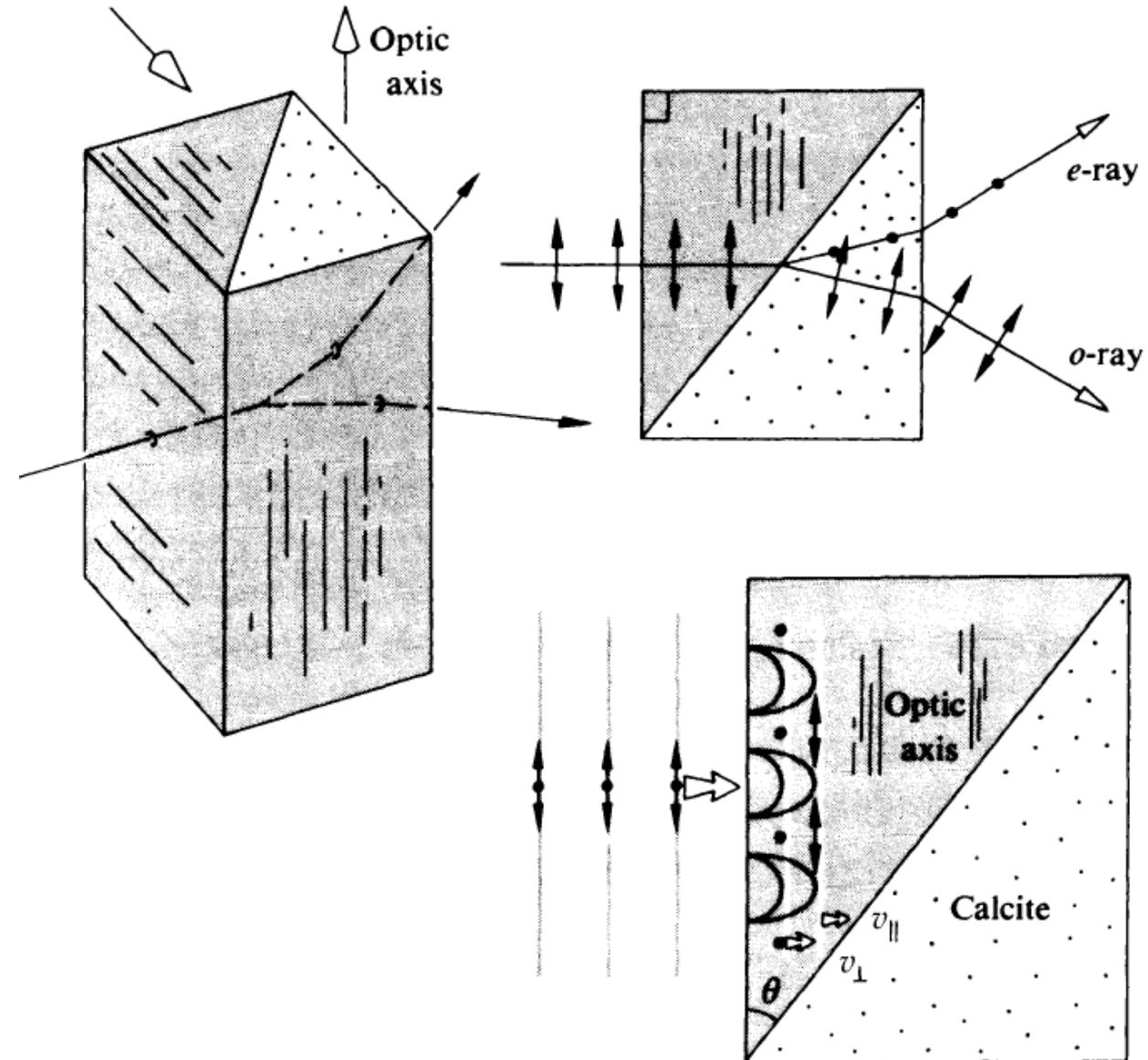
Polarizzatori Birifrangenti

- Prisma di Wollaston

- Divisore di fascio polarizzatore (polarizing beamsplitter)

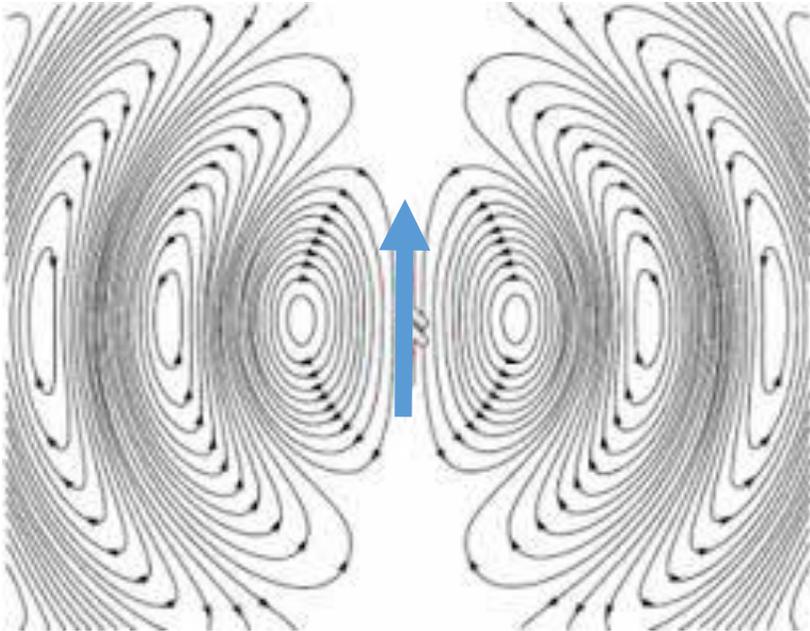
- Due prismi con asse ottico *ortogonale*

