Laboratorio di Fisica Atomica
CdL Fisica e Astrofisica

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Rifrazione – Riflessione - Polarizzazione

- **Testi/articoli di riferimento:**

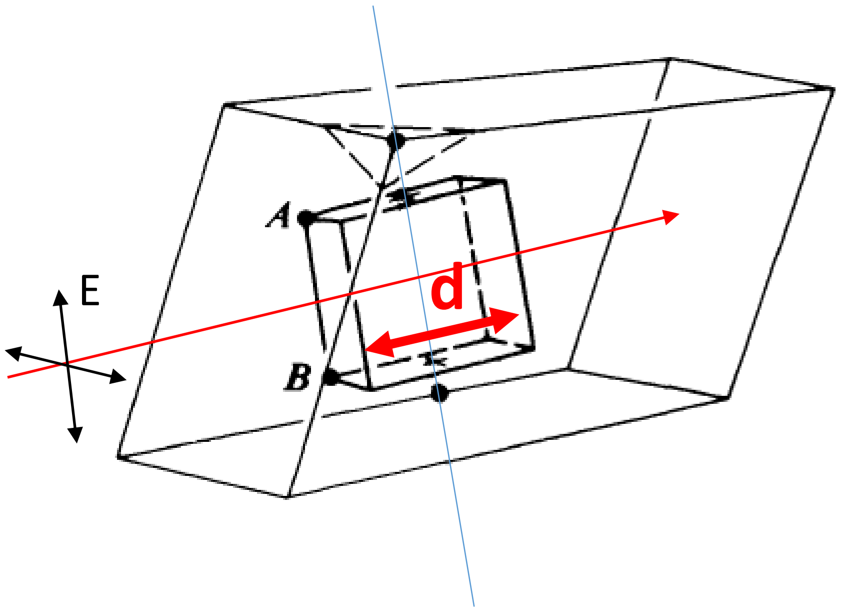
  - E. Hecht «Optics»

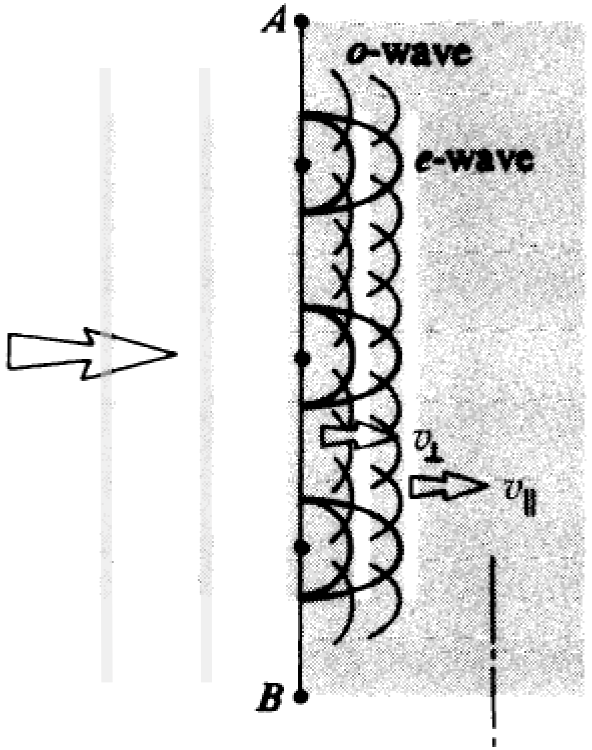
  - G. R. Fowles "Introduction to modern optics"

  - R.D. Guenther «Modern Optics»

Lamine di ritardo

- idea: far acquisire una fase aggiuntiva ad una componente $\mathcal{P}$ rispetto all'altra





$$\Lambda = d(|n_o - n_e|)$$

$$\Delta\varphi = k_0 \Lambda$$

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|)}$$

$$v_\parallel > v_\perp \qquad n_o > n_e$$

Lamine di ritardo

	$\boxed{\Delta\varphi = \frac{2\pi}{\lambda_0} d(\lvert n_o - n_e \rvert)}$	$\Lambda = d(\lvert n_o - n_e \rvert)$
• lamina λ	2π	λ
• lamina $\lambda/2$	π	$\lambda/2$
• lamina $\lambda/4$	$\pi/2$	$\lambda/4$

Lamine di ritardo

$$\Lambda = d(|n_o - n_e|)$$

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|)}$$

$\boxed{€€€}$

- true-zero order

Lamina di spessore minimo
difficile costruzione, fragile

Minore dipendenza da angolo di incidenza, temperatura, lunghezza d'onda

Quarzo
$\Delta n = 9.2 \times 10^{-3}$ (550nm)
$\Delta\phi = \pi/2$ $\boxed{d = 15\ \mu m\ !!}$

$\boxed{€}$

- multi-order

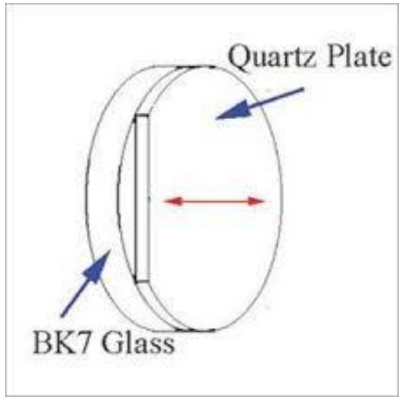
facile costruzione, non fragile

$\Delta\phi_M = \Delta\phi + 2m\pi$

Maggiore dipendenza da angolo di incidenza, temperatura, lunghezza d'onda

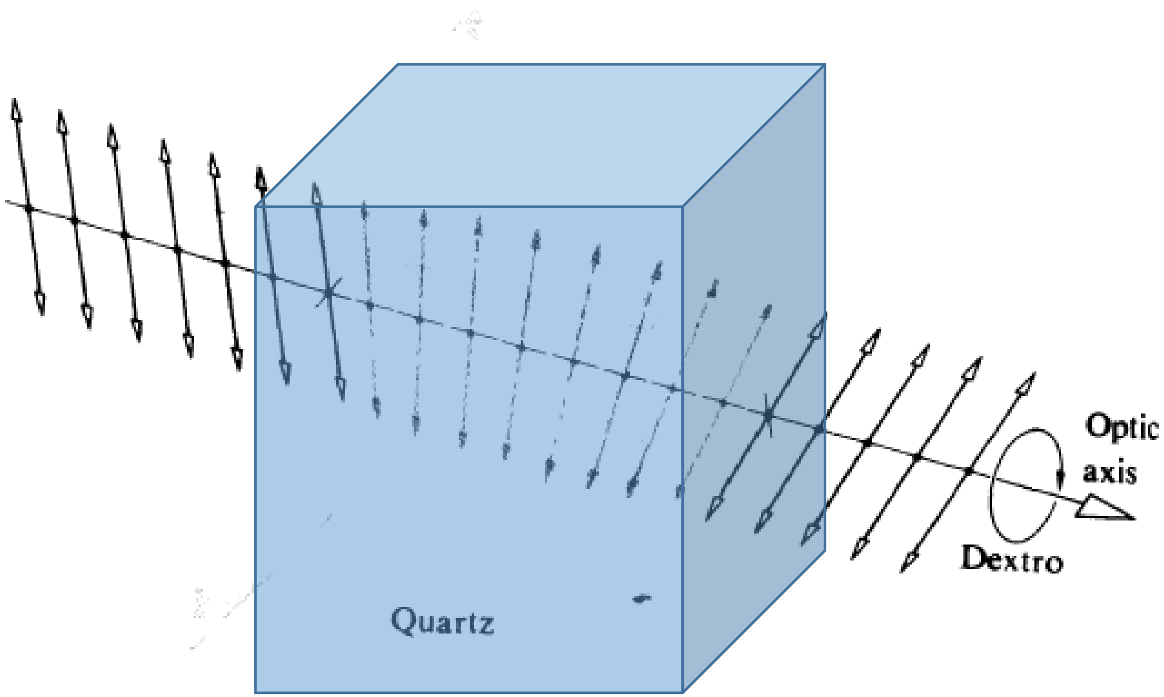
$\boxed{€€}$

- (cemented) zero-order



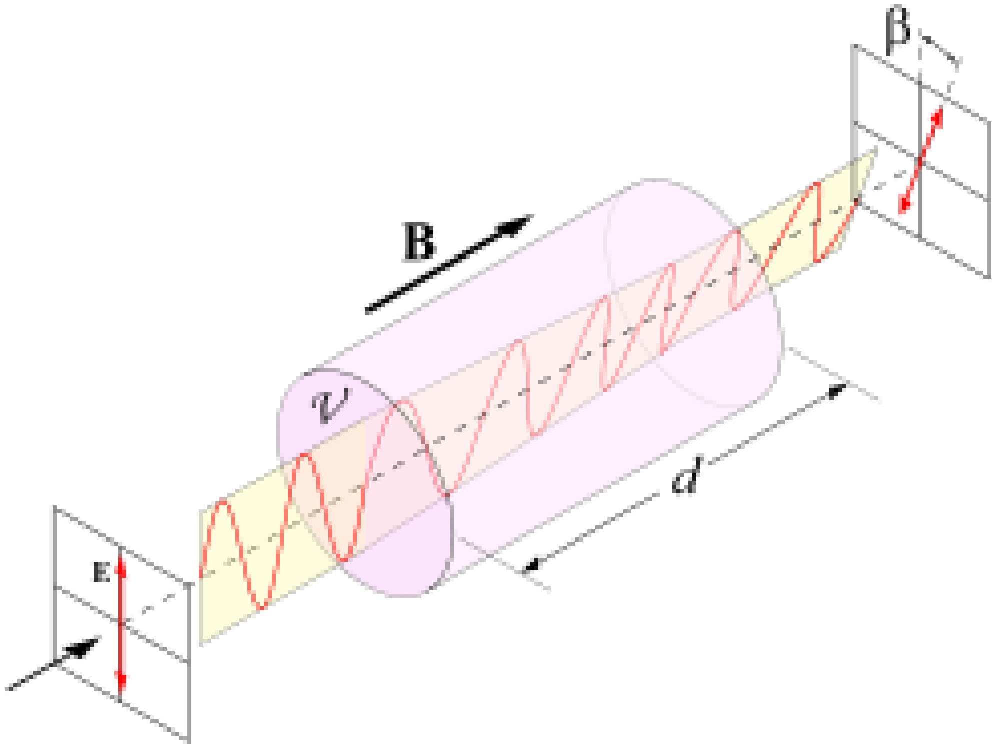
$\Delta\phi = \Delta\phi_{M1} - \Delta\phi_{M2}$

Attività ottica



Diversa velocità di propagazione di componenti con polarizzazione circolare R ed L

- effetto Faraday



$$\beta = VBd$$

V = Costante di Verdet

Attività ottica

$$\vec{\mathbf{E}}_{\mathcal{R}} = \frac{E_0}{2}[\hat{\mathbf{i}}\cos(k_{\mathcal{R}}z - \omega t) + \hat{\mathbf{j}}\sin(k_{\mathcal{R}}z - \omega t)]$$

$$\vec{\mathbf{E}}_{\mathcal{L}} = \frac{E_0}{2}[\hat{\mathbf{i}}\cos(k_{\mathcal{L}}z - \omega t) - \hat{\mathbf{j}}\sin(k_{\mathcal{L}}z - \omega t)]$$

$$\beta = -(k_{\mathcal{R}} - k_{\mathcal{L}})z/2$$

$$\boxed{\beta = \frac{\pi d}{\lambda_0}(n_{\mathcal{L}} - n_{\mathcal{R}})}$$

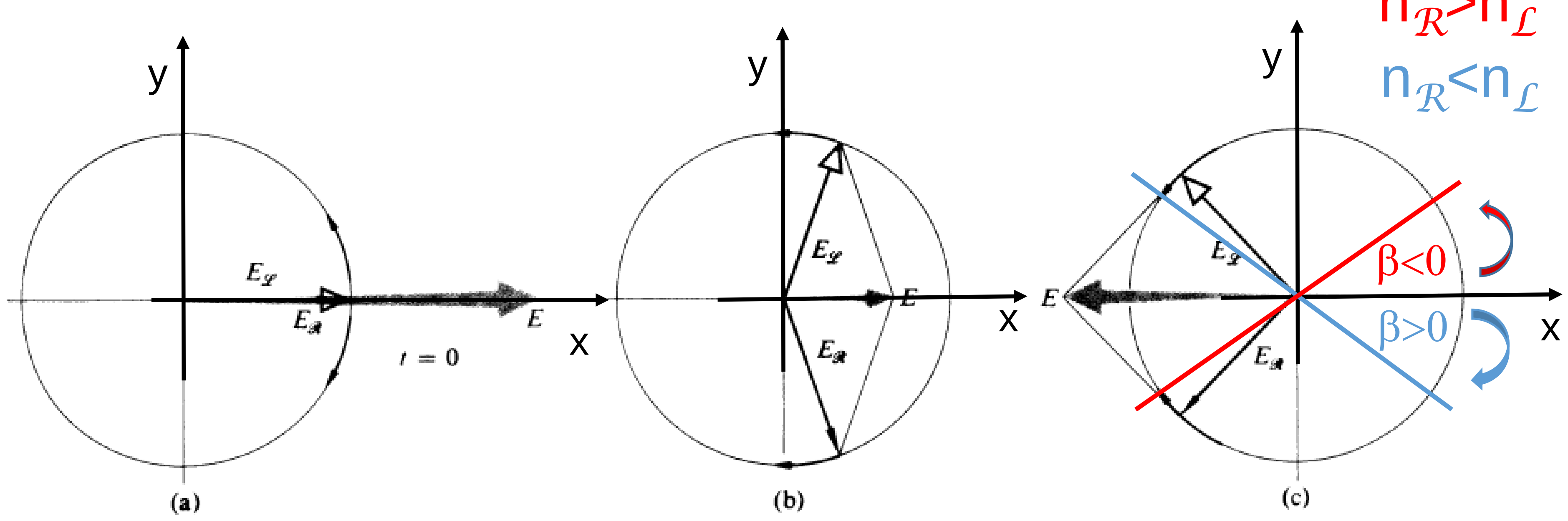
Due componenti in fase!

Uso prostaferesi

$$\vec{\mathbf{E}} = E_0\cos[(k_{\mathcal{R}} + k_{\mathcal{L}})z/2 - \omega t][\hat{\mathbf{i}}\cos(k_{\mathcal{R}} - k_{\mathcal{L}})z/2 + \hat{\mathbf{j}}\sin(k_{\mathcal{R}} - k_{\mathcal{L}})z/2]$$

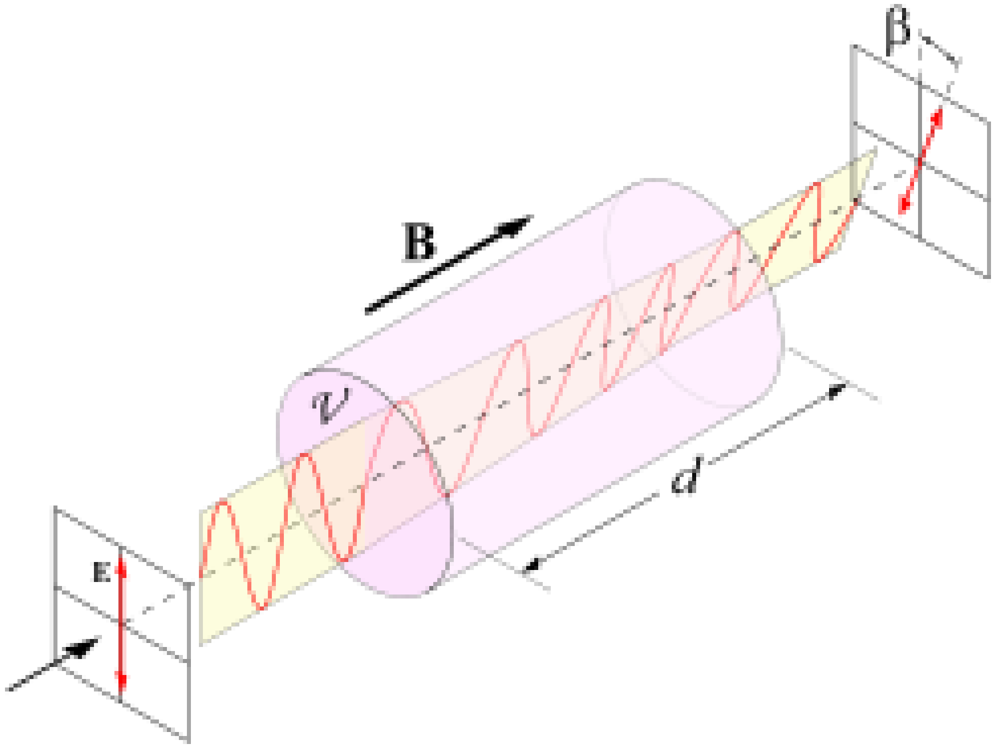
$z = 0$

$$\vec{\mathbf{E}} = E_0\hat{\mathbf{i}}\cos\omega t$$



Attività ottica

- effetto Faraday



$$\beta = VBd$$

V = Costante di Verdet

- effetto Macaluso – Corbino (1898)

Aumento dell'effetto vicino a risonanze atomiche

Onde e-m nei materiali

$$\nabla^2 \mathbf{E} - \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

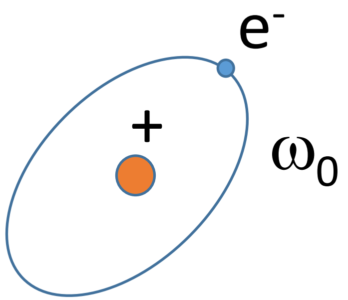
$$\nabla^2 \mathbf{B} - \mu\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{n} \leq c$$

$$\boxed{n = \sqrt{\mu_r \epsilon_r} \approx \sqrt{\epsilon_r} \quad \mathrm{e} \quad \mu_r \approx 1}$$

$$\mathbf{k}^2 = \mu\epsilon\,\omega^2 = \frac{n^2}{c^2}\omega^2$$

- modello semiclassico



$$\mathbf{F}_{\text{totale}} = -m_e\omega_0^2\mathbf{r} - m_e\gamma\dot{\mathbf{r}} + \mathbf{F}_{\text{ext}} = m_e\ddot{\mathbf{r}}$$

$$\ddot{\mathbf{r}} + \gamma\dot{\mathbf{r}} + \omega_0^2\mathbf{r} = \frac{\mathbf{F}_{\text{ext}}}{m_e}$$

$$\ddot{z} + \gamma z + \omega_0^2 z = \frac{q_e}{m_e} \; E_0|_{\text{locale}}\, e^{-i\omega t}$$

$$z(t) = \frac{q_e}{m_e}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \; E_0|_{\text{locale}}\, e^{-i\omega t}$$

Onde e-m nei materiali

$$z(t) = \frac{q_e}{m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \; E_0|_{\text{locale}} \, e^{-i\omega t}$$

momento di dipolo $\mathbf{p} = q_e \mathbf{r}(t)$

$$\begin{aligned}
\mathbf{p} &= \frac{q_e^2}{m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \; \mathbf{E}_0|_{\text{locale}} \, e^{-i\omega t} \\
&= \epsilon_0 \frac{q_e^2}{\epsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \; \mathbf{E}|_{\text{locale}} = \\
&= \epsilon_0 \alpha \; \mathbf{E}|_{\text{locale}} \\
&\approx \epsilon_0 \alpha \, \mathbf{E}
\end{aligned}$$

Polarizzabilità

$$\boxed{\alpha = \frac{q_e^2}{\epsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}}$$

Onde e-m nei materiali

Polarizzabilità

$$\boxed{\alpha = \frac{q_e^2}{\epsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}}$$

$$\begin{aligned} \mathbf{P} &= N\mathbf{p} = \epsilon_0 N\alpha\mathbf{E} = \\ &= \epsilon_0 \left[\frac{Nq_e^2}{\epsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right] \mathbf{E} = \\ &= \epsilon_0 \, \chi_e \, \mathbf{E} \, , \end{aligned}$$

Indice di rifrazione complesso

$$n = |n| e^{i\psi} = \sqrt{n_R^2 + n_I^2} \, e^{i\psi}$$

$$\begin{aligned} \chi_e &= \left[\frac{Nq_e^2}{\epsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right] \\ &= (\epsilon_r - 1) = n^2 - 1 = \\ &= (n+1)(n-1) \end{aligned}$$