- Entrambi i raggi deviati e trasmessi



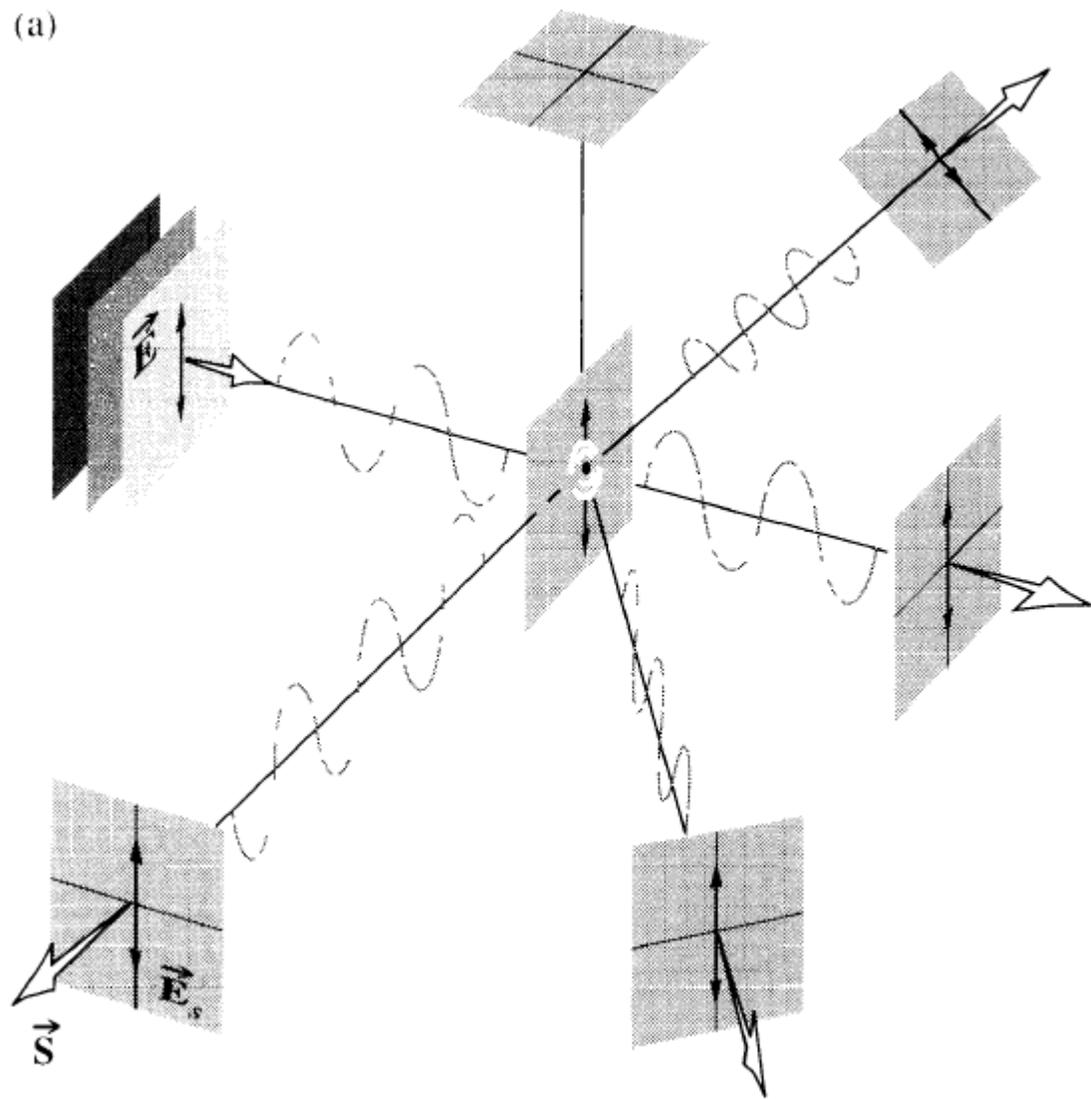
Polarizzazione per scattering

- Radiazione di dipolo

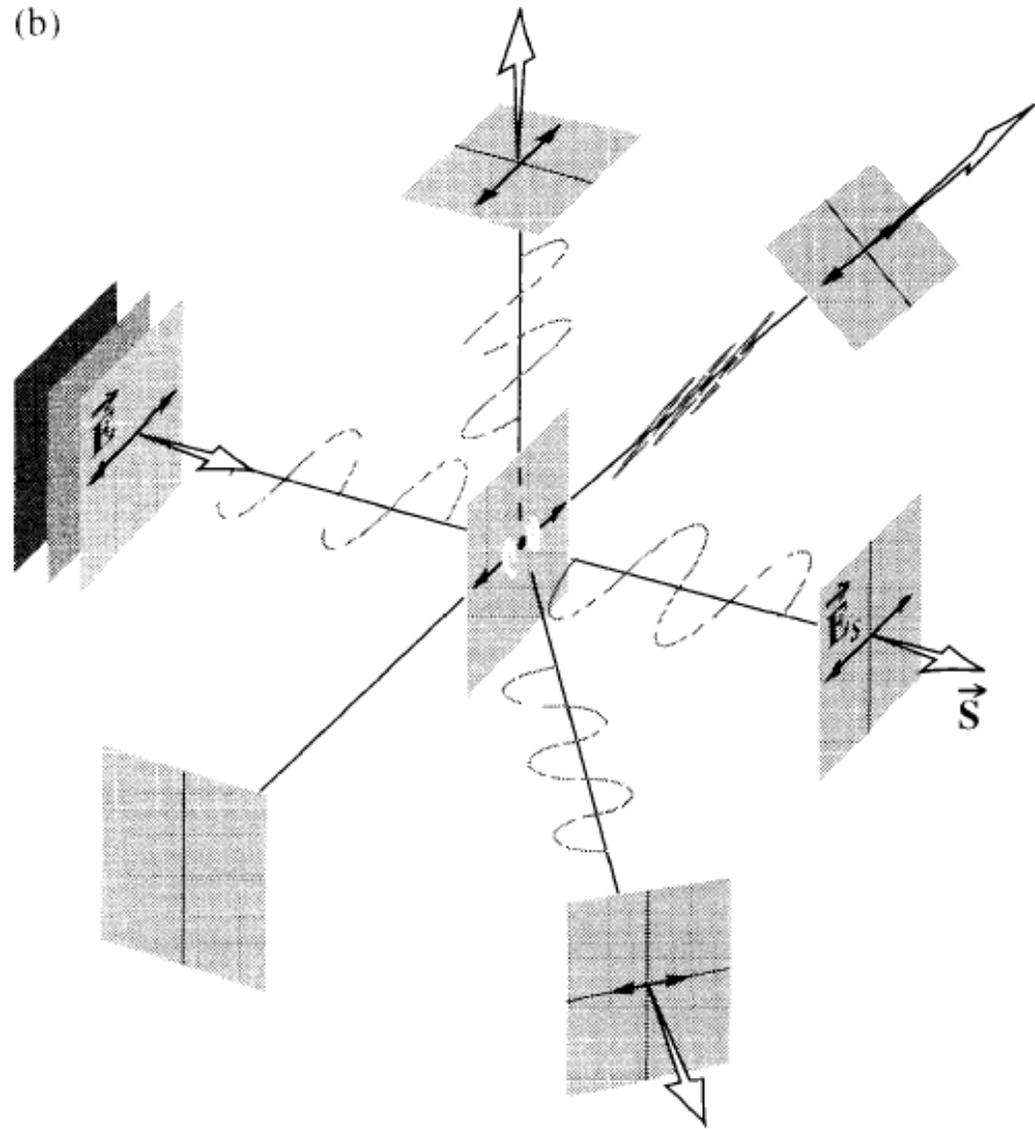


Polarizzazione per scattering

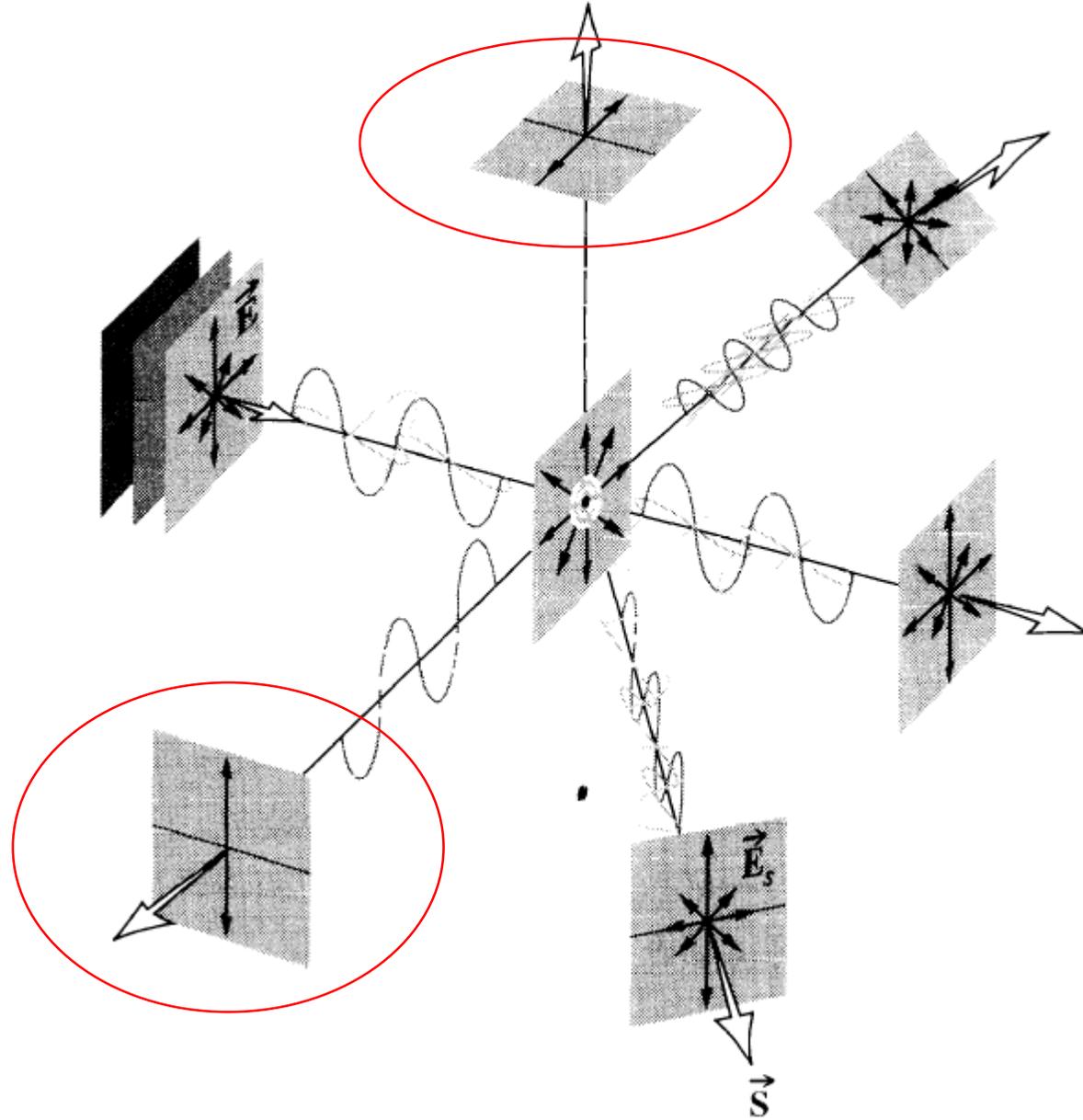
(a)



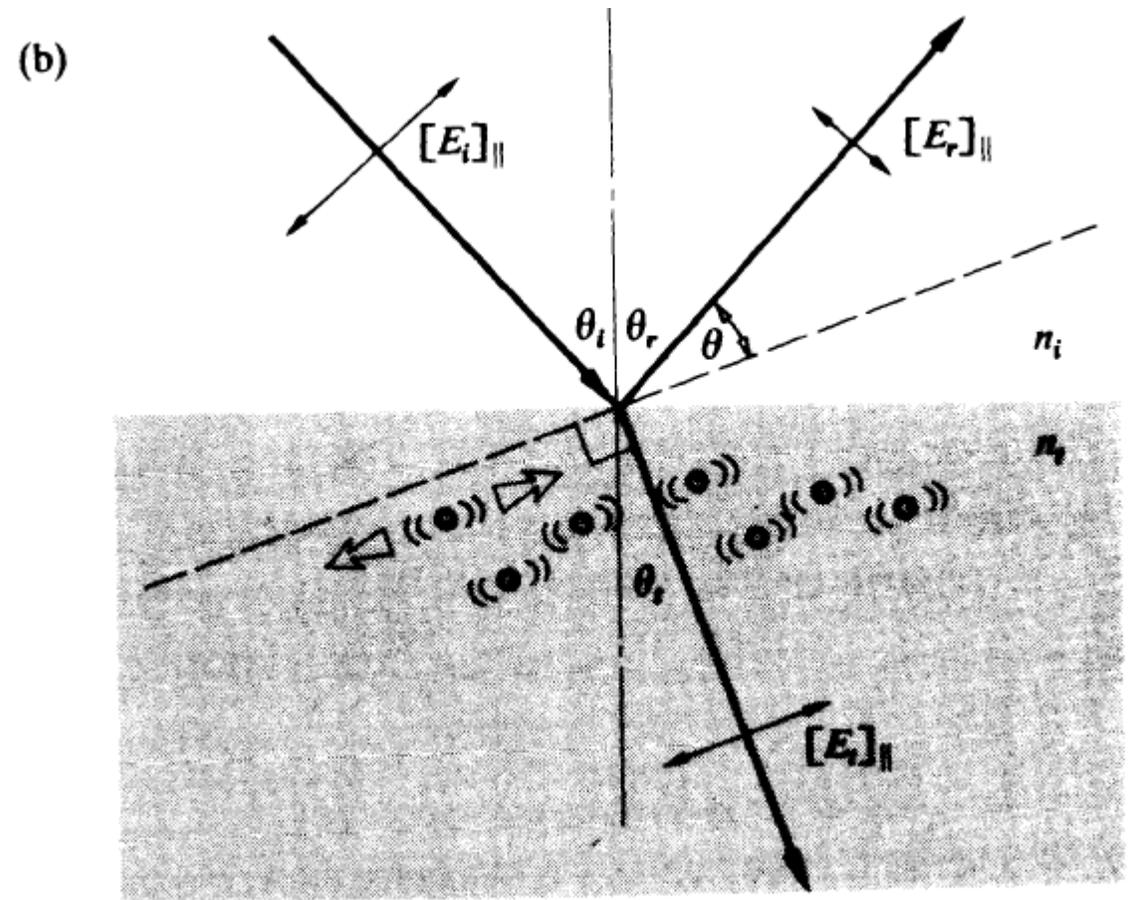
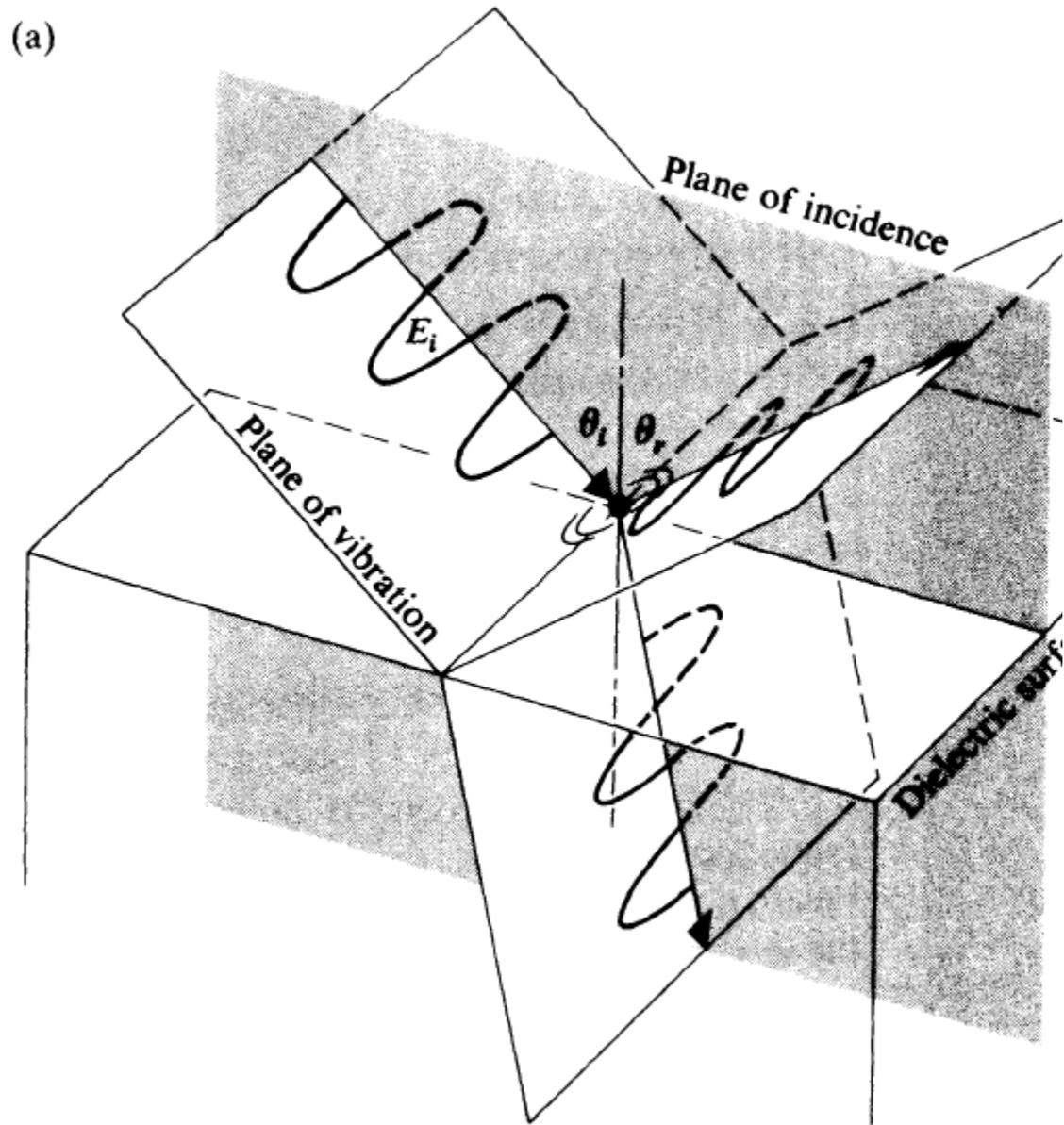
(b)