Per $n \sim 1$

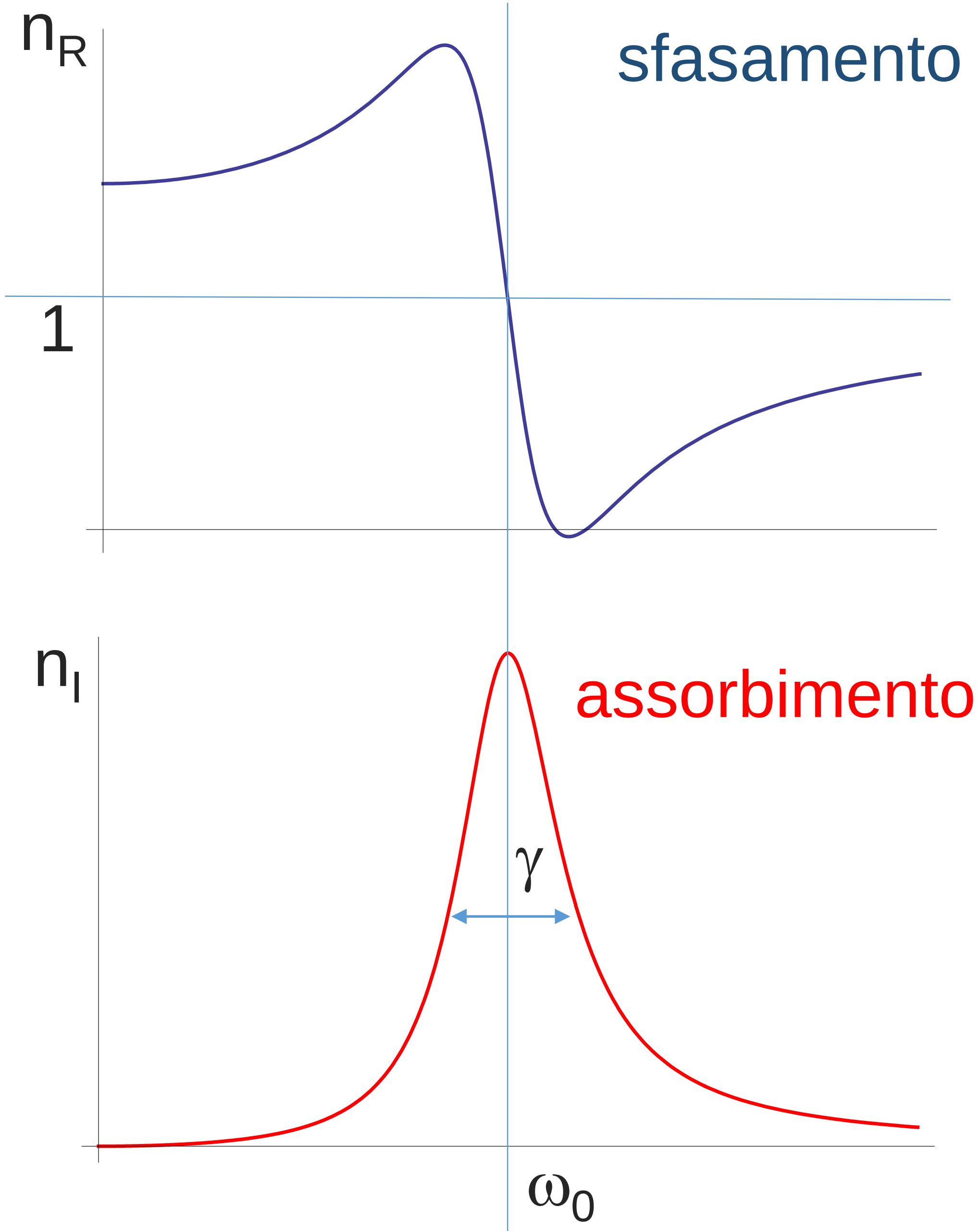
$$\boxed{\begin{aligned} n_R &= 1 + \frac{1}{2} \frac{Nq_e^2}{\epsilon_0 m_e} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2} \\ n_I &= \frac{1}{2} \frac{Nq_e^2}{\epsilon_0 m_e} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2} \end{aligned}}$$

Onde e-m nei materiali

$$\begin{aligned} n_R &= 1 + \frac{1}{2}\frac{Nq_e^2}{\epsilon_0 m_e}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \\ n_I &= \frac{1}{2}\frac{Nq_e^2}{\epsilon_0 m_e}\frac{\gamma\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \end{aligned}$$

$$\begin{aligned} k_R &= \frac{\omega}{c} n_R \\ k_I &= \frac{\omega}{c} n_I \end{aligned}$$

$$\mathbf{E}(y,t) = \hat{z}\, E_0\, \underline{e^{-\frac{n_I}{c}\omega y}}\, \underline{e^{i(\frac{n_R}{c}\omega y - \omega t)}}$$



Onde e-m nei materiali (Clausius -Mossotti)

Contributo al campo da atomi vicini

$$\mathbf{E}|_{\text{locale}} = \mathbf{E} + \frac{1}{3}\frac{\mathbf{P}}{\epsilon_0}$$

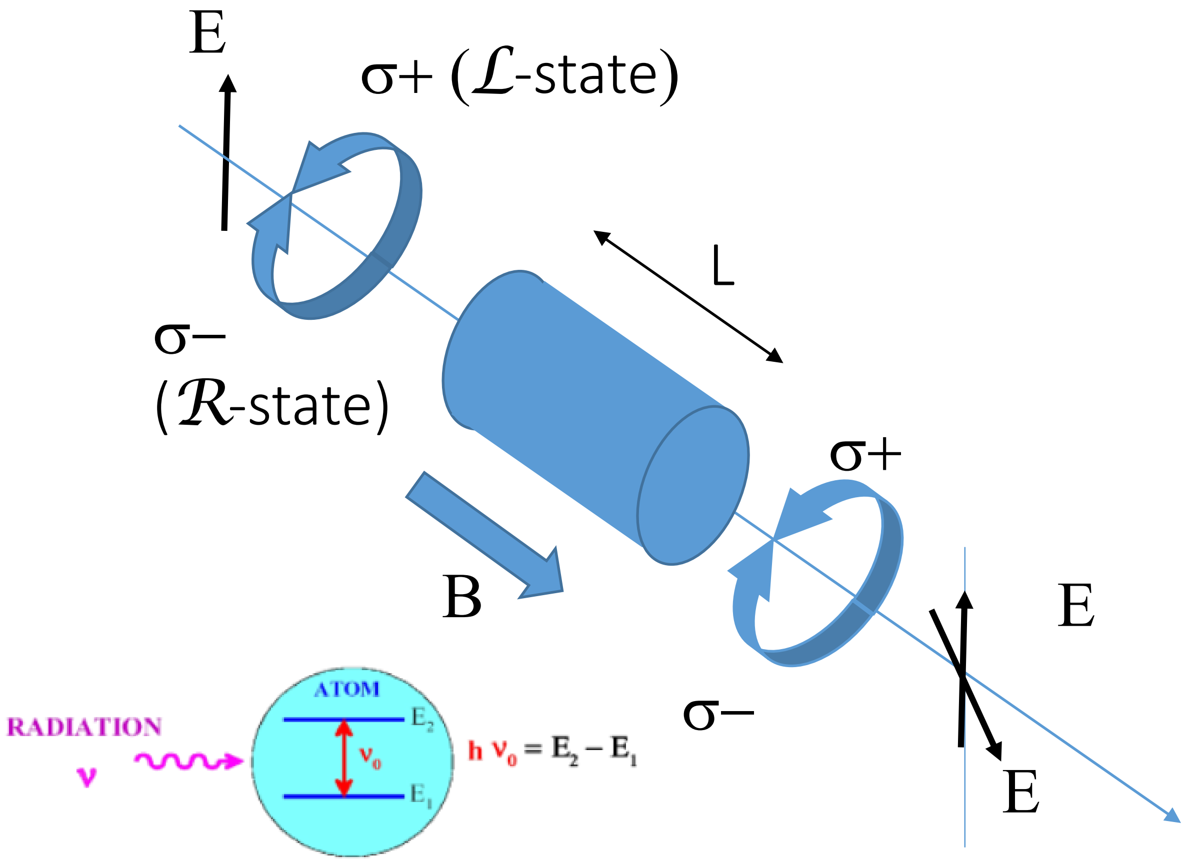
$$\begin{aligned}\mathbf{p} &= \frac{q_e^2}{m_e}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\;\mathbf{E}_0|_{\text{locale}}\, e^{-i\omega t} \\ &= \epsilon_0 \alpha\;\mathbf{E}|_{\text{locale}}\;,\end{aligned}$$

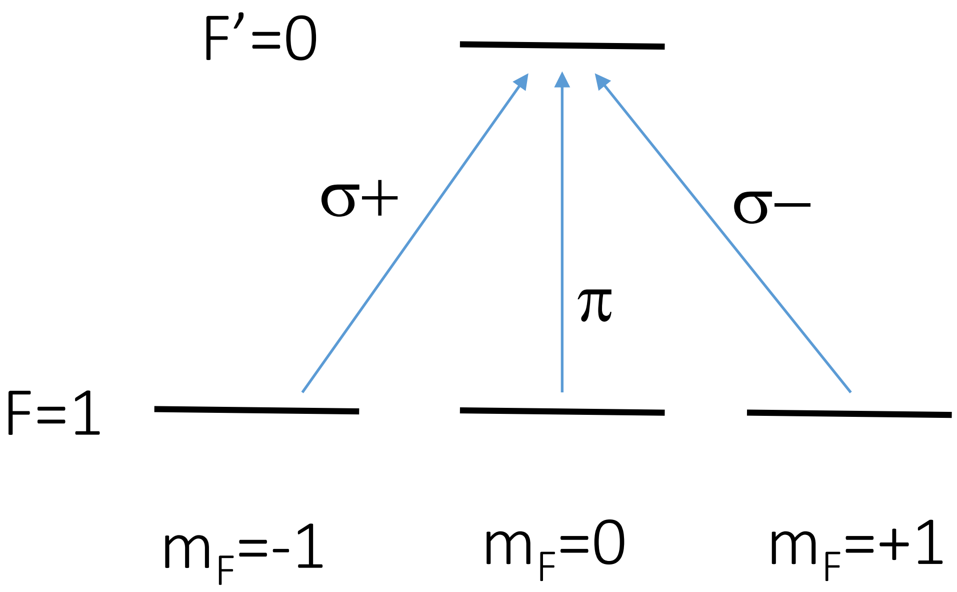
$$\begin{aligned}\mathbf{P} &= N\mathbf{p} \\ &= \epsilon_0 N\alpha\left[\mathbf{E} + \frac{1}{3}\frac{\mathbf{P}}{\epsilon_0}\right] \\ &= \epsilon_0 \frac{N\alpha}{1 - \frac{1}{3}N\alpha}\,\mathbf{E} \\ &= \epsilon_0\,\chi_e\,\mathbf{E} \\ &= \epsilon_0(\epsilon_r - 1)\,\mathbf{E}\end{aligned}$$

$$(n^2 - 1) = \frac{N\alpha}{1 - \frac{1}{3}N\alpha}\;,$$

$$\boxed{3\frac{n^2 - 1}{n^2 + 2} = N\alpha\;.}$$

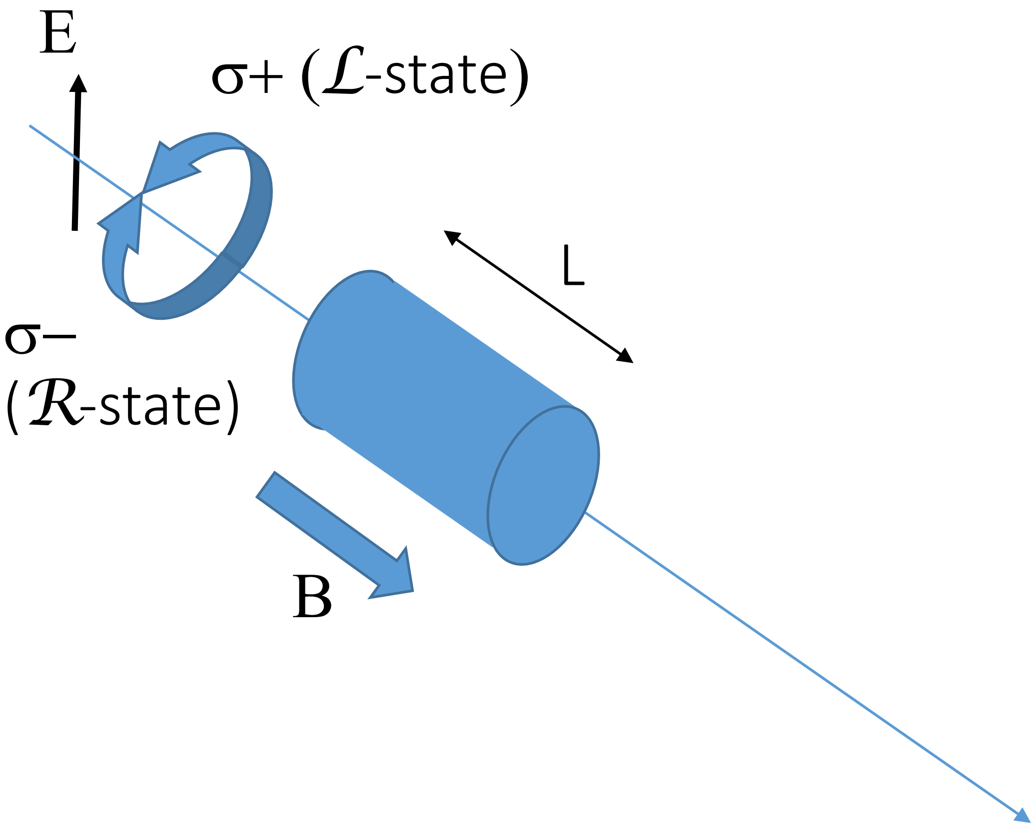
Effetto Faraday (Macaluso-Corbino)

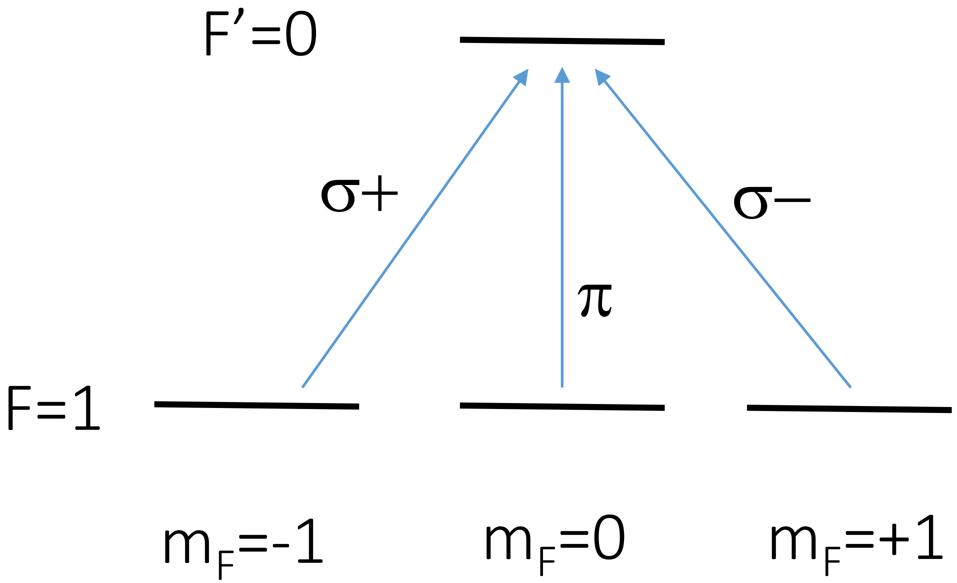




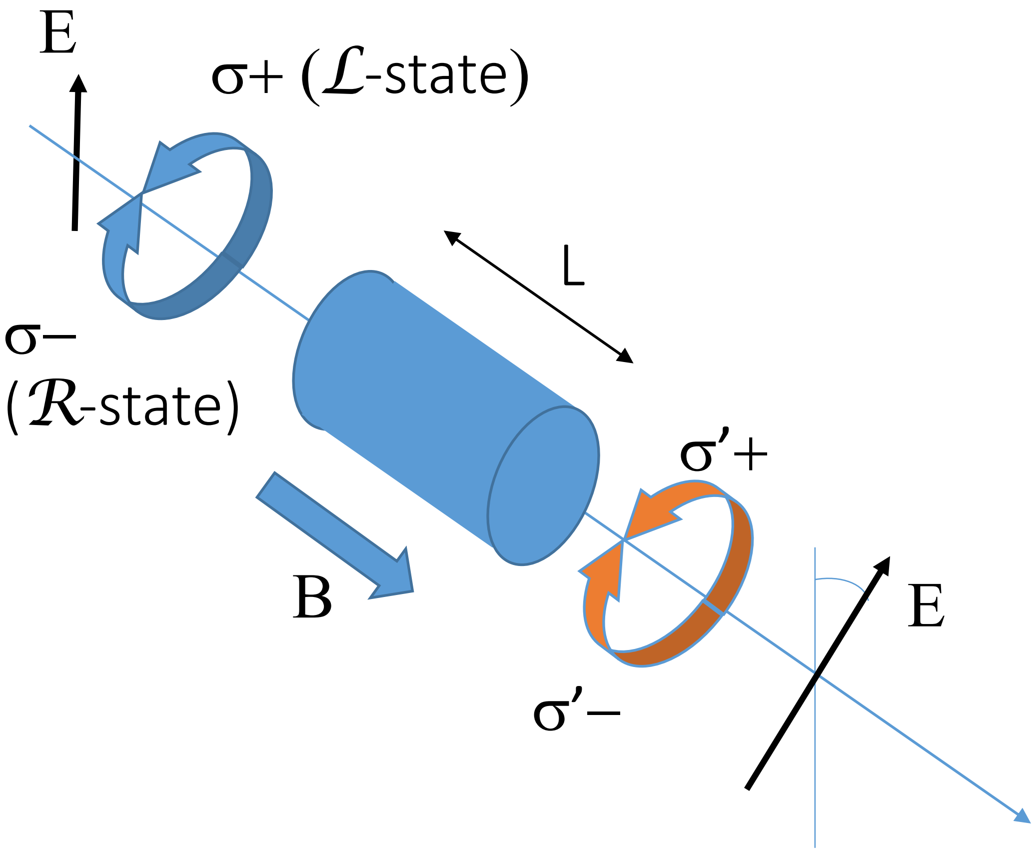
- Vetri flint $V \simeq 3 \times 10^{-5}$ rad G^{-1} cm^{-1}
- Rubidio ($\omega \simeq \omega_0$) $V \simeq 10^4$ rad G^{-1} cm^{-1}

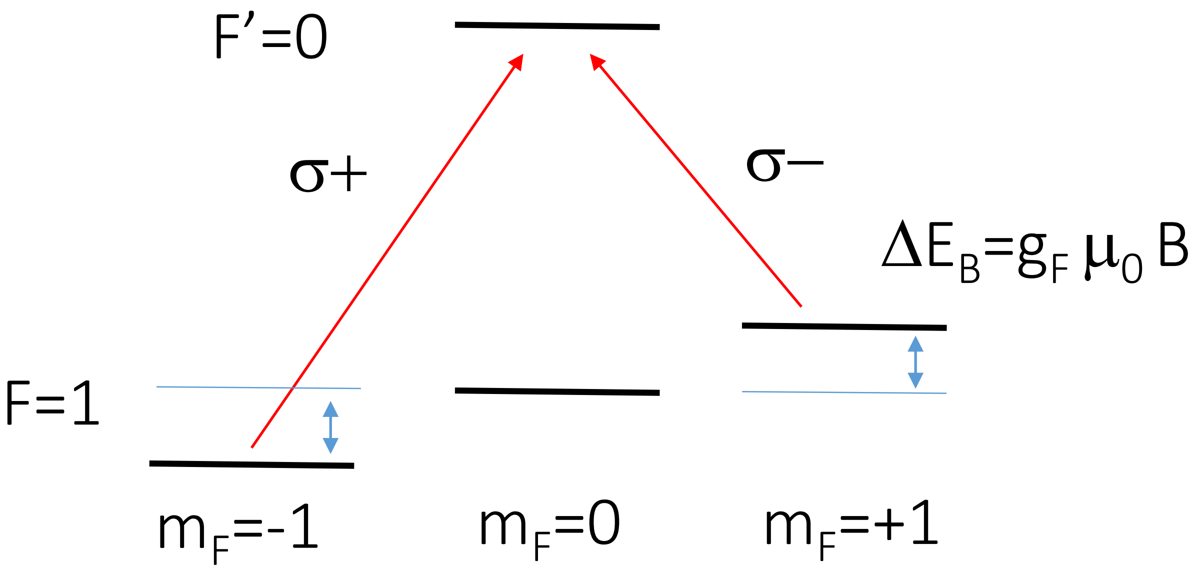
Effetto Faraday (Macaluso-Corbino)





Effetto Faraday (Macaluso-Corbino)

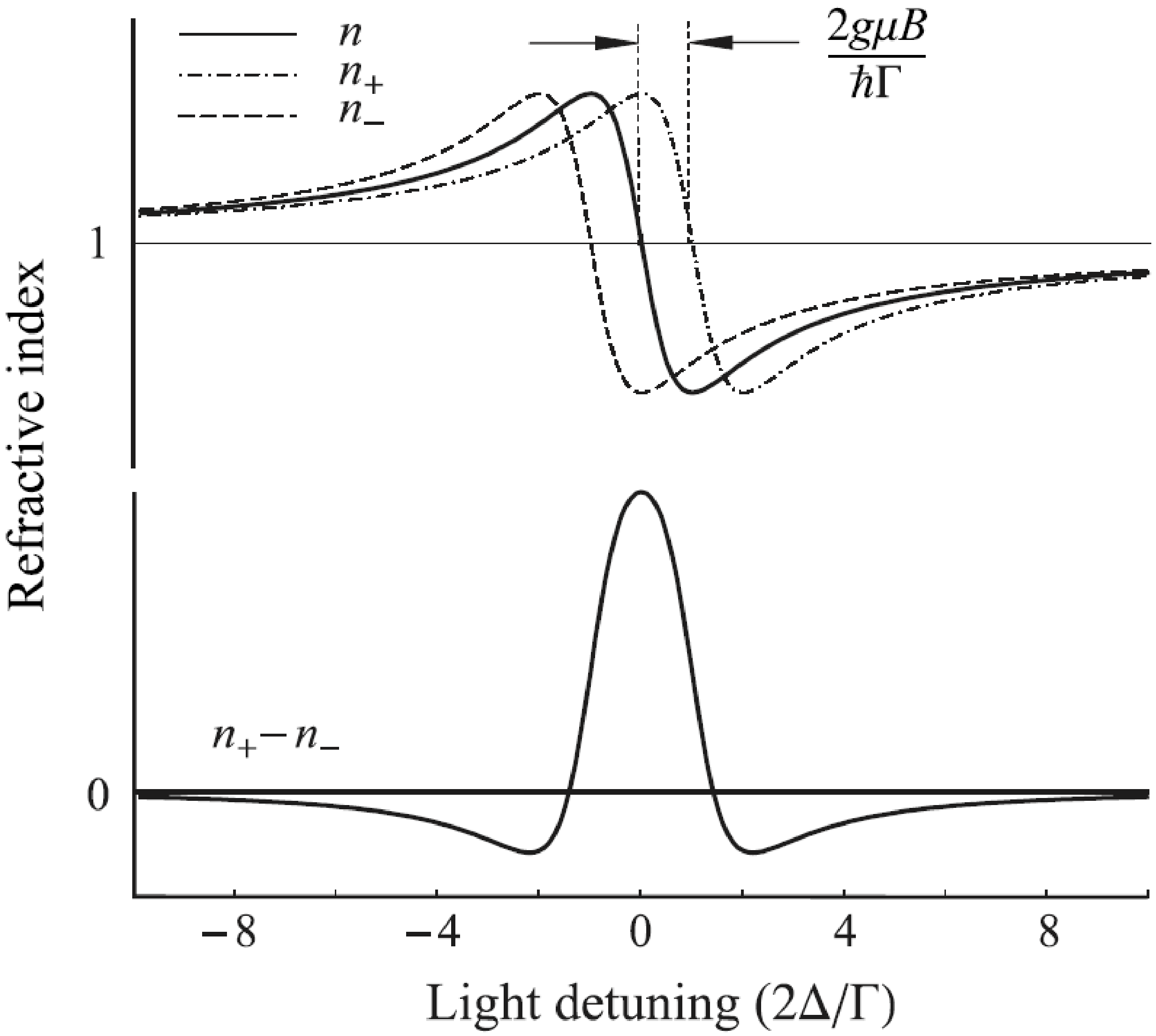


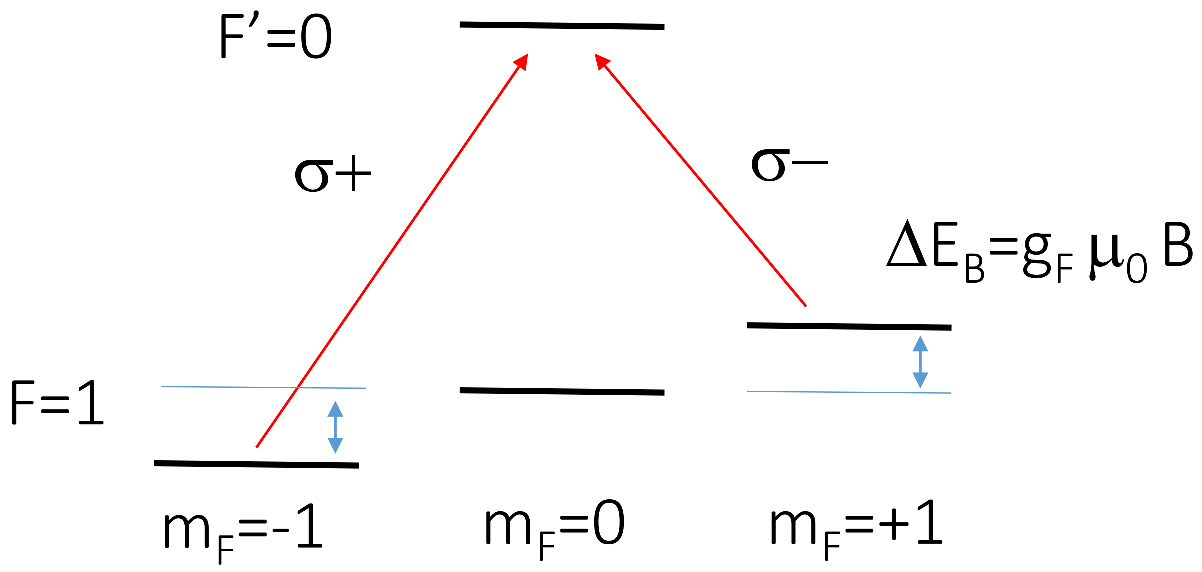


$$\boxed{\beta \;=\; \pi\, L\, (n_{\mathcal{L}} - n_{\mathcal{R}})/\lambda_0}$$

$$\boxed{\beta \;=\; \pi\, L\, \mathit{Re}\{n_+ - n_-\}/\lambda_0}$$

Effetto Faraday (Macaluso-Corbino)





$$\beta = \pi\, L\, (n_{\mathcal{L}} - n_{\mathcal{R}})/\lambda_0$$

$$\beta = \pi\, L\, Re\{n_+ - n_-\}/\lambda_0$$

Effetto Faraday (Macaluso-Corbino)

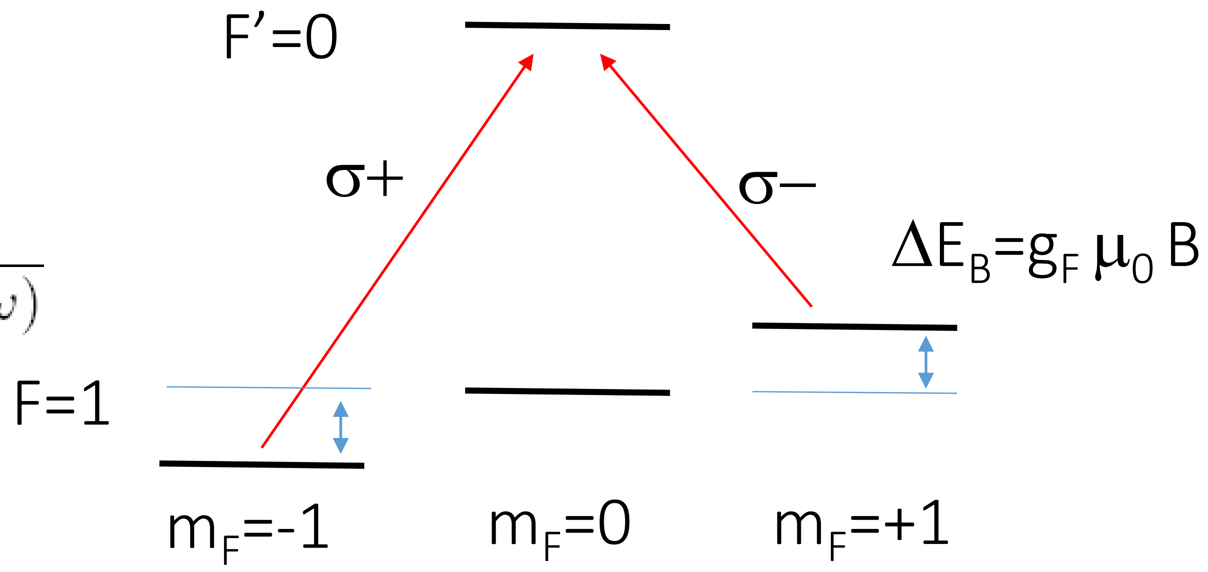
$$\boxed{\beta = \pi L \, Re\{n_+ - n_-\}/\lambda_0}$$

$$n(\omega) = 1 + \frac{1}{2}\frac{N\, q_e^2}{\epsilon_0\, m_e}\frac{1}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

Estensione modello semiclassico:

$$n_\pm(\omega) = 1 + \frac{1}{2}\frac{N\, q_e^2}{\epsilon_0\, m_e}\frac{1}{((\omega_0 \pm g_F \mu_0 B/\hbar)^2 - \omega^2 - i\gamma\omega)}$$

$$\gamma \quad \longrightarrow \quad \Gamma = \frac{\omega_0^3}{3\pi\epsilon_0 \hbar c^2}|\langle e|\mu|g\rangle|^2$$



$$\omega = \omega_0$$

$$\boxed{\beta = \frac{L}{2L_0}\frac{b}{1+b^2}}$$

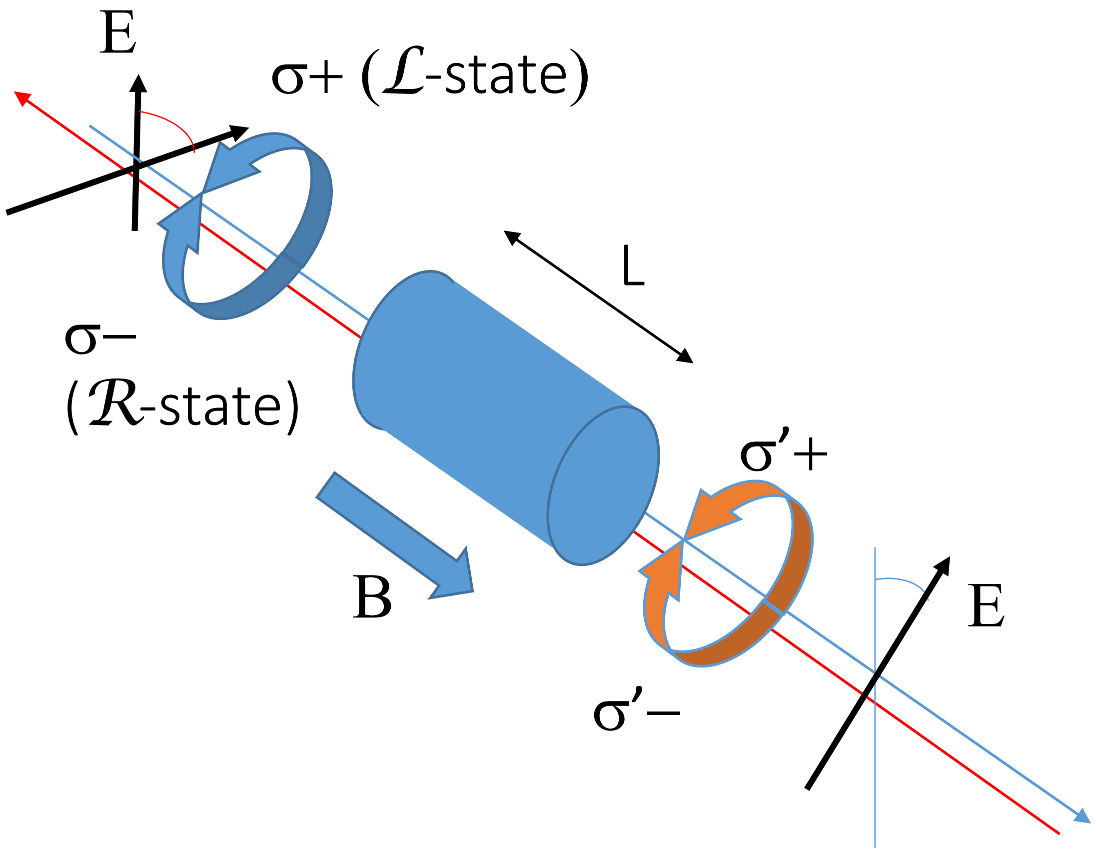
$$b = 2\frac{\delta\omega_B}{\Gamma} = 2\frac{g_F \mu_0 B}{\hbar\,\Gamma}$$

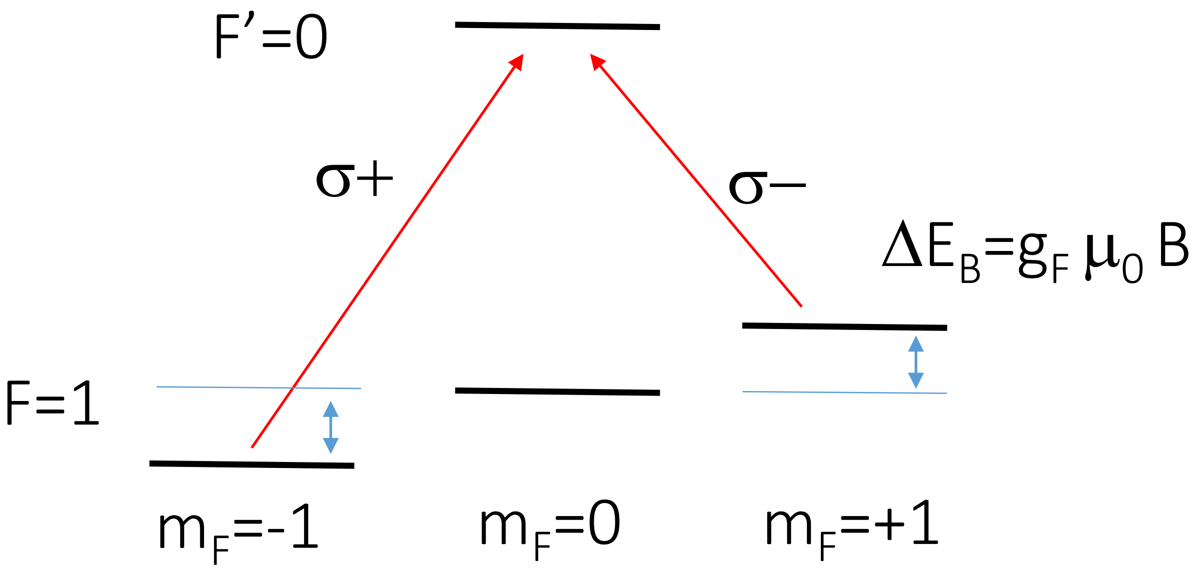
Shift Zeeman normalizzato

$$L_0 = \frac{N q_e^2}{\epsilon_0\, m_e\, c\, \Gamma}$$

Lunghezza di assorbimento in risonanza

Effetto Faraday (Macaluso-Corbino)



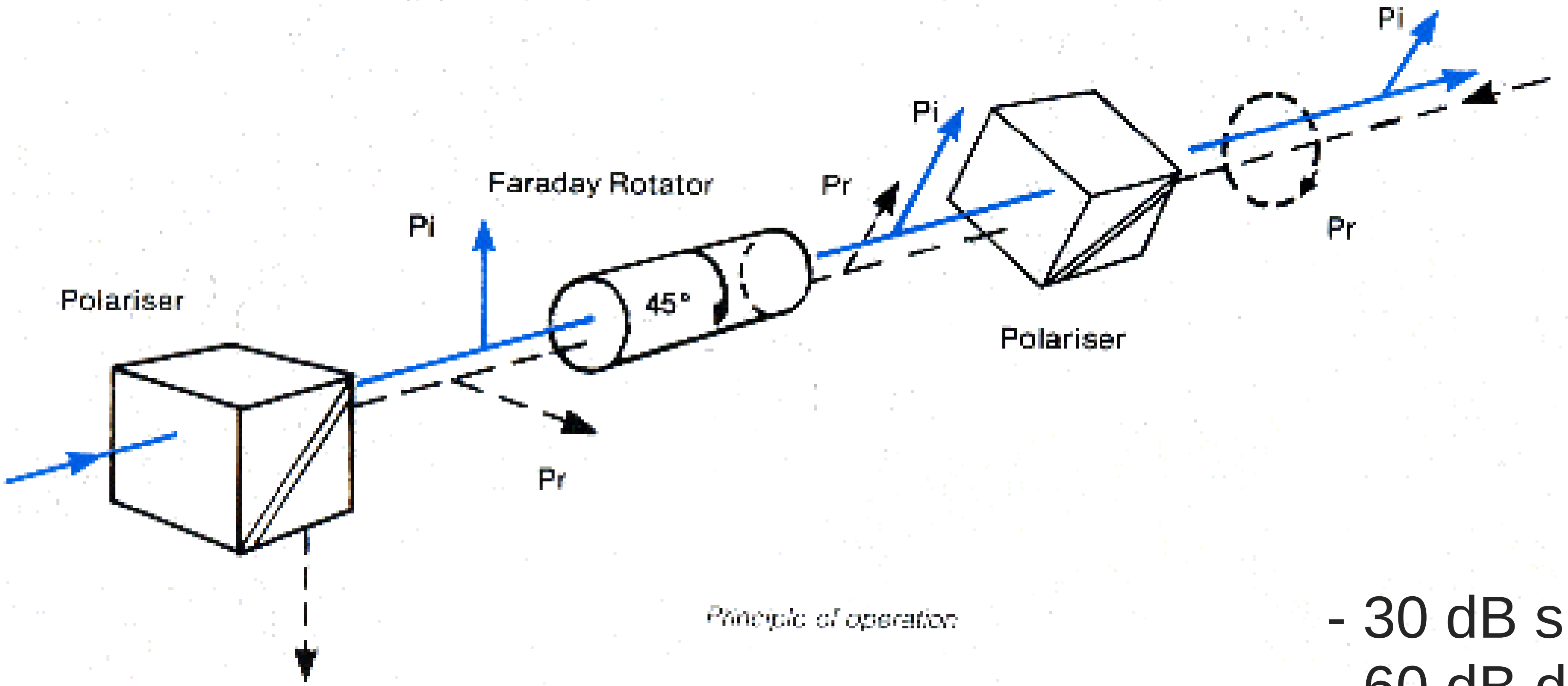


$$\boxed{\beta = \frac{L}{2L_0}\frac{b}{1+b^2}}$$

$$b = 2\frac{\delta\omega_B}{\Gamma} = 2\frac{g_F \mu_0 B}{\hbar\,\Gamma}$$

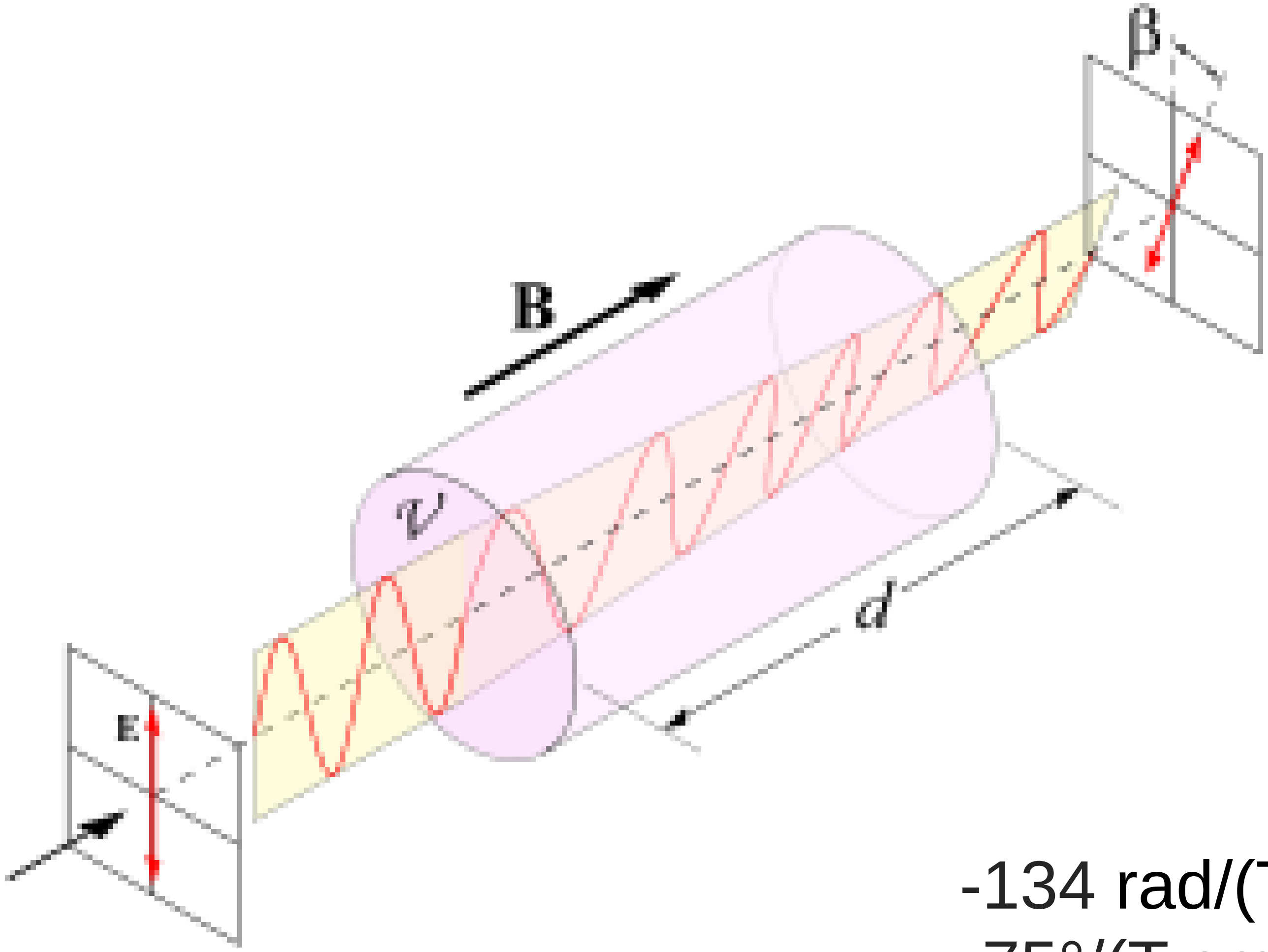
Rotazione nello stesso verso per direzioni opposte !