Polarizzazione per scattering



Polarizzazione per riflessione



Solo componente con polarizzazione normale al piano di incidenza (parallela alla superficie) è riflessa

Polarizzazione per Riflessione

- Angolo di Brewster

$$\theta_p + \theta_t = 90^\circ$$

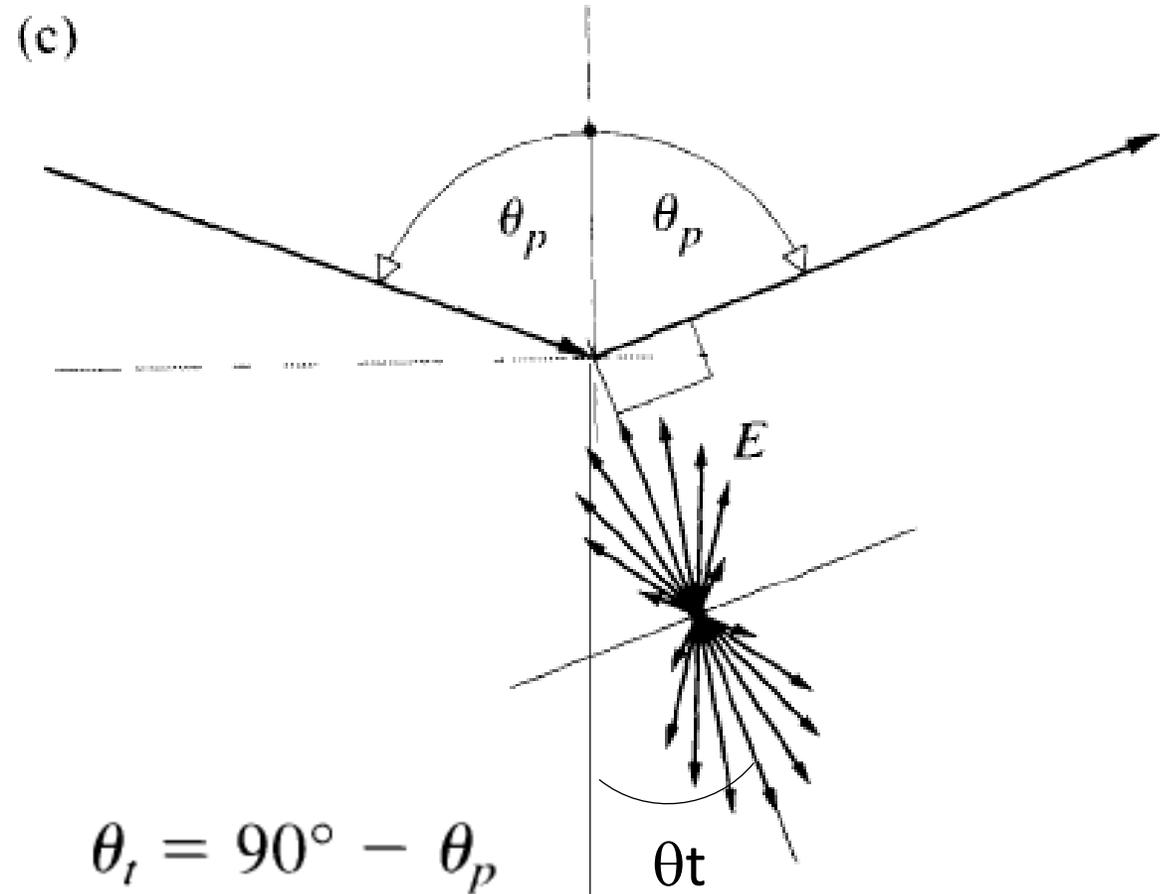
$$n_i \sin \theta_p = n_t \sin \theta_t$$