Isolatore ottico



- 30 dB singoli
- 60 dB doppio stadio

Rotatore di Faraday

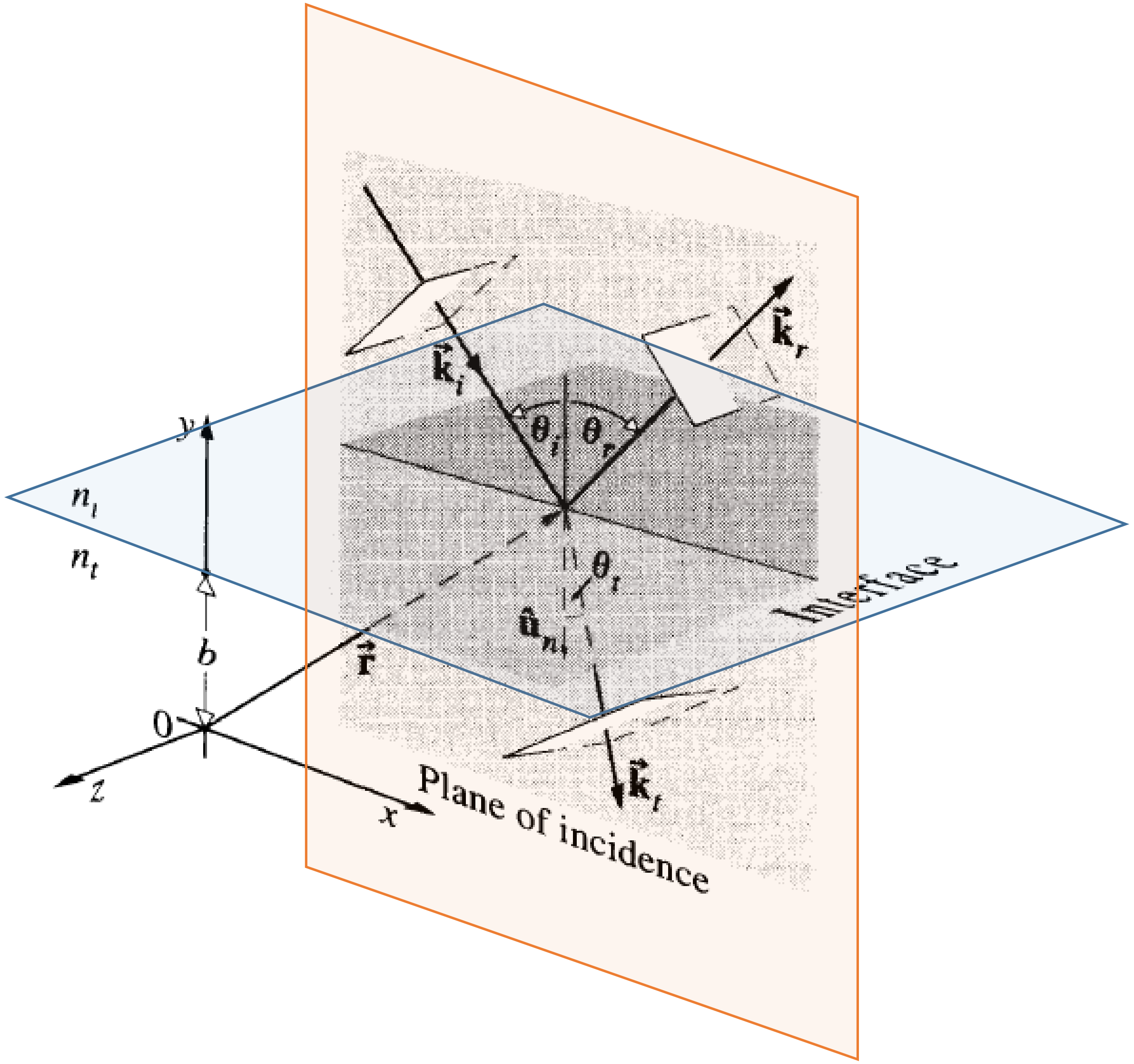


$$\beta = VBd$$

V = Costante di Verdet

-134 rad/(T m) per Terbium Gallium garnet
-75°/(T cm)

Riflessione Rifrazione



$$\vec{E}_i = \vec{E}_{0i}\cos(\vec{k}_i \cdot \vec{r} - \omega_i t)$$

$$\vec{E}_r = \vec{E}_{0r}\cos(\vec{k}_r \cdot \vec{r} - \omega_r t + \varepsilon_r)$$

$$\vec{E}_t = \vec{E}_{0t}\cos(\vec{k}_t \cdot \vec{r} - \omega_t t + \varepsilon_t)$$

- continuità dei campi all'interfaccia

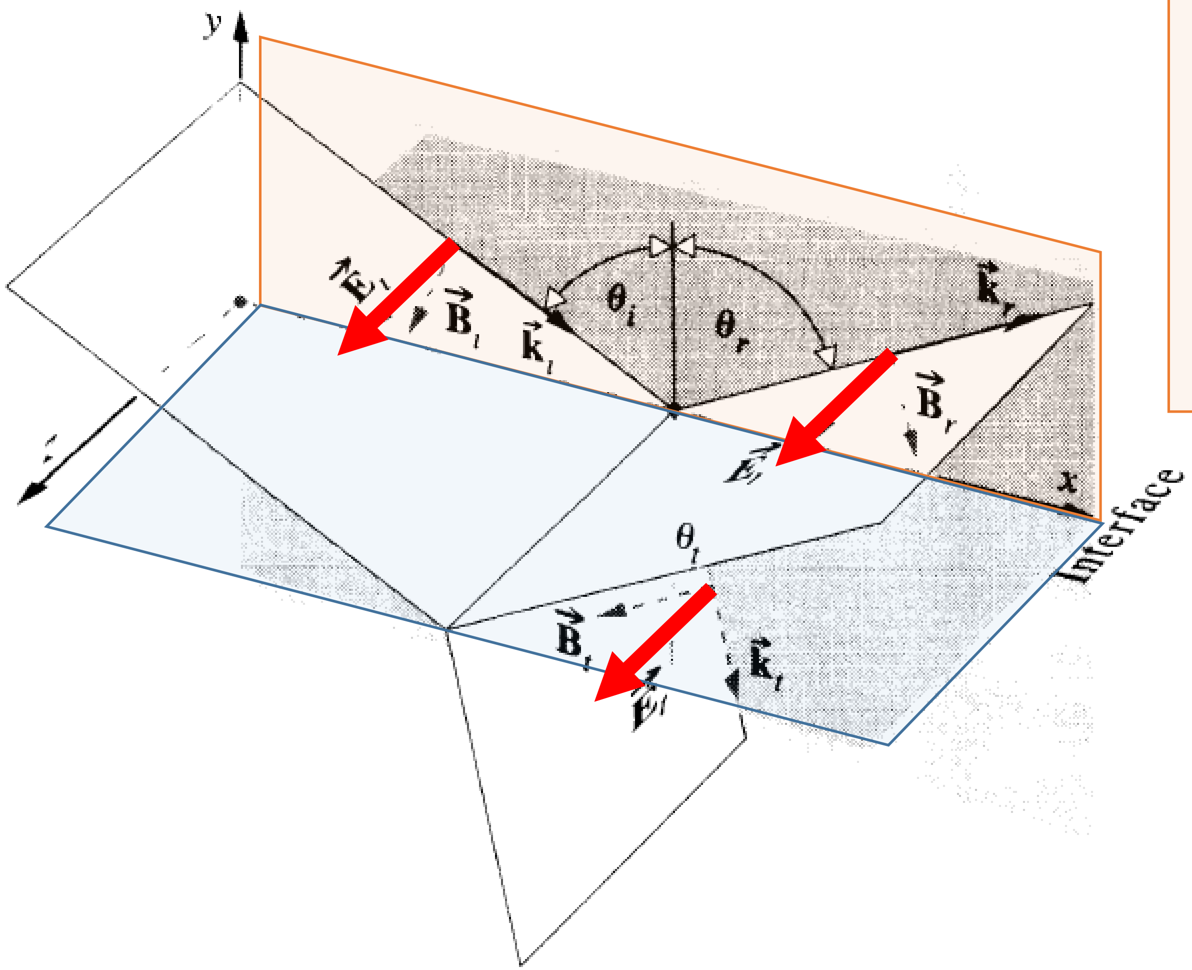
$$\theta_i = \theta_r$$

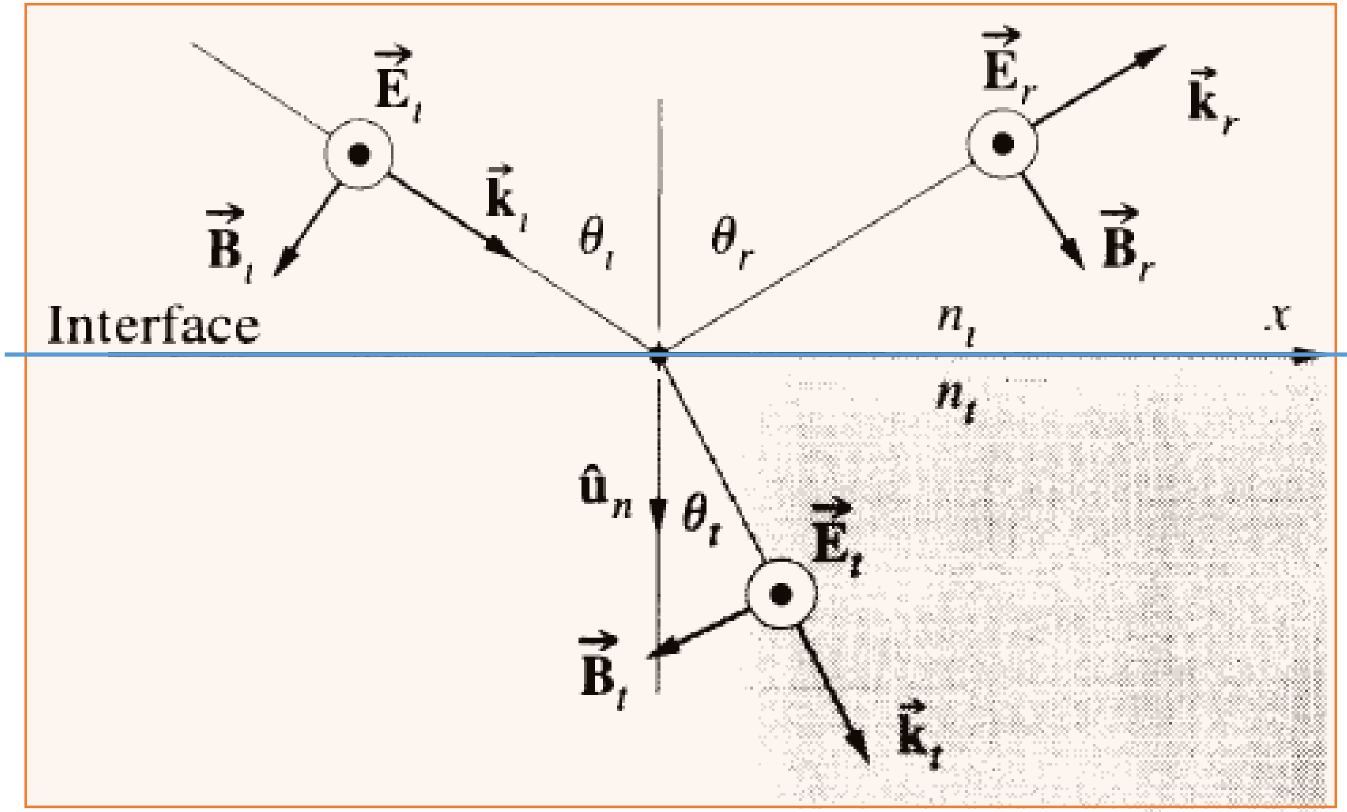
$$\omega_i = \omega_r = \omega_t$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Riflessione Rifrazione

- E *ortogonale* a piano di incidenza (s)





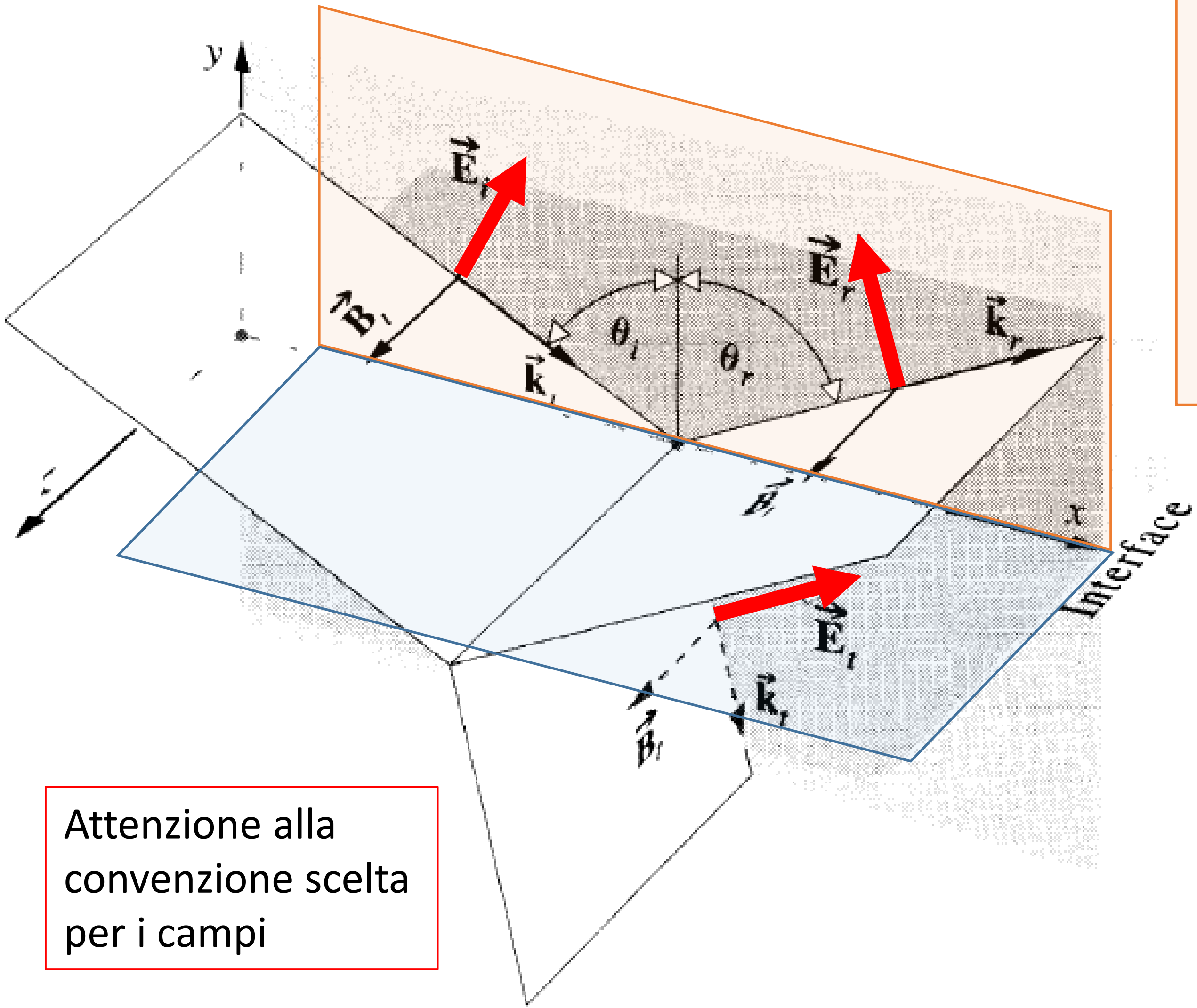
- Relazioni di Fresnel

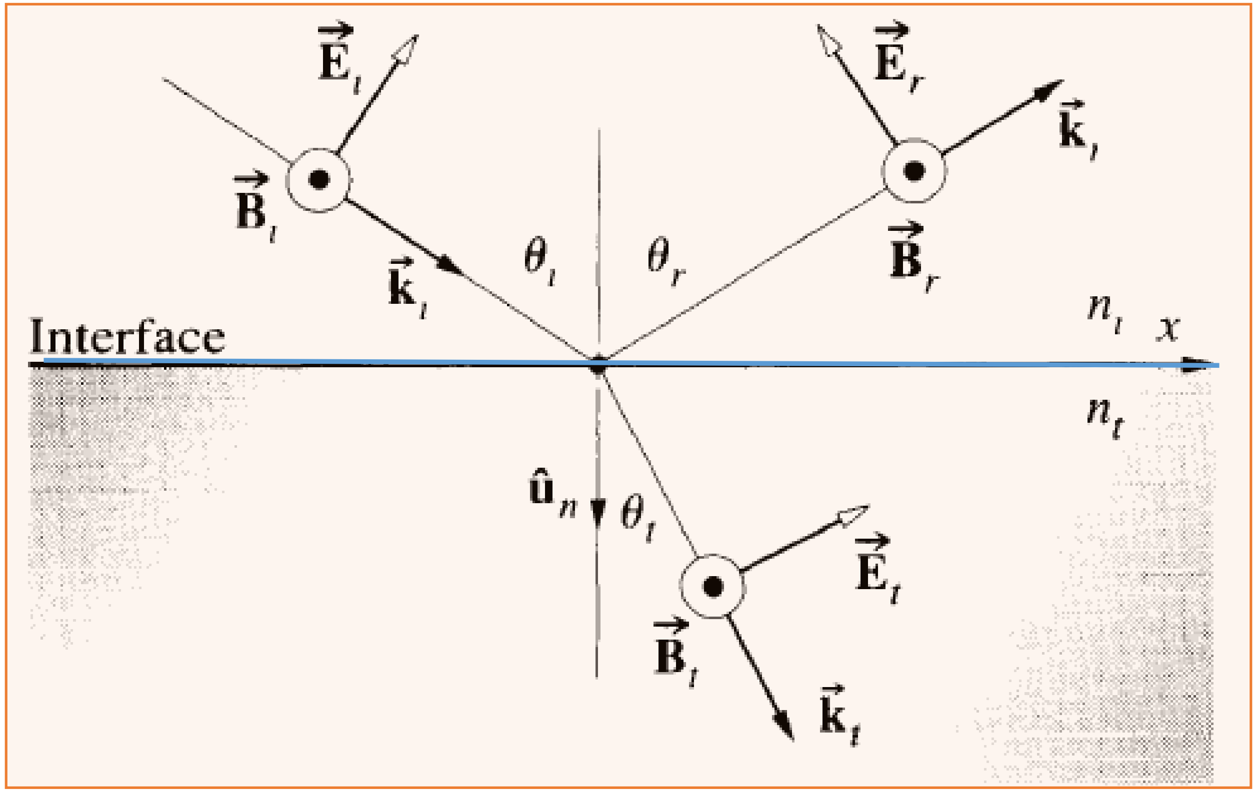
$$r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Riflessione Rifrazione

- E *parallelo* a piano di incidenza (p)





Attenzione alla convenzione scelta per i campi

- Relazioni di Fresnel

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$t_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

Riflessione Rifrazione

- Relazioni di Fresnel

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

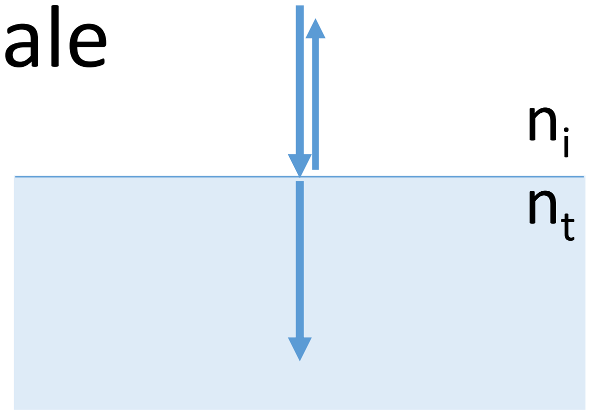
$$r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\parallel} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

- incidenza normale

$\theta_i=0, \theta_t=0$



$$[r_{\parallel}]_{\theta_i=0} = [-r_{\perp}]_{\theta_i=0} = \frac{n_t - n_i}{n_t + n_i} = \pm 0.2$$

$n_i=1$
$n_t=1.5$

$$[t_{\parallel}]_{\theta_i=0} = [t_{\perp}]_{\theta_i=0} = \frac{2 n_i}{n_i + n_t} = +0.8$$

Riflessione Rifrazione

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} < 0 \qquad \begin{matrix} = -0.2 \\ = -1 \end{matrix}$$

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad \begin{matrix} = +0.2 \\ = -1 \end{matrix}$$