$$n_i \sin \theta_p = n_t \cos \theta_p$$

$$\tan \theta_p = n_t/n_i$$

$$n_i = 1 \quad n_t \approx 1.5$$

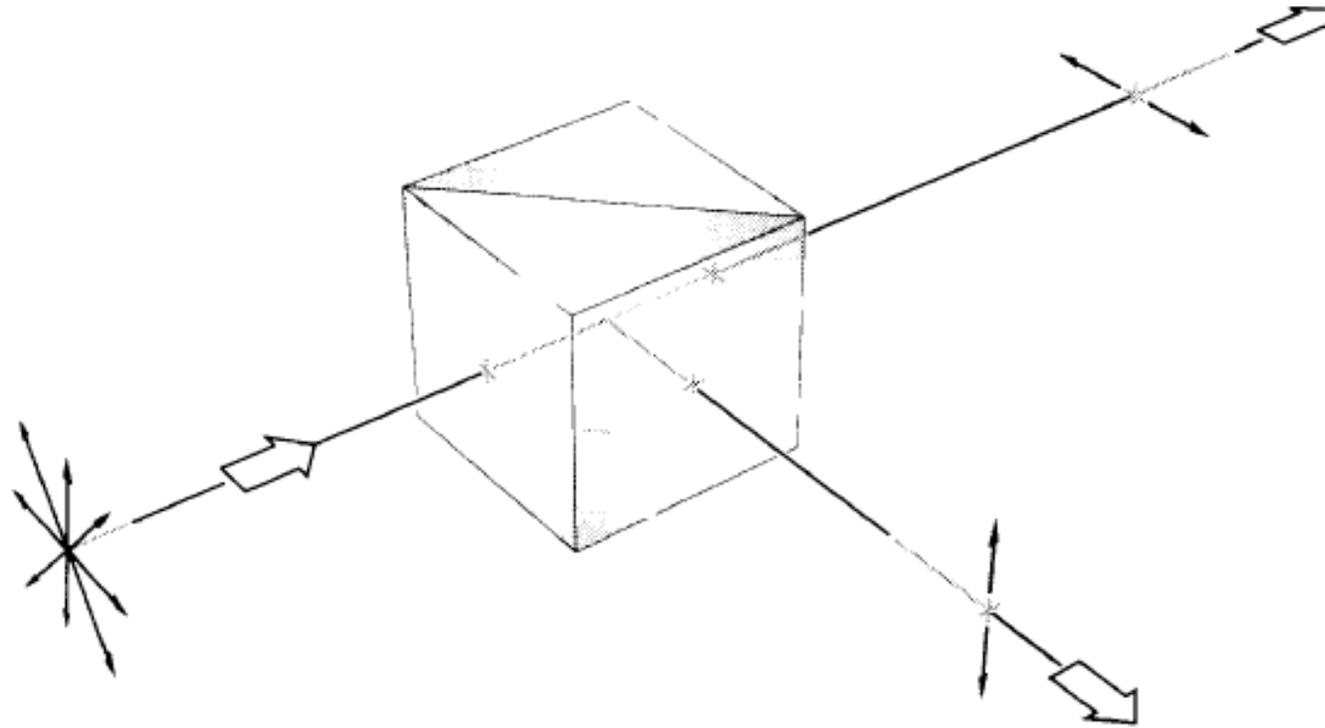
$$\theta_p \approx 56^\circ$$



- problema: il fascio riflesso è debole

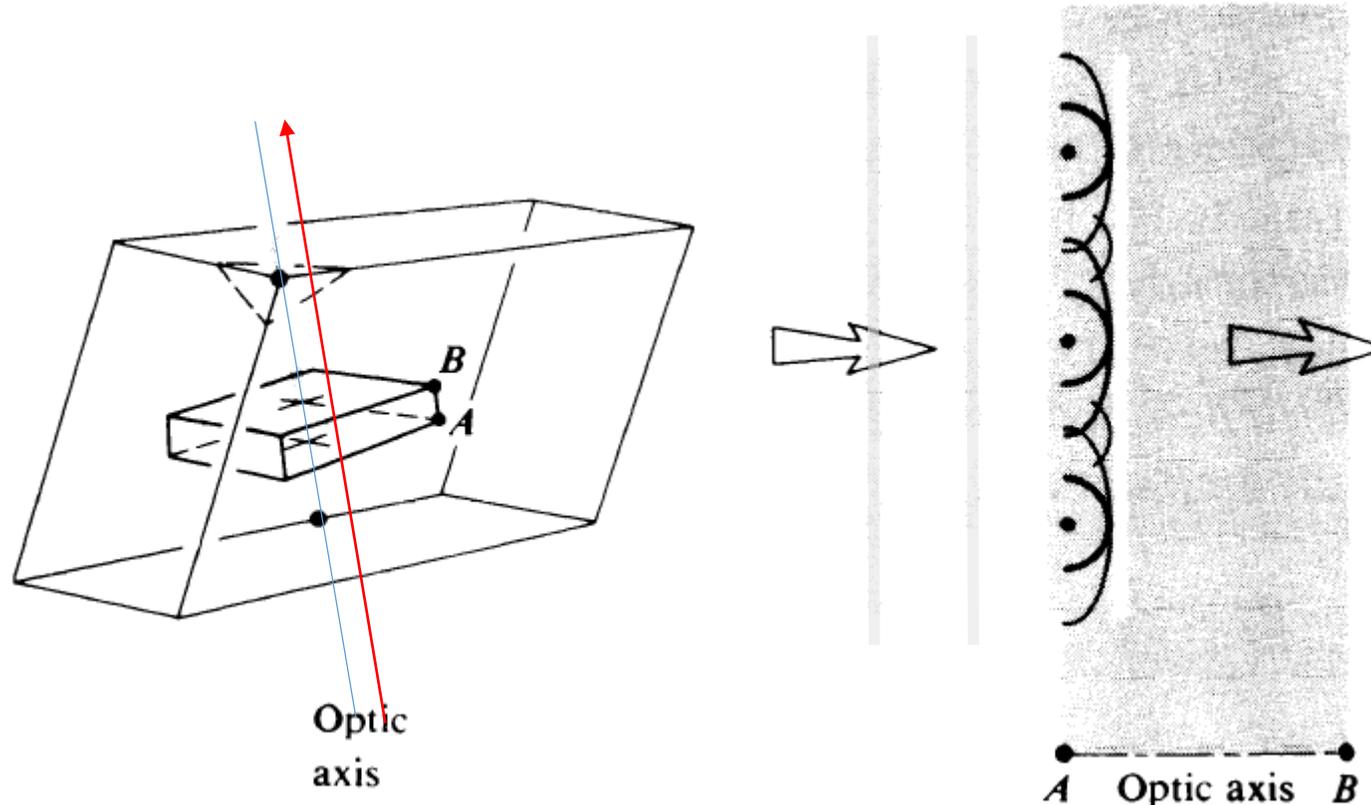
Polarizzazione per riflessione

- Cubo polarizzatore



Lamine di ritardo

- idea: far acquisire una fase aggiuntiva ad una componente \mathcal{P} rispetto all'altra



Lamine di ritardo

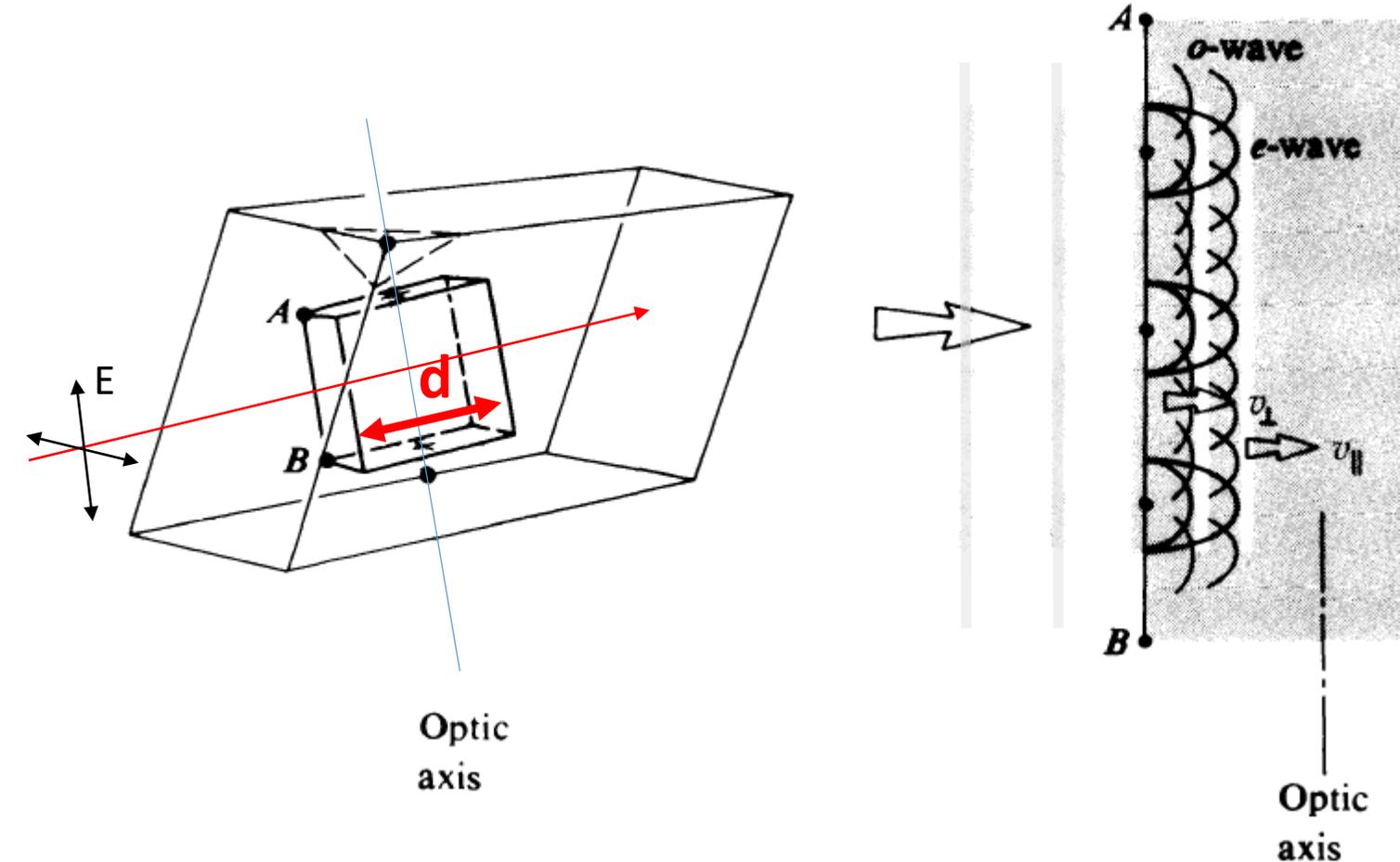
- idea: far acquisire una fase aggiuntiva ad una componente \mathcal{P} rispetto all'altra

$$\Lambda = d(|n_o - n_e|)$$

$$\Delta\varphi = k_0\Lambda$$

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|)$$

$$v_{\parallel} > v_{\perp} \quad n_o > n_e$$



Lamine di ritardo

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|)$$

$$\Lambda = d(|n_o - n_e|)$$

- | | | |
|----------------------|---------|-------------|
| • lamina λ | 2π | λ |
| • lamina $\lambda/2$ | π | $\lambda/2$ |
| • lamina $\lambda/4$ | $\pi/2$ | $\lambda/4$ |

Lamine di ritardo

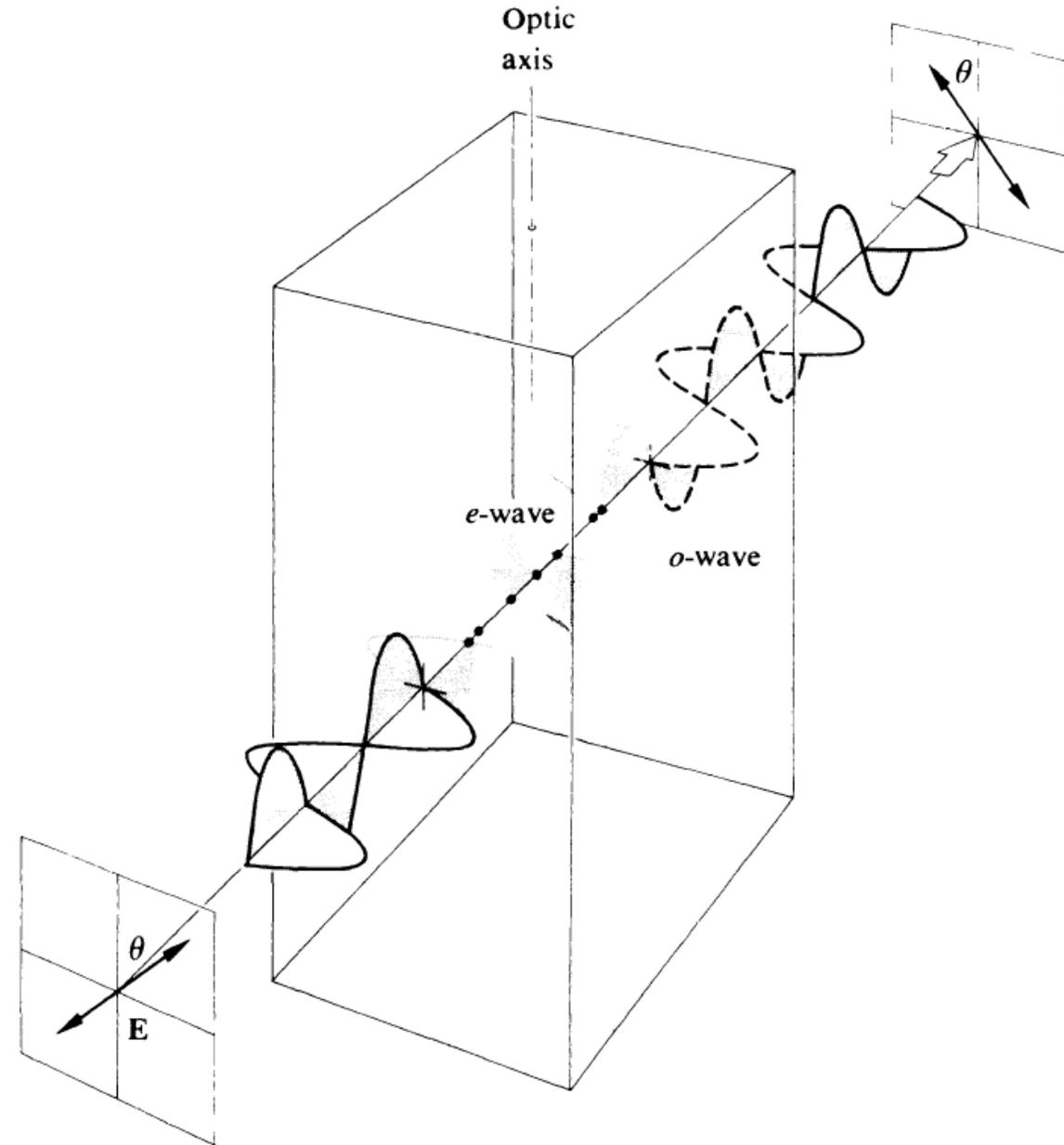
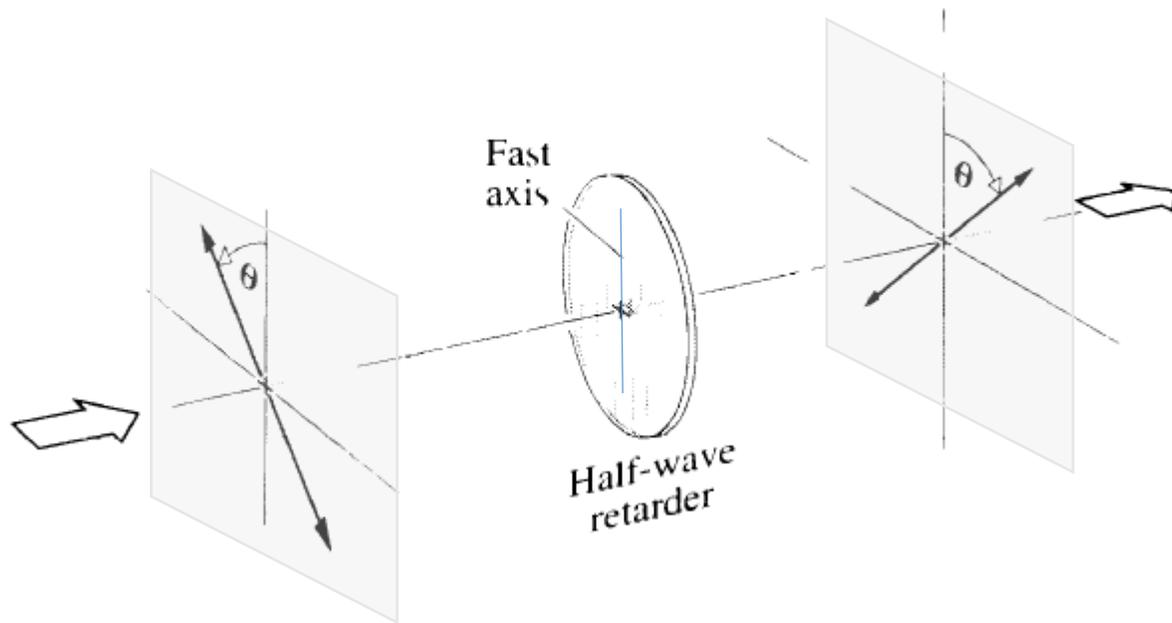
- lamina $\lambda/2$