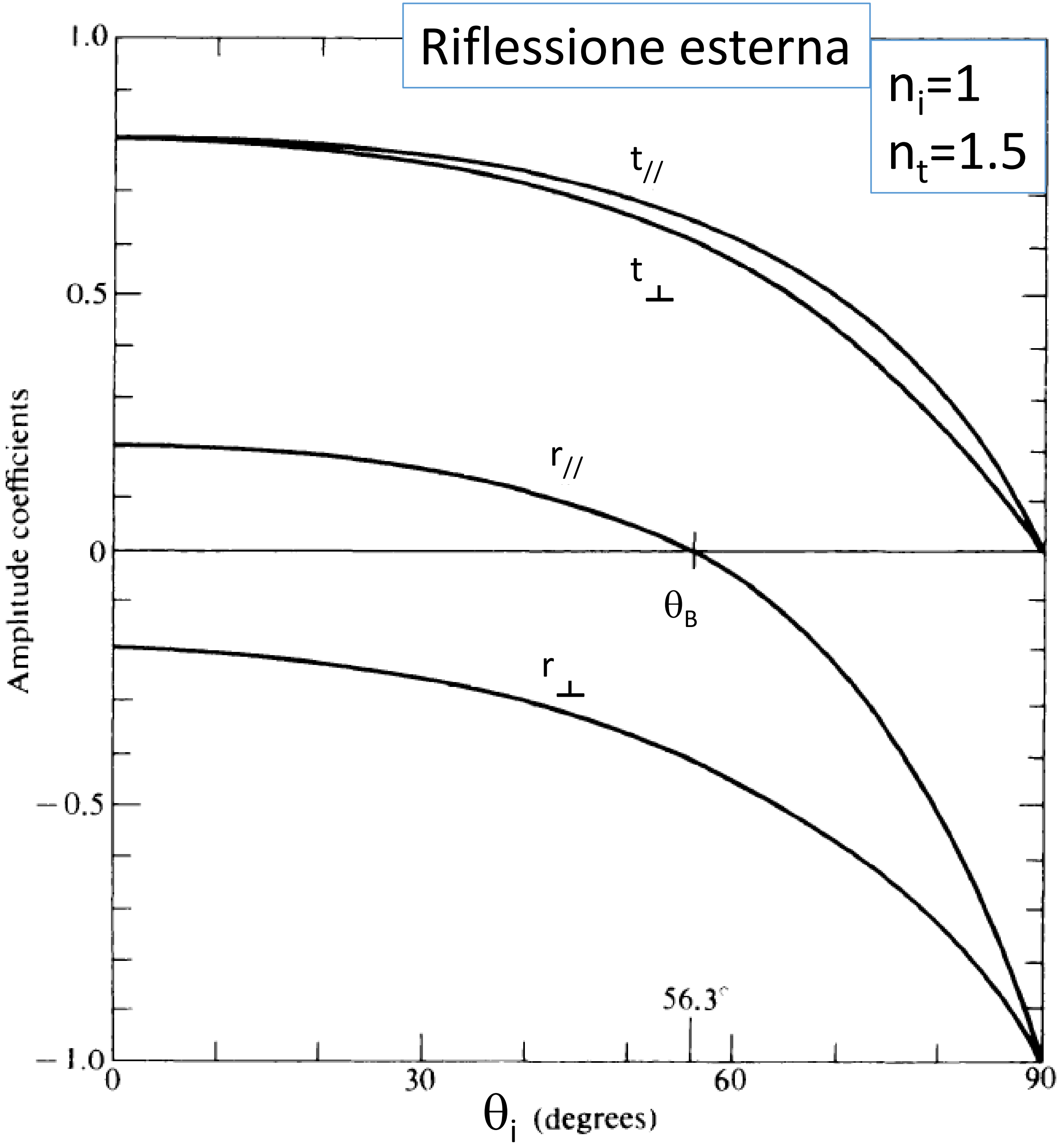
$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad \begin{matrix} = 0.8 \\ = 0 \end{matrix}$$

$$t_\parallel = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad \begin{matrix} = 0.8 \\ = 0 \end{matrix}$$

$$t_\perp + (-r_\perp) = 1 \qquad\qquad t_\parallel + r_\parallel = 1$$

Per ogni θi

Solo per θi=0

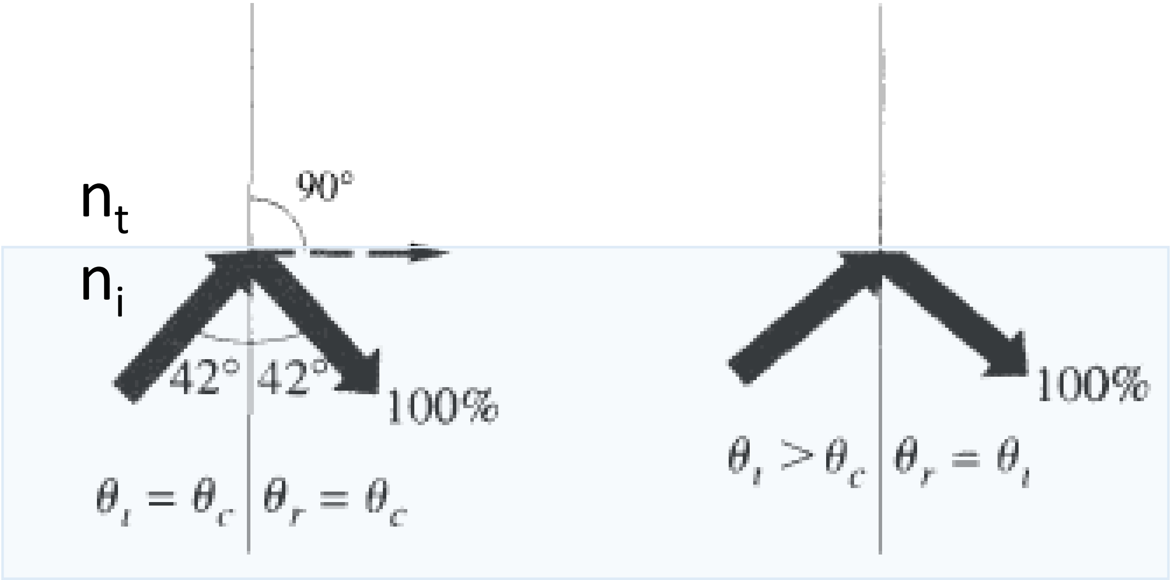


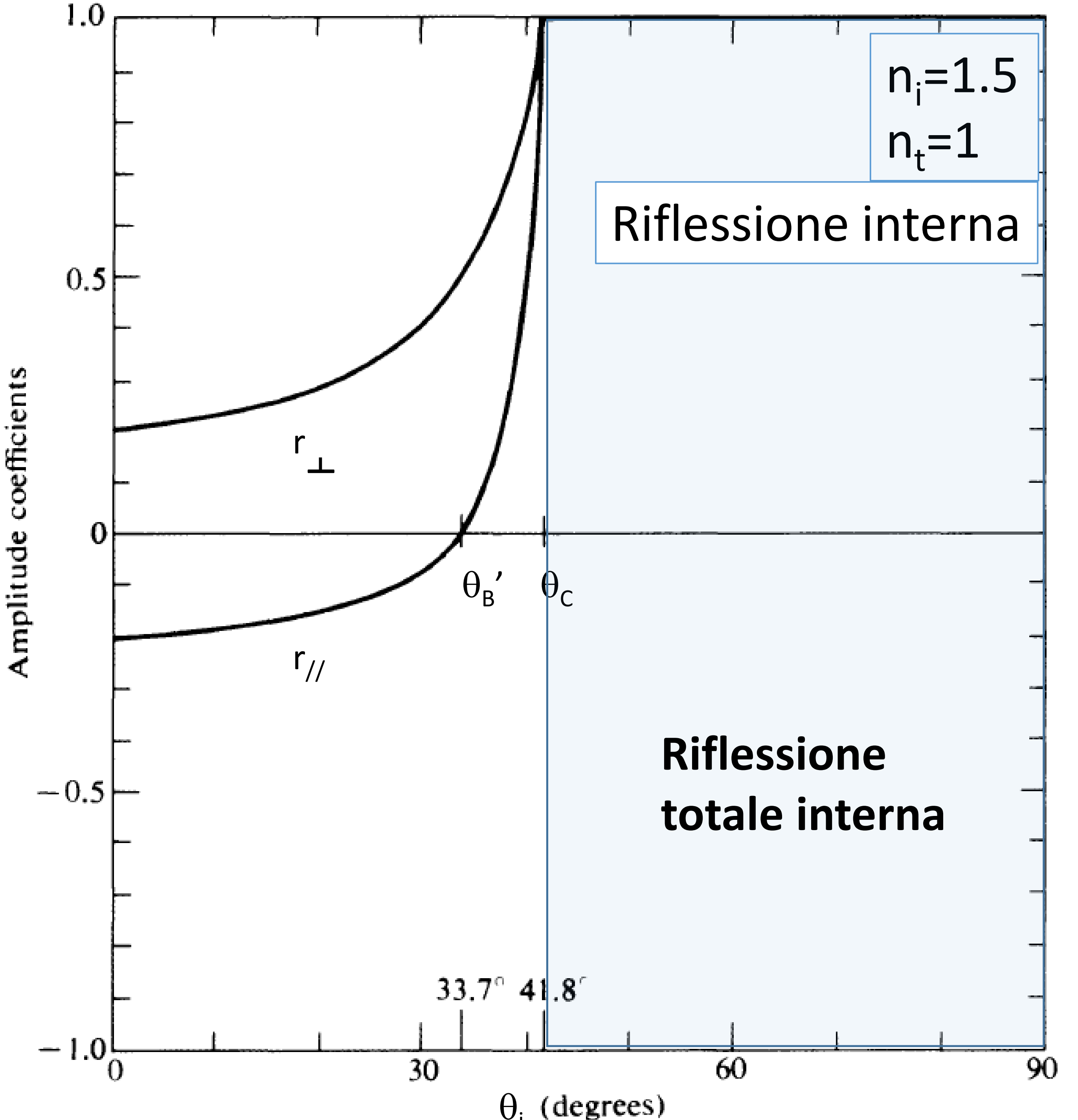
Riflessione Rifrazione

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

- θ'_B e θ_B complementari
- $\sin\theta_C = n_t/n_i$



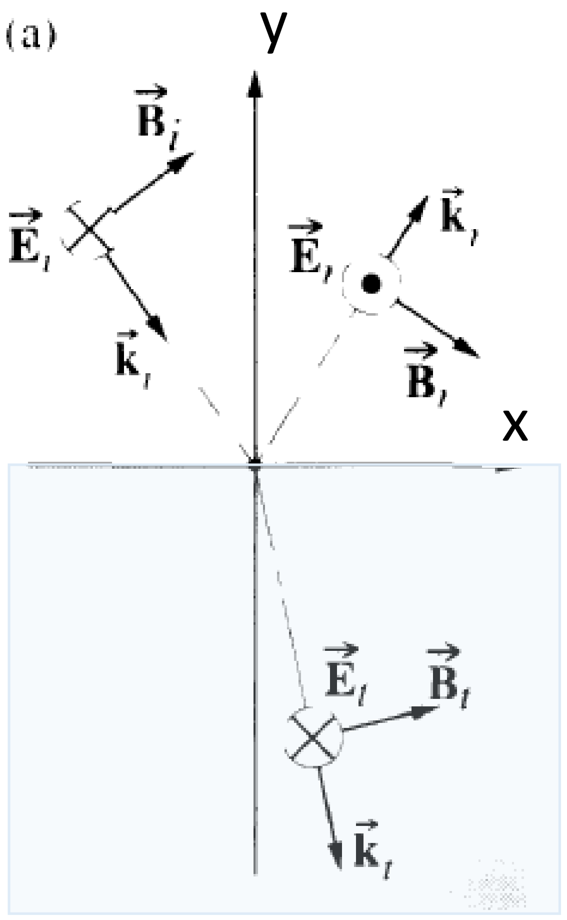


Riflessione Rifrazione

- shift di fase

$E_{//i}$ in fase con $E_{//t}$ se componenti *y* sono parallele

		$\Delta\phi$
$r_\perp = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$	< 0	π
$t_\perp = \dfrac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$	> 0	0
$t_\parallel = \dfrac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$	> 0	0
$r_\parallel = \dfrac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$	$\pm$	?