lin. - $\mathcal{P}(\theta)$

lin. - $\mathcal{P}(-\theta)$

cir. - \mathcal{R}

cir. - \mathcal{L}



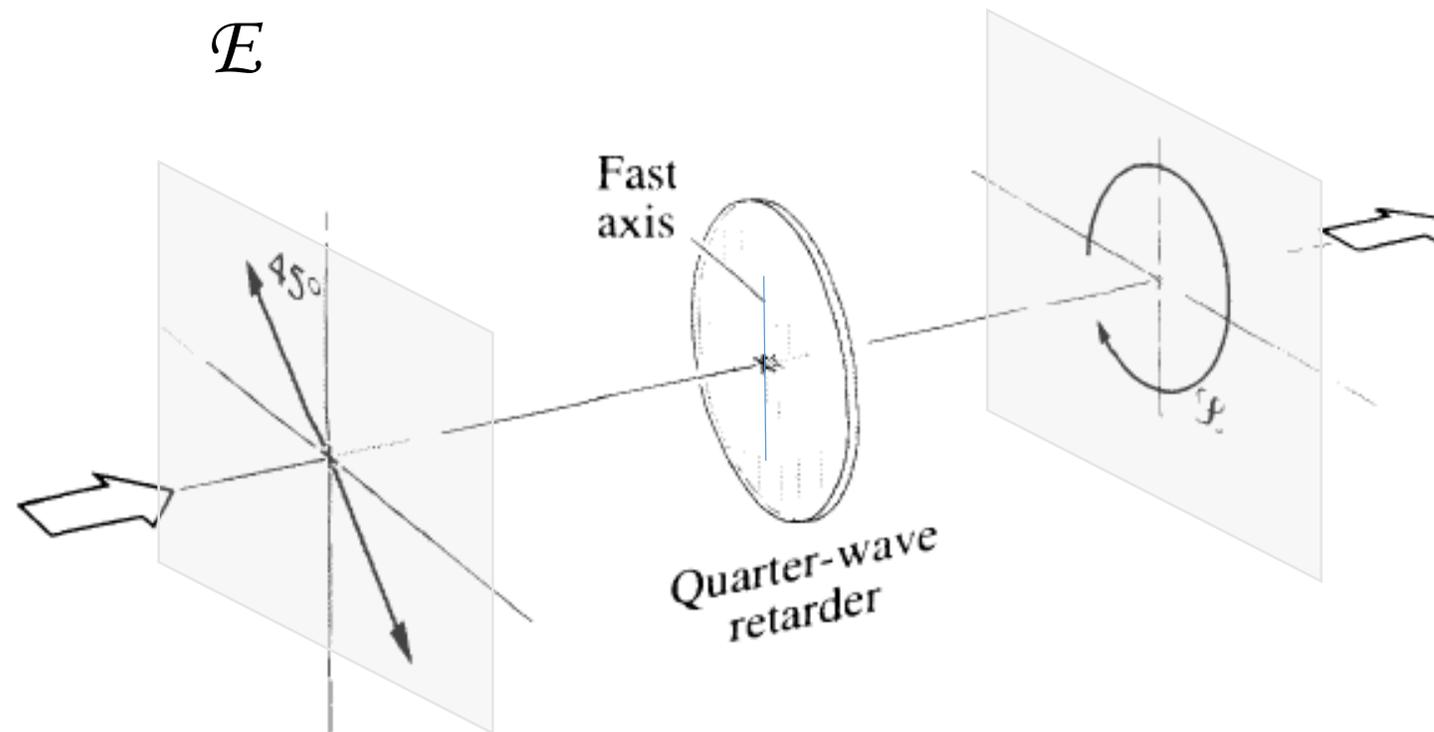
Lamine di ritardo

- lamina $\lambda/4$

lin. - $\mathcal{P} (-45^\circ)$ cir. - \mathcal{L}

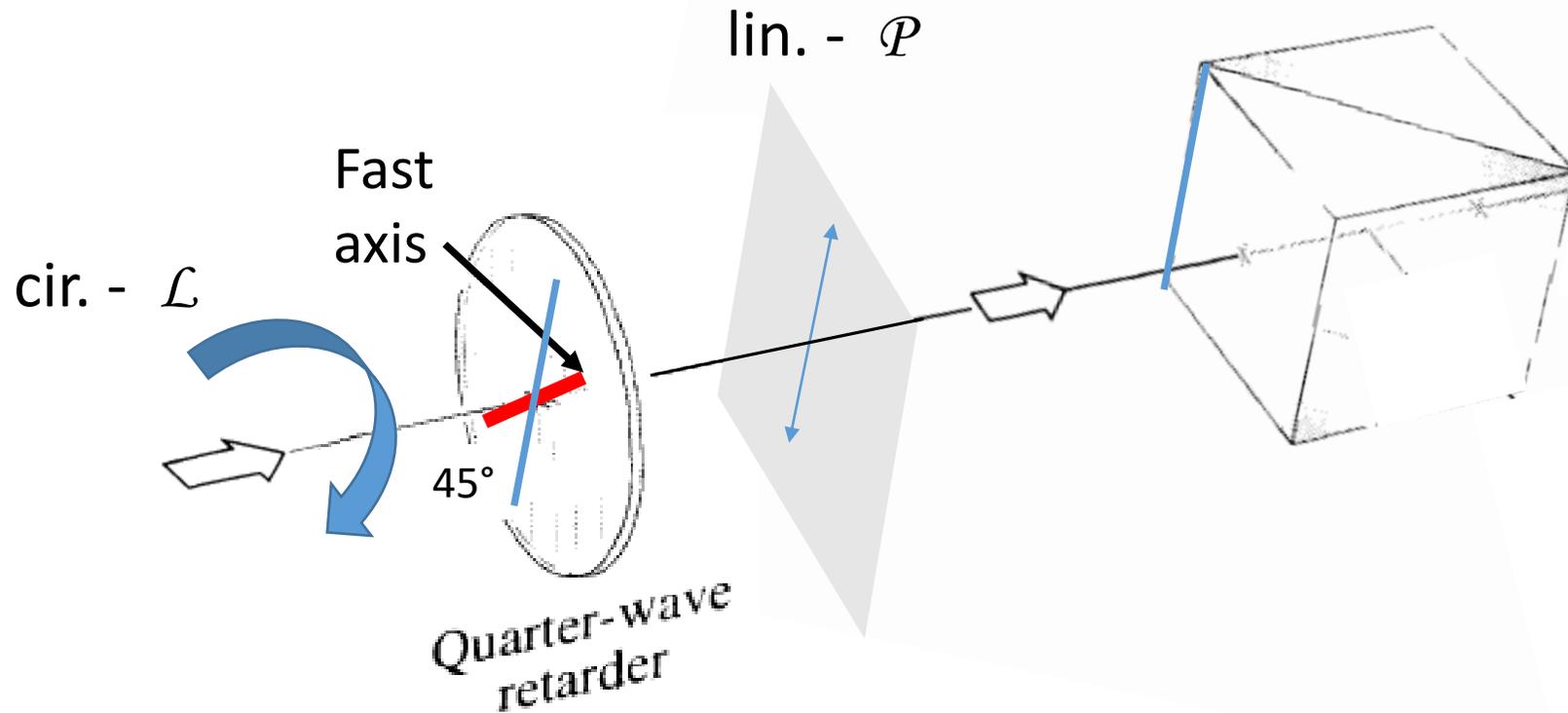
lin. - $\mathcal{P} (+45^\circ)$ cir. - \mathcal{R}