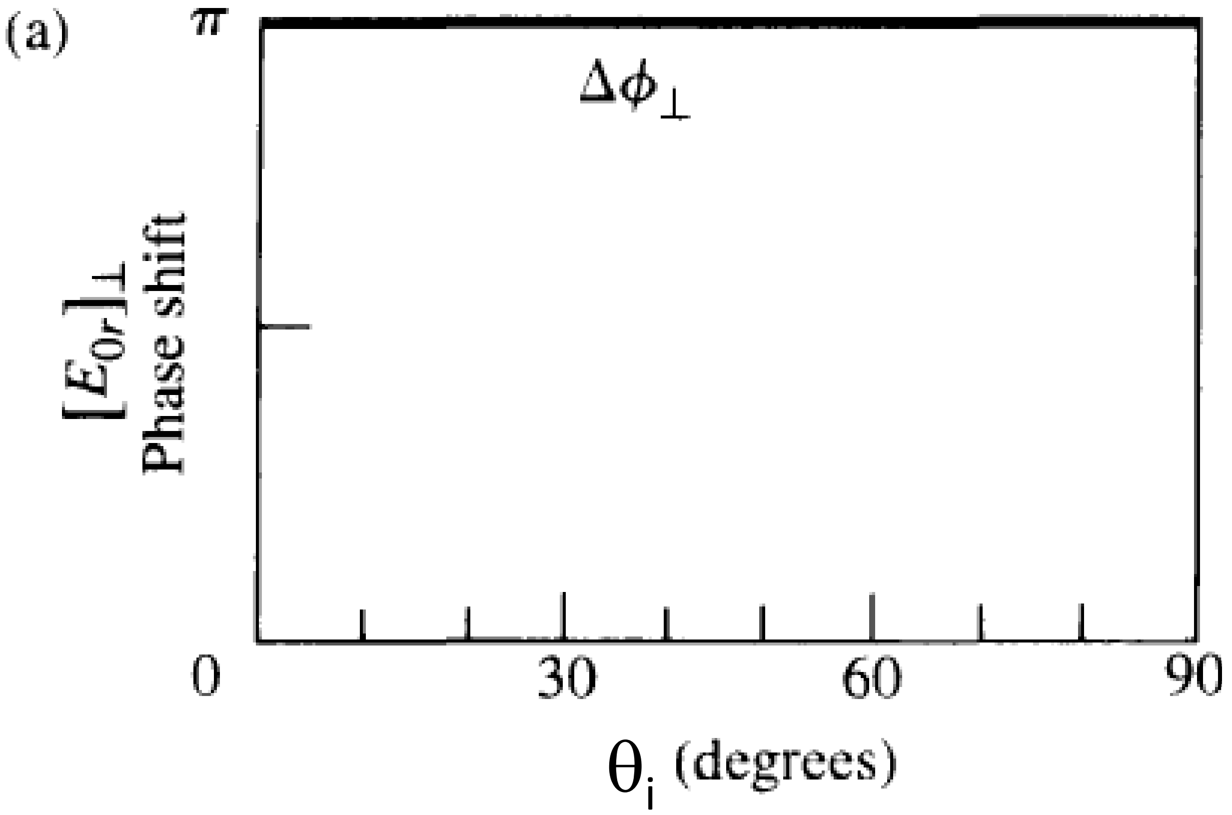


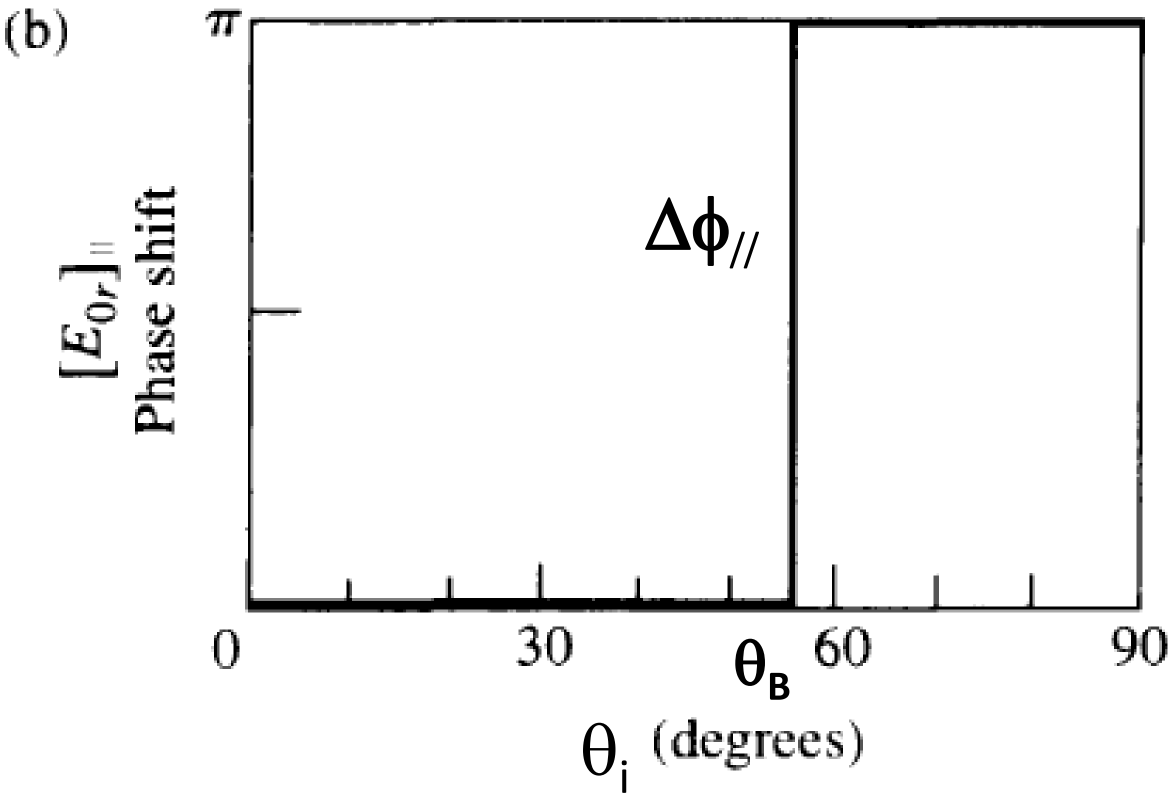
Riflessione Rifrazione

- shift di fase

$n_i=1$
$n_t=1.5$

Riflessione esterna



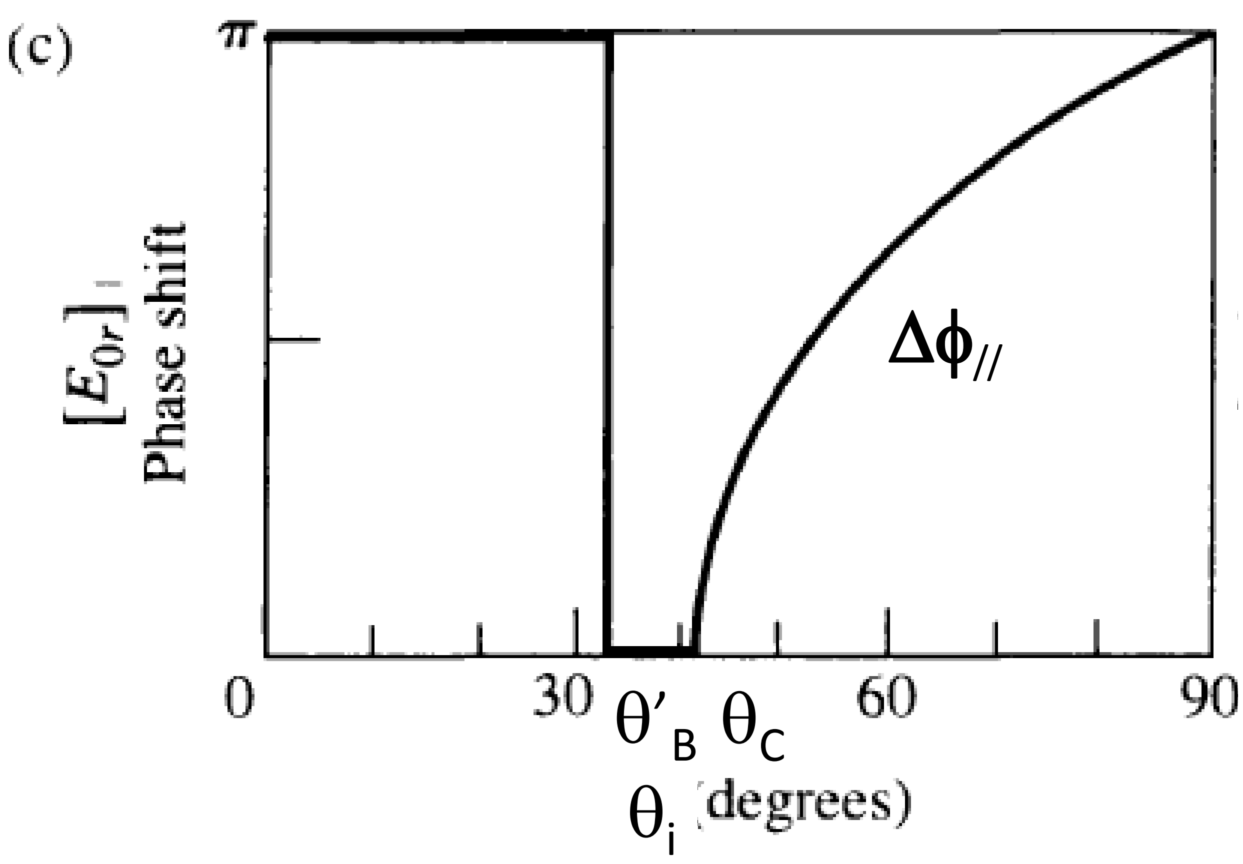


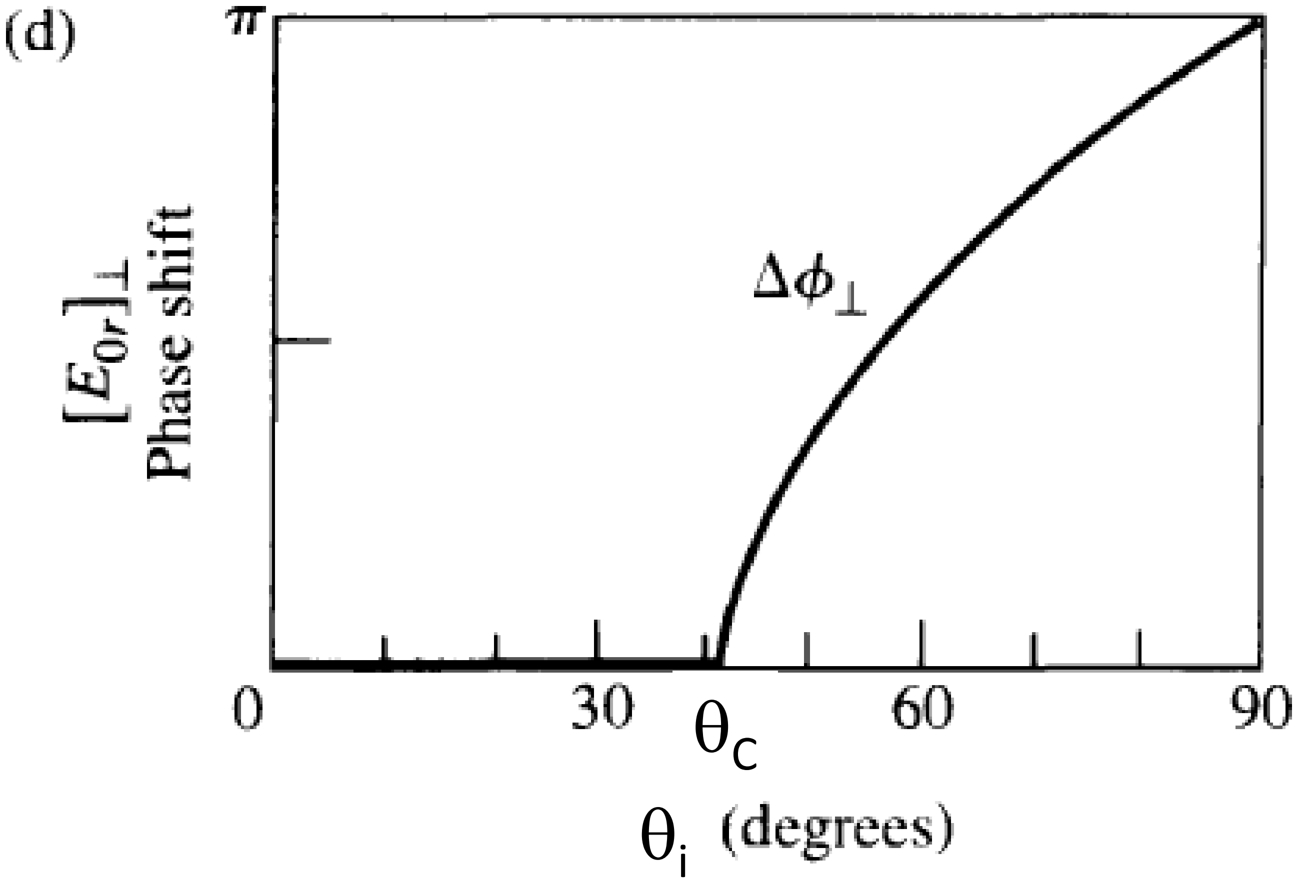
Riflessione Rifrazione

- shift di fase

n_i=1.5
n_t=1

Riflessione interna

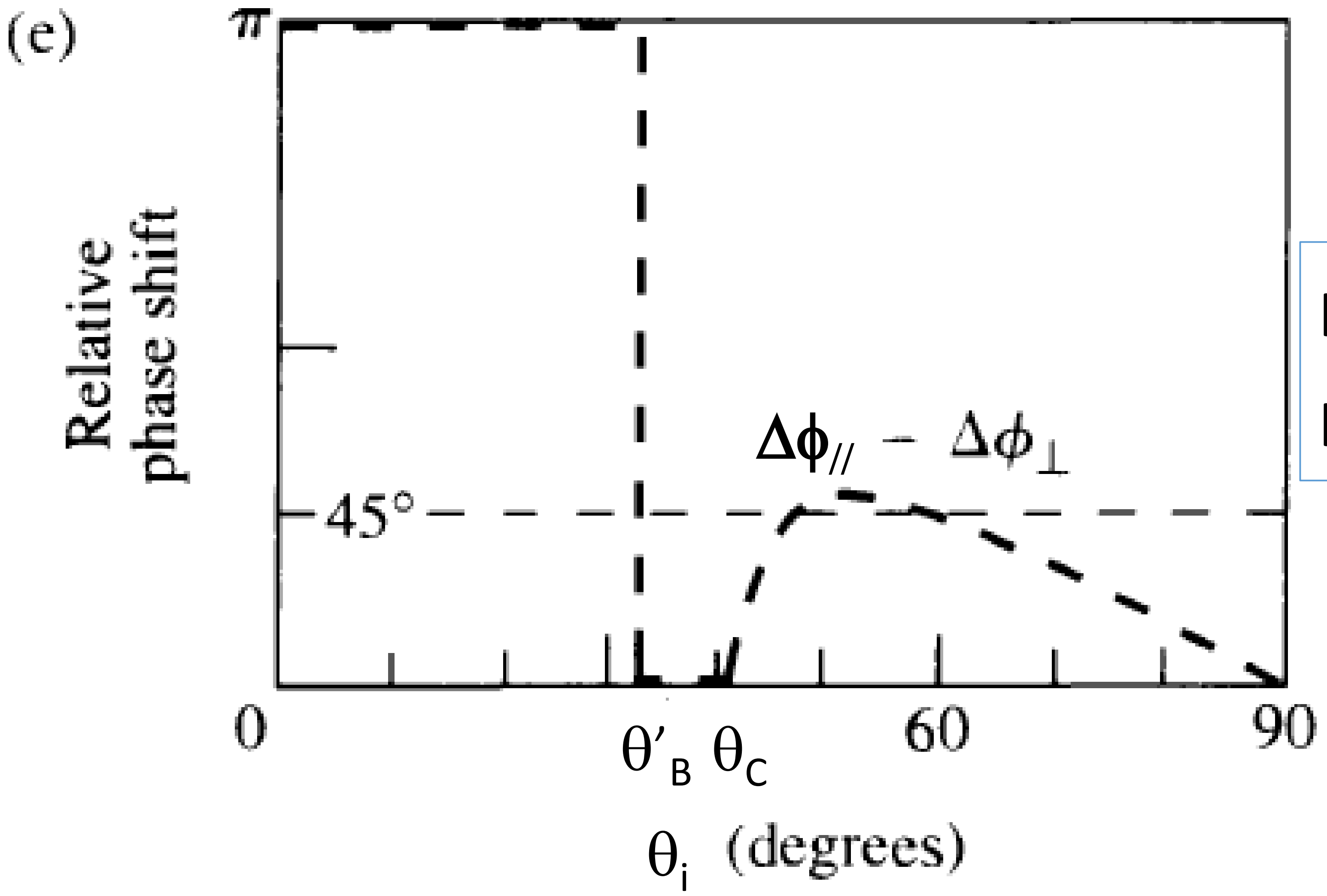




Riflessione Rifrazione

- shift di fase

Riflessione interna

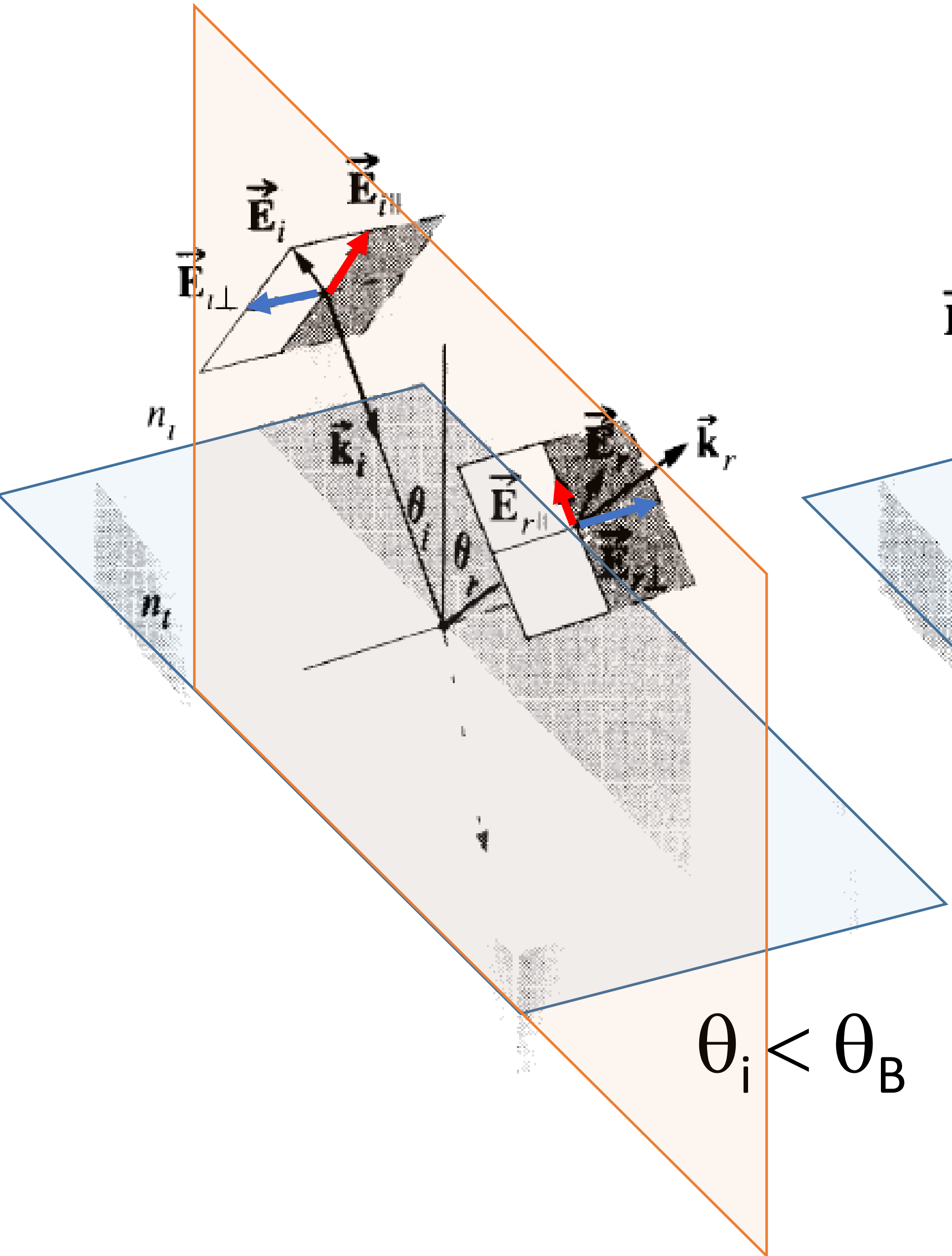


$n_i=1.5$
$n_t=1$

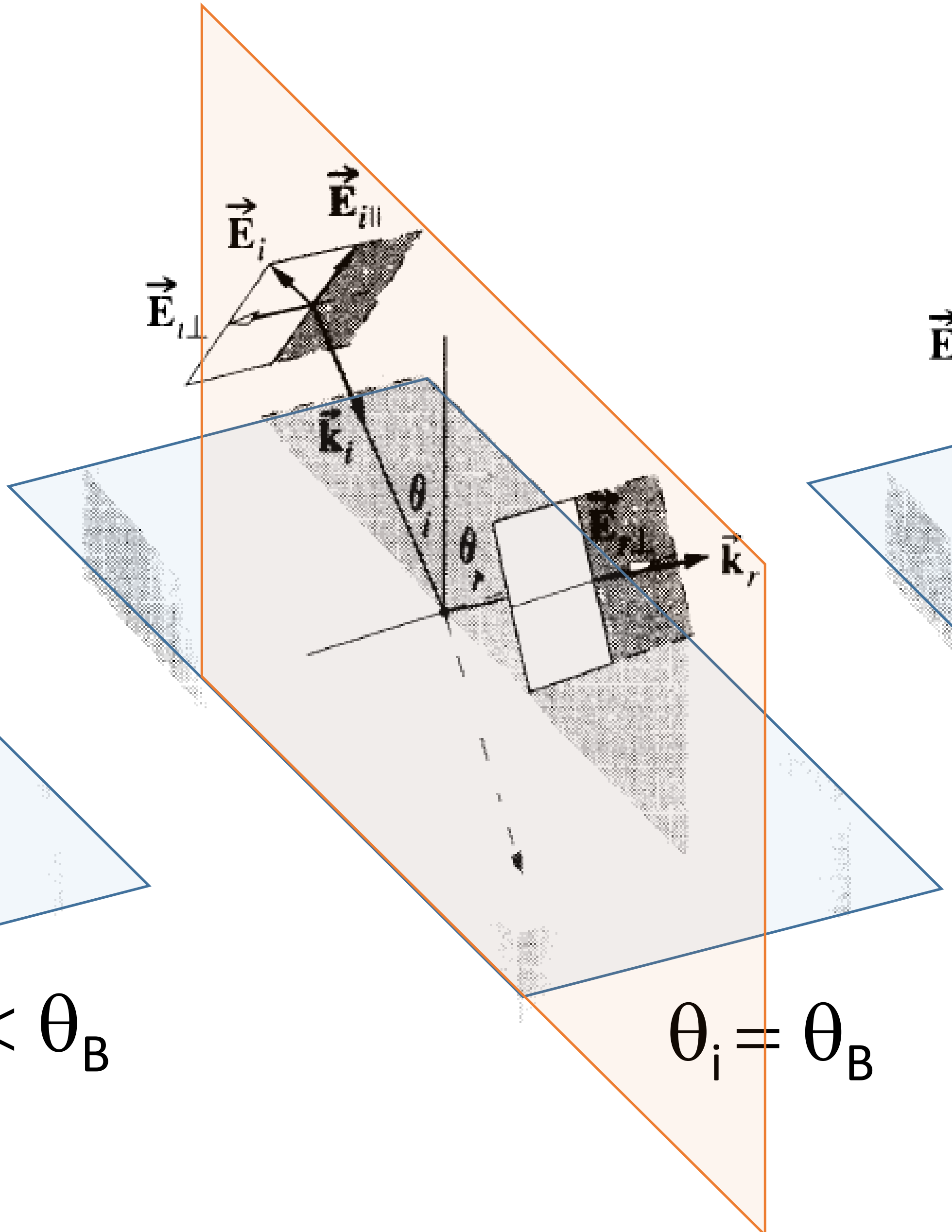
Riflessione Rifrazione

Riflessione esterna

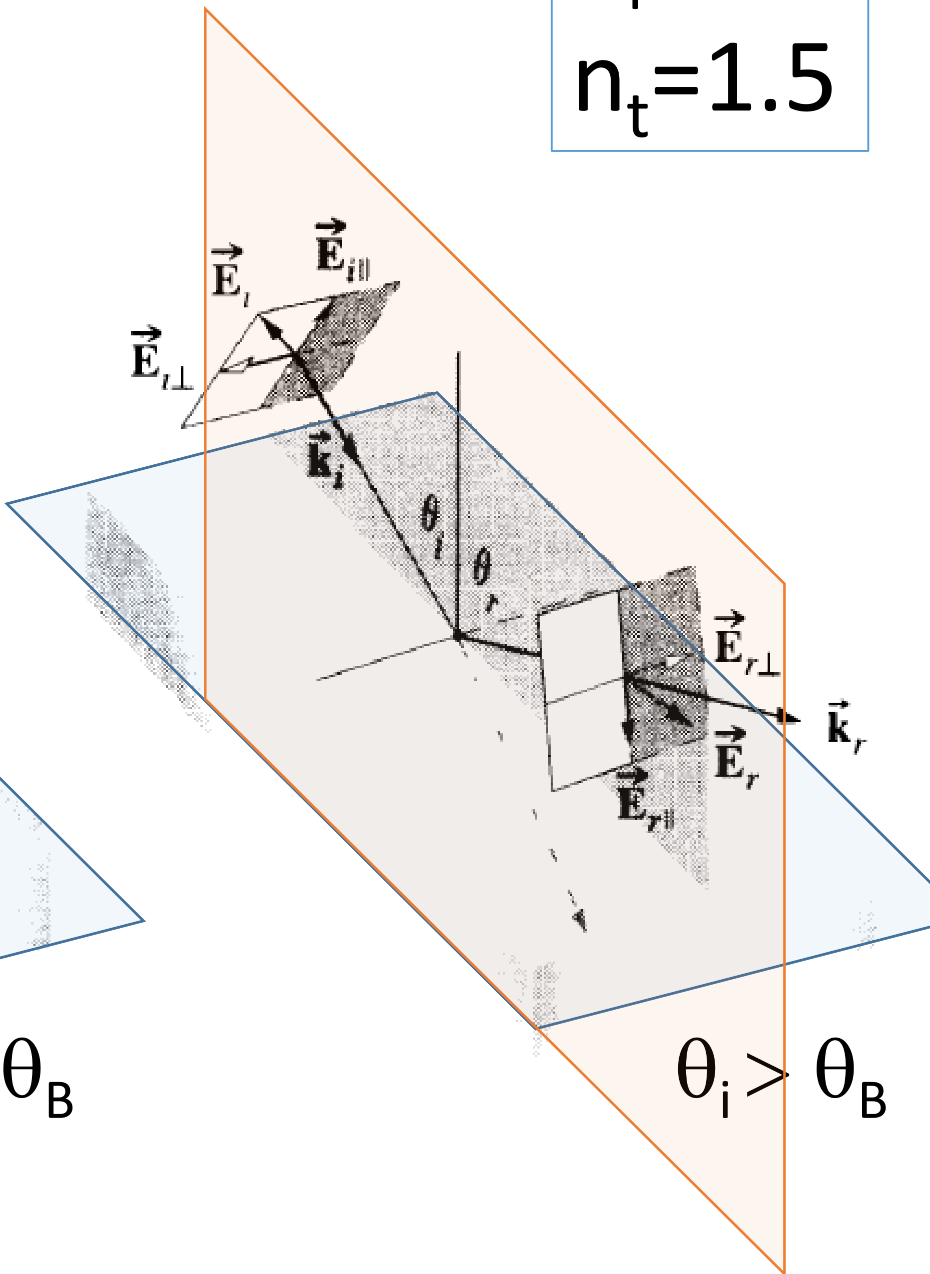
$n_i=1$
$n_t=1.5$



$\theta_i < \theta_B$



$\theta_i = \theta_B$

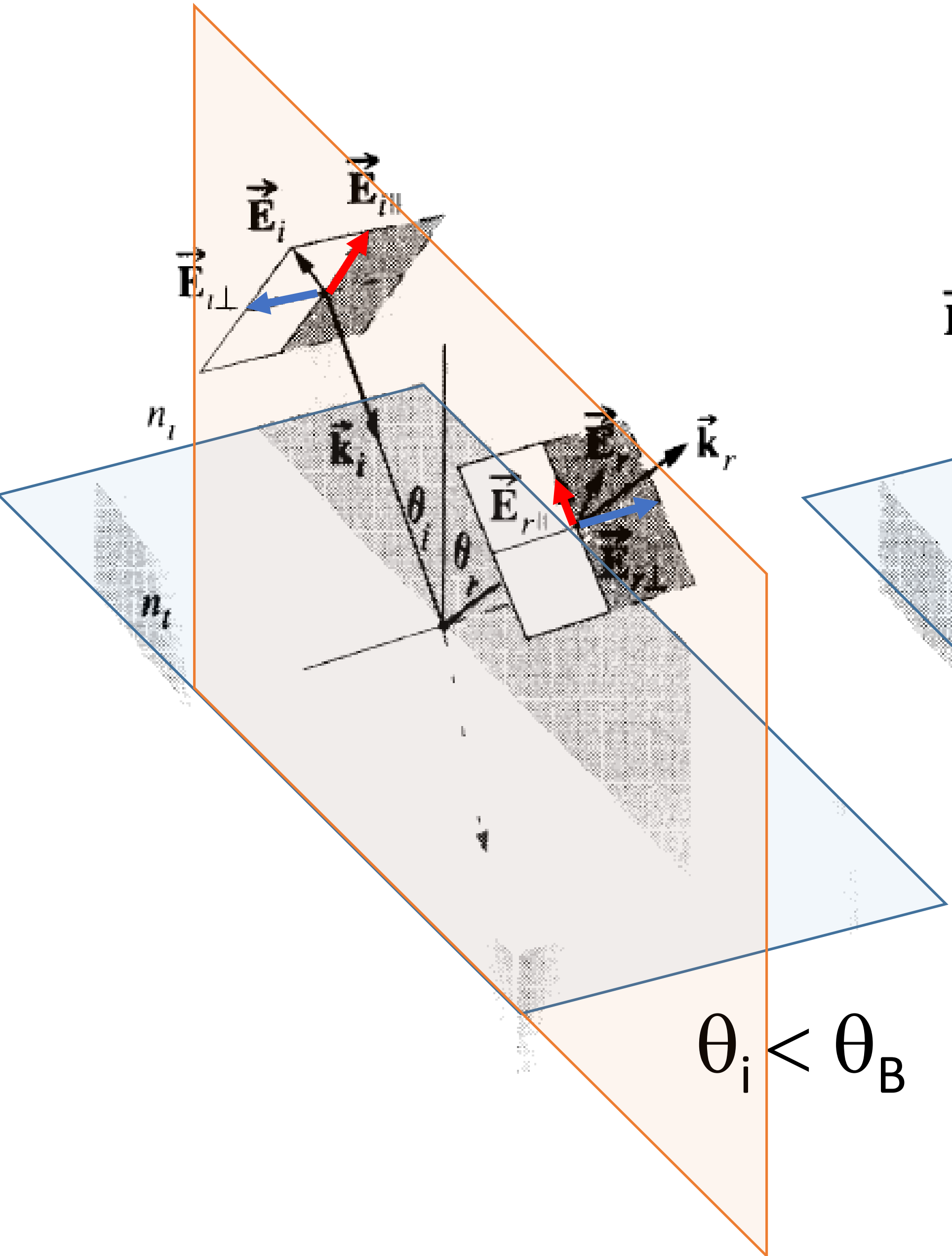


$\theta_i > \theta_B$