lin. - $\mathcal{P} (\theta)$ \mathcal{E}



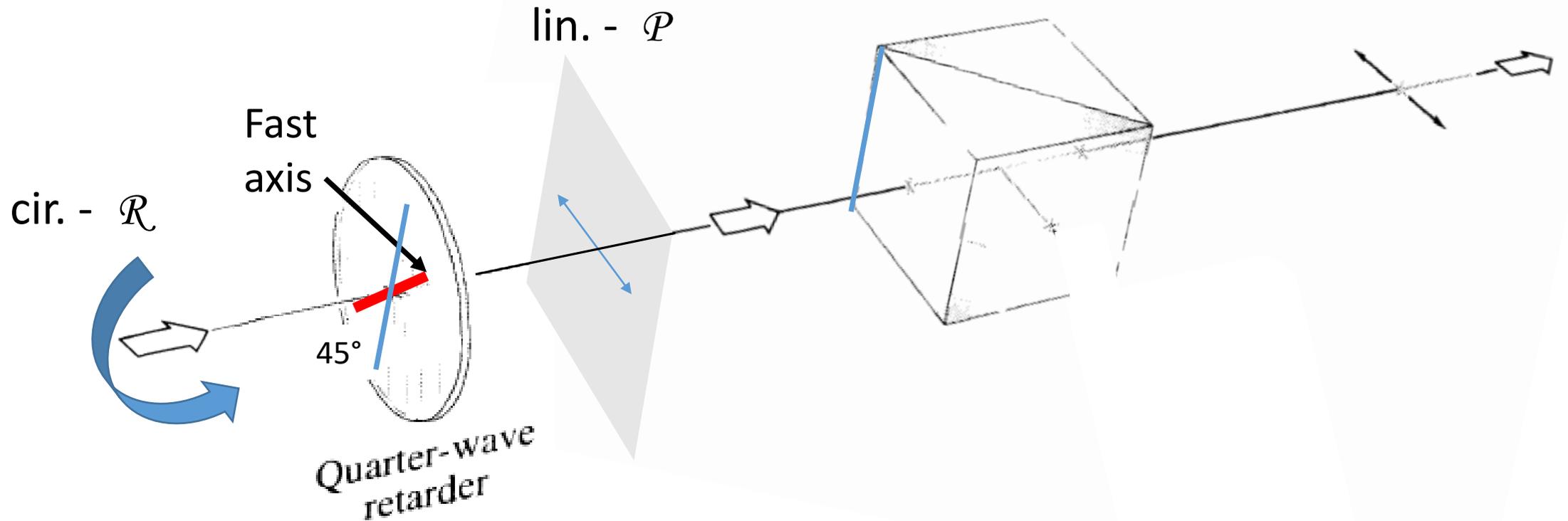
Analizzatore di polarizzazione

- lamina $\lambda/4$ + cubo



Analizzatore di polarizzazione

- lamina $\lambda/4$ + cubo



Vettori di Jones

- descrizione vettoriale dello stato di polarizzazione

$$\vec{\mathbf{E}} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \quad \tilde{\mathbf{E}} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix}$$

- polarizzazione lineare - \mathcal{P}

$$\tilde{\mathbf{E}}_h = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ 0 \end{bmatrix} \quad \text{and} \quad \tilde{\mathbf{E}}_v = \begin{bmatrix} 0 \\ E_{0y} e^{i\varphi_y} \end{bmatrix} \quad \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_h + \tilde{\mathbf{E}}_v$$

per $E_{0x} = E_{0y}$
 $\varphi_x = \varphi_y$

$$\tilde{\mathbf{E}} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0x} e^{i\varphi_x} \end{bmatrix} \quad \tilde{\mathbf{E}} = E_{0x} e^{i\varphi_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

\mathcal{P} - state (45°)

Vettori di Jones

- descrizione vettoriale dello stato di polarizzazione

\mathcal{P} -state (45°)

$$\tilde{\mathbf{E}} = E_{0x} e^{i\varphi_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \vec{\mathbf{E}}_{45} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- similmente per lin.

$$\vec{\mathbf{E}}_h = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{\mathbf{E}}_v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Vettori di Jones

- circolare - R

$$\tilde{\mathbf{E}}_{\mathcal{R}} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0x} e^{i(\varphi_x - \pi/2)} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ e^{-i\pi/2} \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

- vettori normalizzati circ.

$$\tilde{\mathbf{E}}_{\mathcal{R}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{and similarly} \quad \tilde{\mathbf{E}}_{\mathcal{L}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Vettori di Jones

$$\tilde{\mathbf{E}}_{\mathcal{R}} + \tilde{\mathbf{E}}_{\mathcal{L}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 + 1 \\ -i + i \end{bmatrix} = \frac{2}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

lin. P-state!
Ampiezza doppia!

- vettori normalizzati circ.

$$\tilde{\mathbf{E}}_{\mathcal{R}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{and similarly} \quad \tilde{\mathbf{E}}_{\mathcal{L}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Vettori di Jones

$$\tilde{\mathbf{E}}_{\mathcal{R}} + \tilde{\mathbf{E}}_{\mathcal{L}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 + 1 \\ -i + i \end{bmatrix} = \frac{2}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

lin. P-state!
Ampiezza doppia!

- es. pol. ellittica

$$\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -i \end{bmatrix}$$

Vettori di Jones

- ortogonalità

$$\tilde{\mathbf{E}}_{\mathcal{R}} \cdot \tilde{\mathbf{E}}_{\mathcal{L}}^* = \frac{1}{2}[(1)(1)^* + (-i)(i)^*] = 0$$

$$\tilde{\mathbf{E}}_h \cdot \tilde{\mathbf{E}}_v^* = [(1)(0)^* + (0)(1)^*] = 0$$

$$\tilde{\mathbf{E}}_{\mathcal{R}} \cdot \tilde{\mathbf{E}}_{\mathcal{R}} = \tilde{\mathbf{E}}_{\mathcal{L}} \cdot \tilde{\mathbf{E}}_{\mathcal{L}}^* = 1$$

$$\tilde{\mathbf{E}}_{\mathcal{R}} \cdot \tilde{\mathbf{E}}_{\mathcal{L}}^* = \tilde{\mathbf{E}}_{\mathcal{L}} \cdot \tilde{\mathbf{E}}_{\mathcal{R}}^* = 0$$

Set ortonormale

Vettori di Jones

- Matrici di Mueller

$$\begin{bmatrix} \tilde{E}_{tx} \\ \tilde{E}_{ty} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \tilde{E}_{ix} \\ \tilde{E}_{iy} \end{bmatrix}$$

Es: stato-p (45°) + $\lambda/4$ (fast axis vertical)

$$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{E}_{tx} \\ \tilde{E}_{ty} \end{bmatrix}$$

$$\tilde{\mathbf{E}}_t = \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Circ. R -state ($\sigma+$)

Vettori di Jones

- Matrici di Mueller

Horizontal linear polarizer \leftrightarrow $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Vertical linear polarizer \updownarrow $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$\tilde{\mathbf{E}}_f = \mathcal{A}_n \cdots \mathcal{A}_2 \mathcal{A}_1 \tilde{\mathbf{E}}_i$$

Linear polarizer at $+45^\circ$ \nearrow $\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Linear polarizer at -45° \nwarrow $\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Quarter-wave plate, fast axis vertical $e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

Quarter-wave plate, fast axis horizontal $e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

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