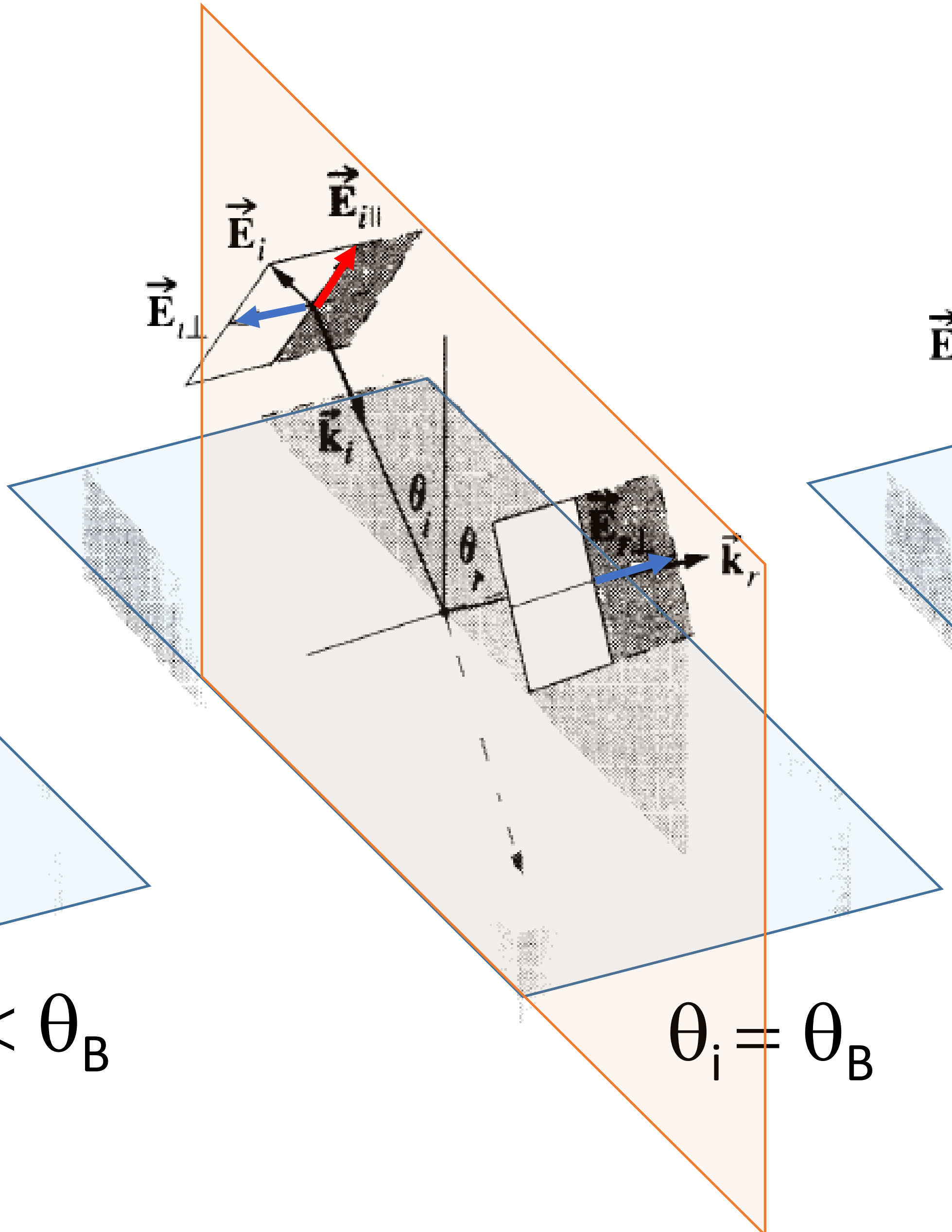
Riflessione Rifrazione

Riflessione esterna

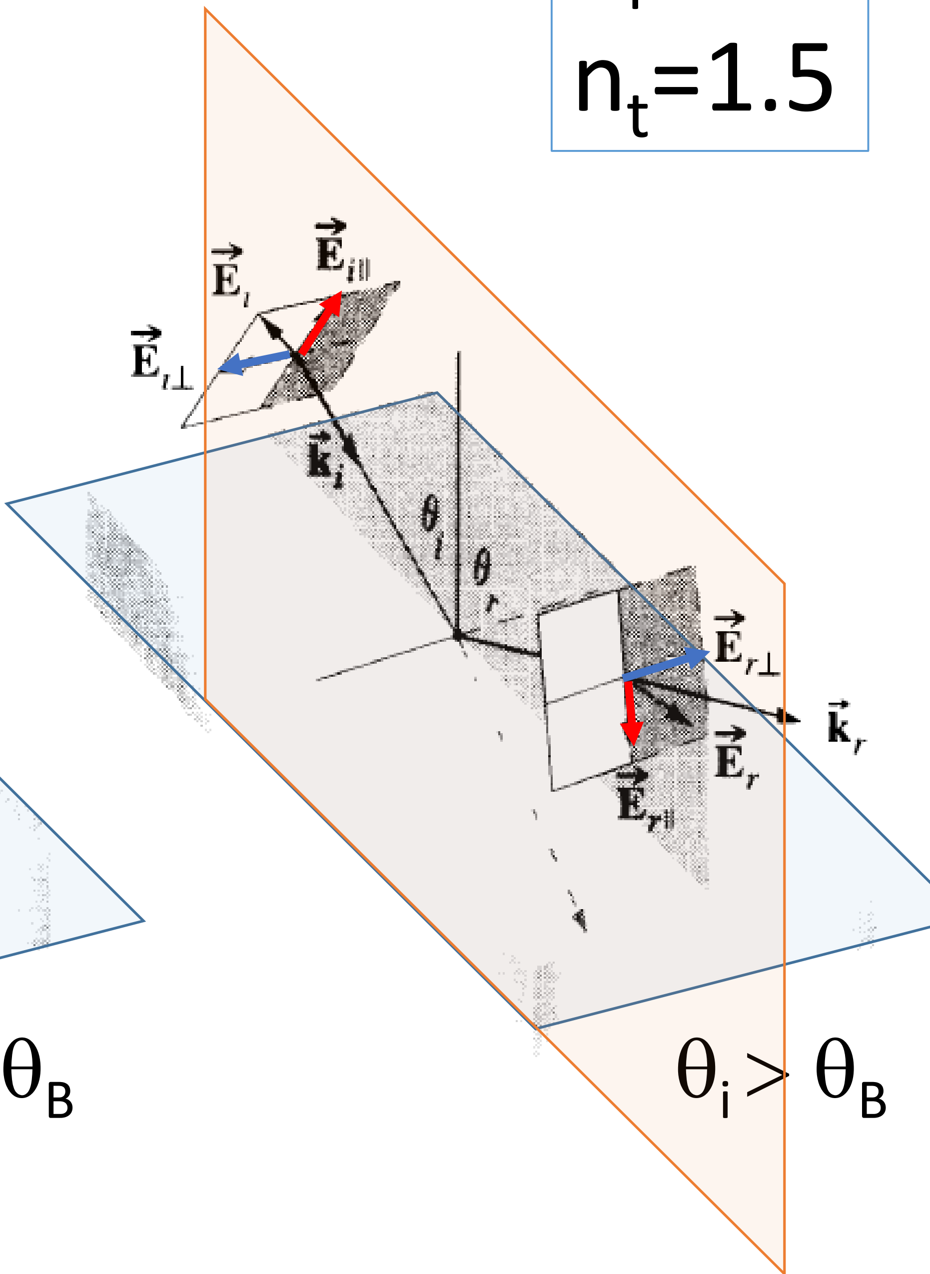
$n_i=1$
$n_t=1.5$



$\theta_i < \theta_B$



$\theta_i = \theta_B$

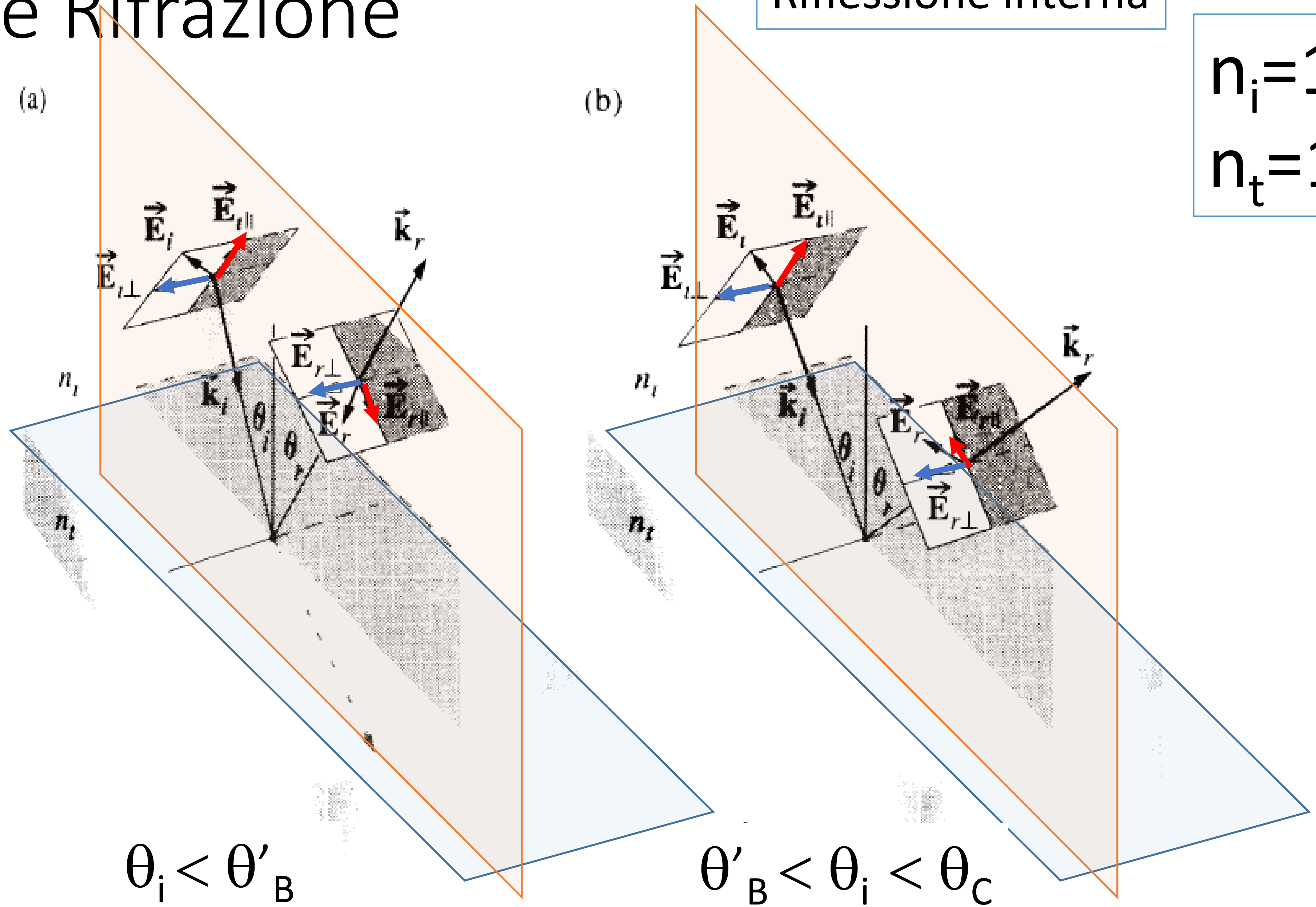


$\theta_i > \theta_B$

Riflessione Rifrazione

Riflessione interna

n_i=1.5
n_t=1

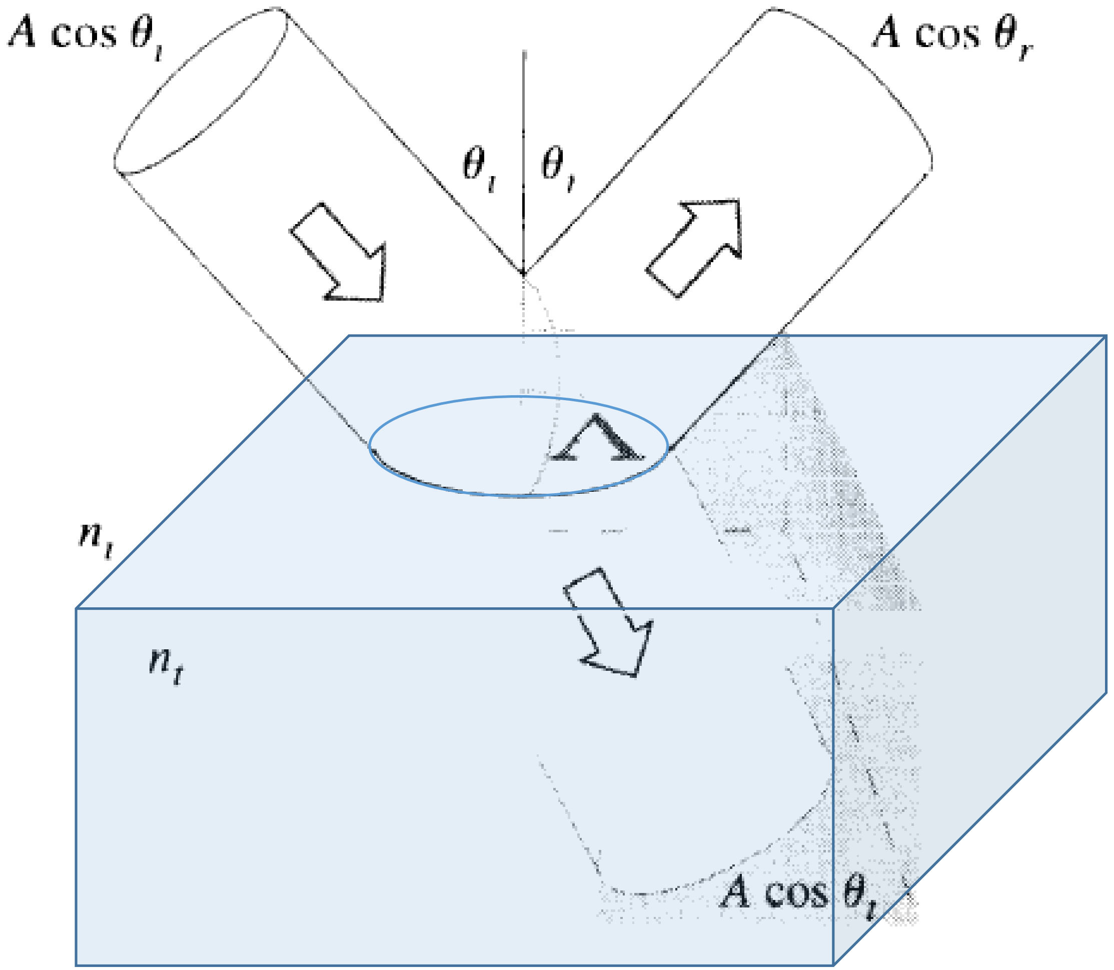


$\theta_i < \theta'_B$

$\theta'_B < \theta_i < \theta_C$

Riflessione Rifrazione

- Riflettanza e Trasmittanza



$$R_\perp = r_\perp^2$$

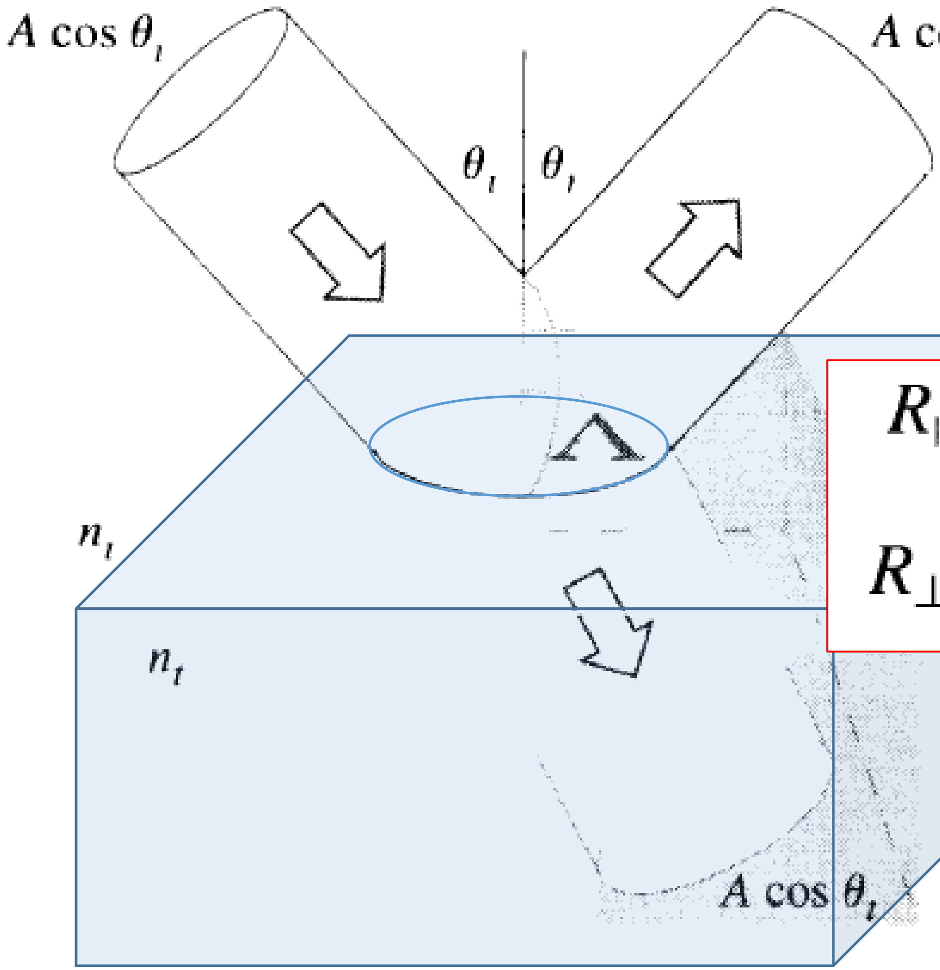
$$R_\parallel = r_\parallel^2$$

$$T_\perp = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i}\right) t_\perp^2$$

$$T_\parallel = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i}\right) t_\parallel^2$$

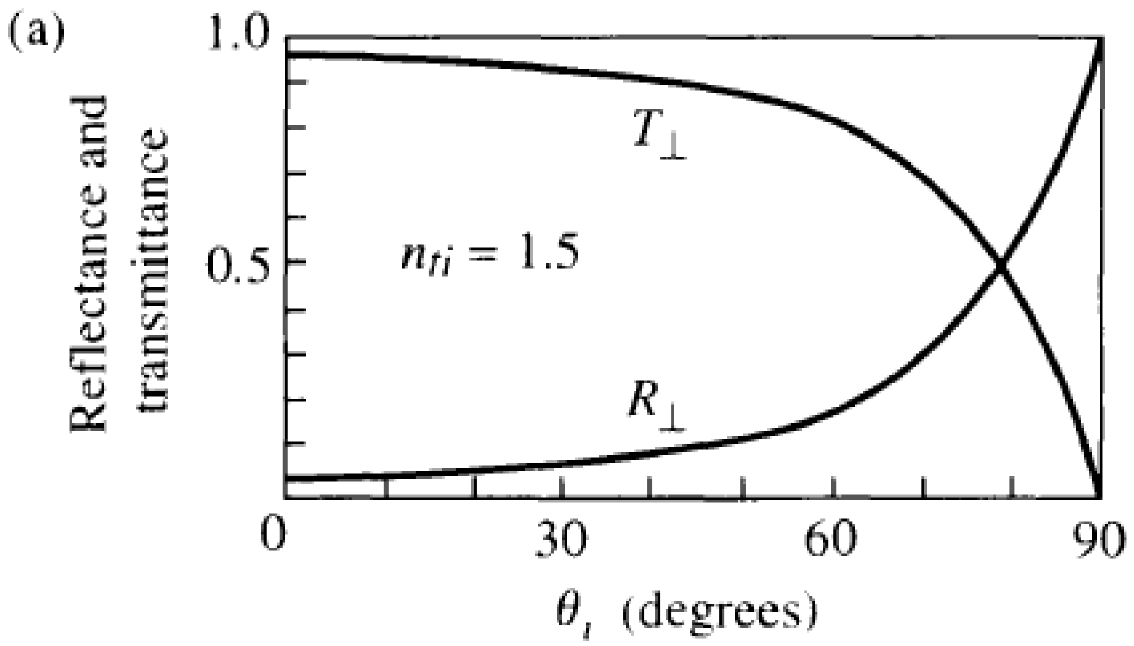
Riflessione Rifrazione

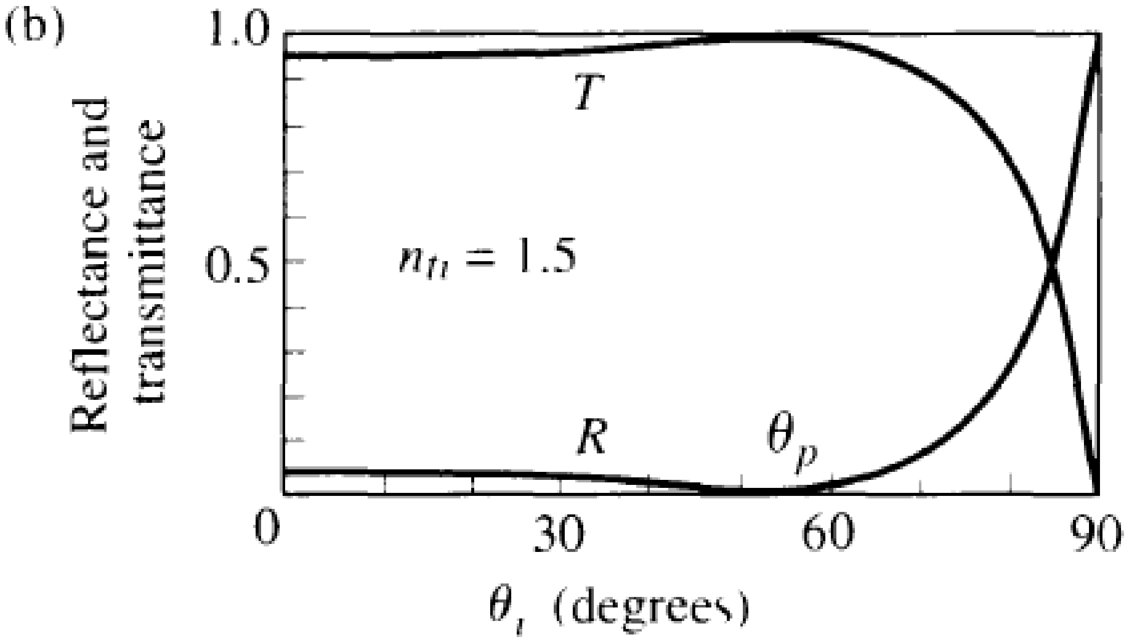
- Riflettanza e Trasmittanza



$$R_{\parallel} + T_{\parallel} = 1$$

$$R_{\perp} + T_{\perp} = 1$$



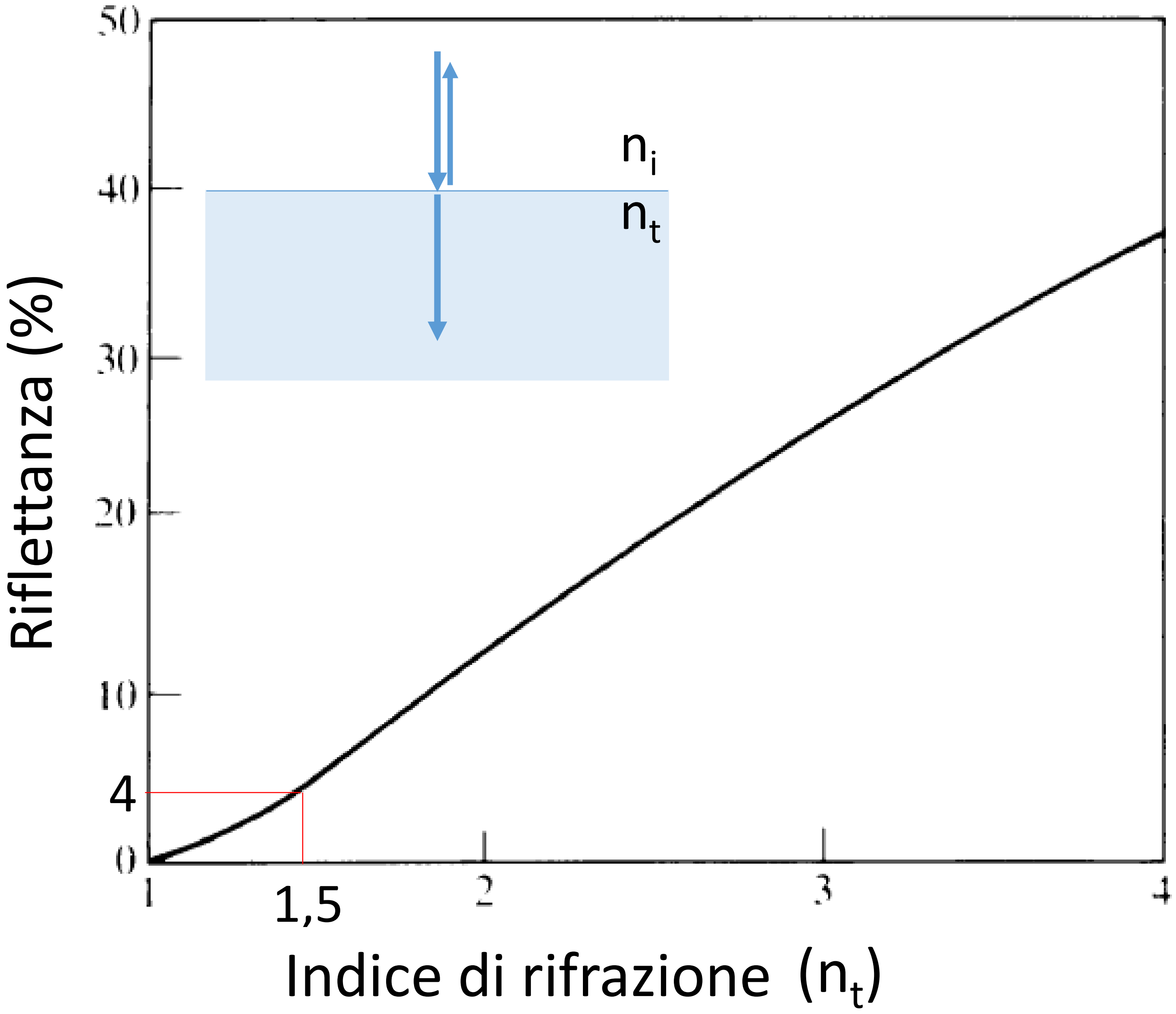


Riflessione Rifrazione

- riflettanza ad incidenza normale (n_i=1)

$$R = R_{\parallel} = R_{\perp} = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

$$T = T_{\parallel} = T_{\perp} = \frac{4n_t n_i}{(n_t + n_i)^2}$$

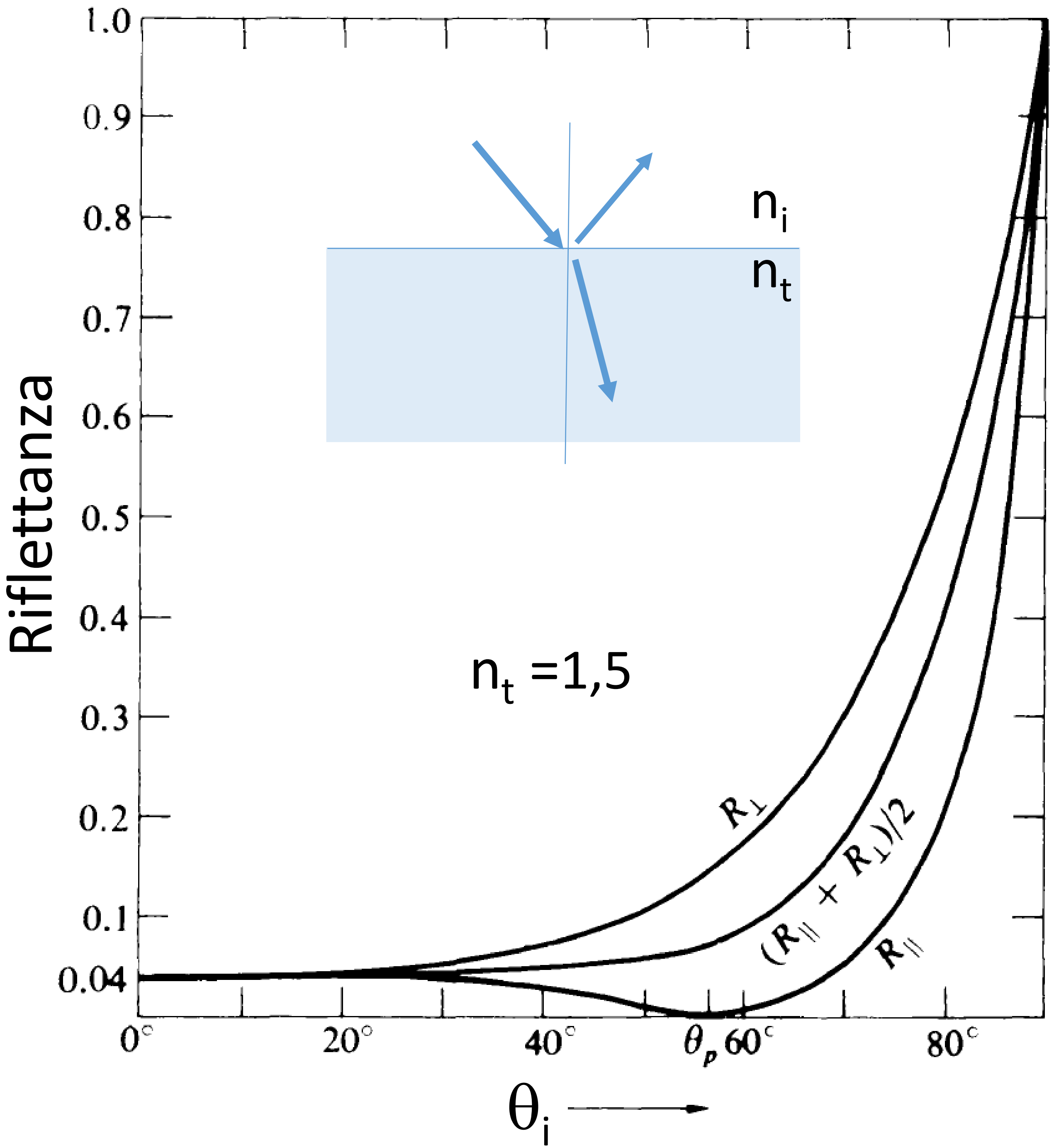


Riflessione Rifrazione

- riflettanza (n_i=1)

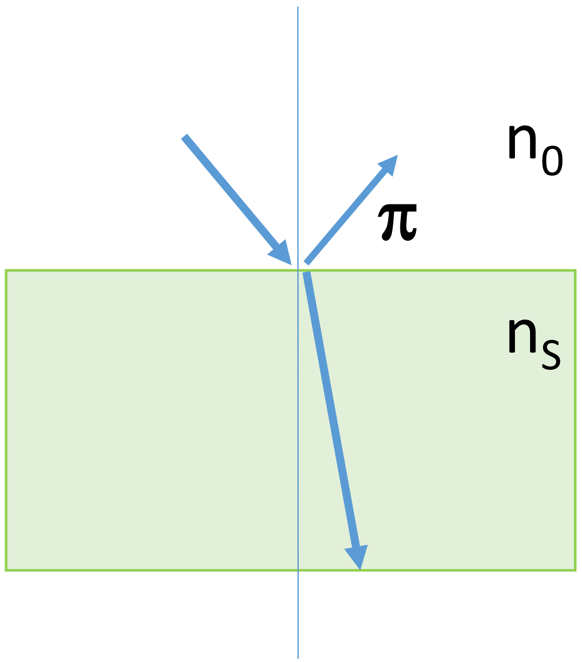
$$R_\parallel = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

$$R_\perp = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$$

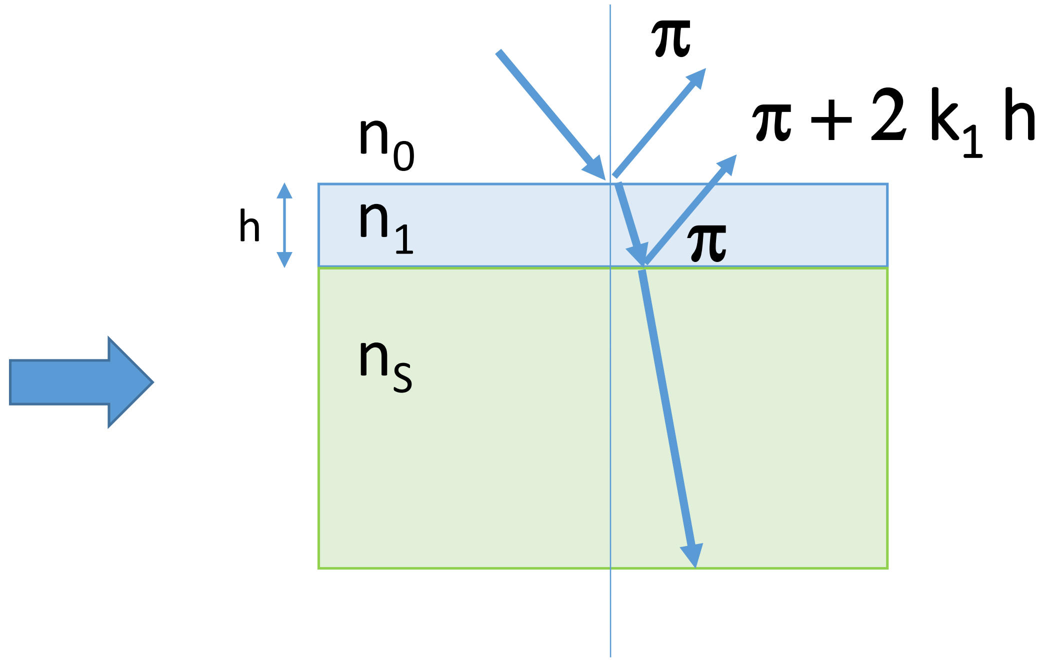


Trattamenti AR

Incidenza quasi normale
E ortogonale



- Doppio strato $n_0 < n_1 < n_S$



- Interferenza distruttiva per :

$$\delta = 2\,k_1\,h = (2m+1)\,\pi \longrightarrow h = \lambda_0 /4\,(2m+1)$$

Trattamenti AR

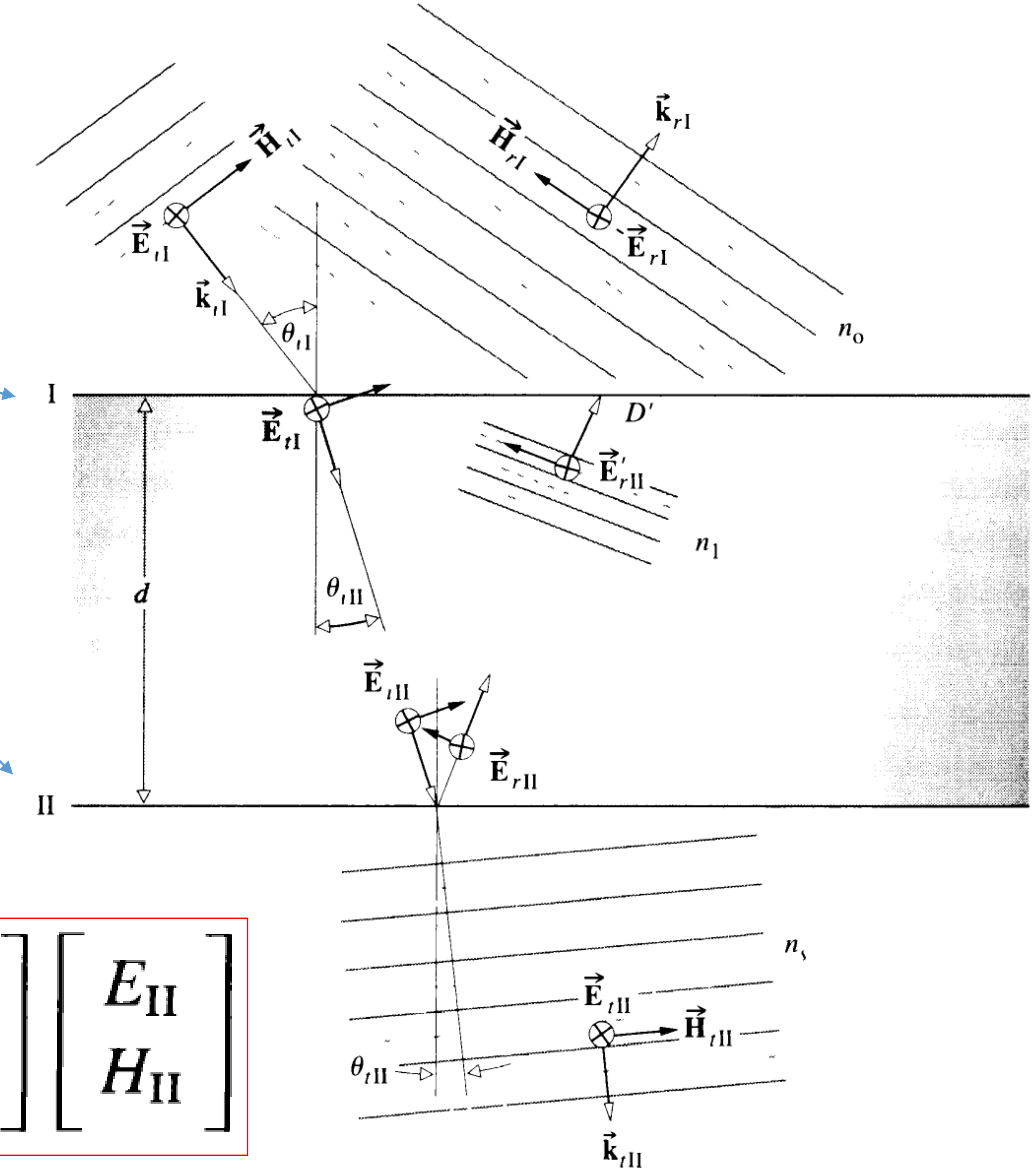
E ortogonale

$$E_{\mathrm{I}} = E_{i\mathrm{I}} + E_{r\mathrm{I}} = E_{t\mathrm{I}} + E'_{r\mathrm{II}}$$

$$E_{\mathrm{II}} = E_{i\mathrm{II}} + E_{r\mathrm{II}} = E_{t\mathrm{II}}$$

$$\Upsilon_1 \equiv \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_1/\cos\,\theta_{i\mathrm{II}}$$

$$\boxed{\begin{bmatrix} E_{\mathrm{I}} \\ H_{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} \cos k_0 h & (i \sin k_0 h)/\Upsilon_1 \\ \Upsilon_1 i \sin k_0 h & \cos k_0 h \end{bmatrix} \begin{bmatrix} E_{\mathrm{II}} \\ H_{\mathrm{II}} \end{bmatrix}}$$



Trattamenti AR

E ortogonale

$$R_1 = \frac{n_1^2(n_0 - n_s)^2 \cos^2 k_0 h + (n_0 n_s - n_1^2)^2 \sin^2 k_0 h}{n_1^2(n_0 + n_s)^2 \cos^2 k_0 h + (n_0 n_s + n_1^2)^2 \sin^2 k_0 h}$$

Per $k_0 h = \frac{1}{2}\pi$

$$\boxed{R_1 = \frac{(n_0 n_s - n_1^2)^2}{(n_0 n_s + n_1^2)^2}}$$

$$n_1^2 = n_0 n_s$$

MgF_2

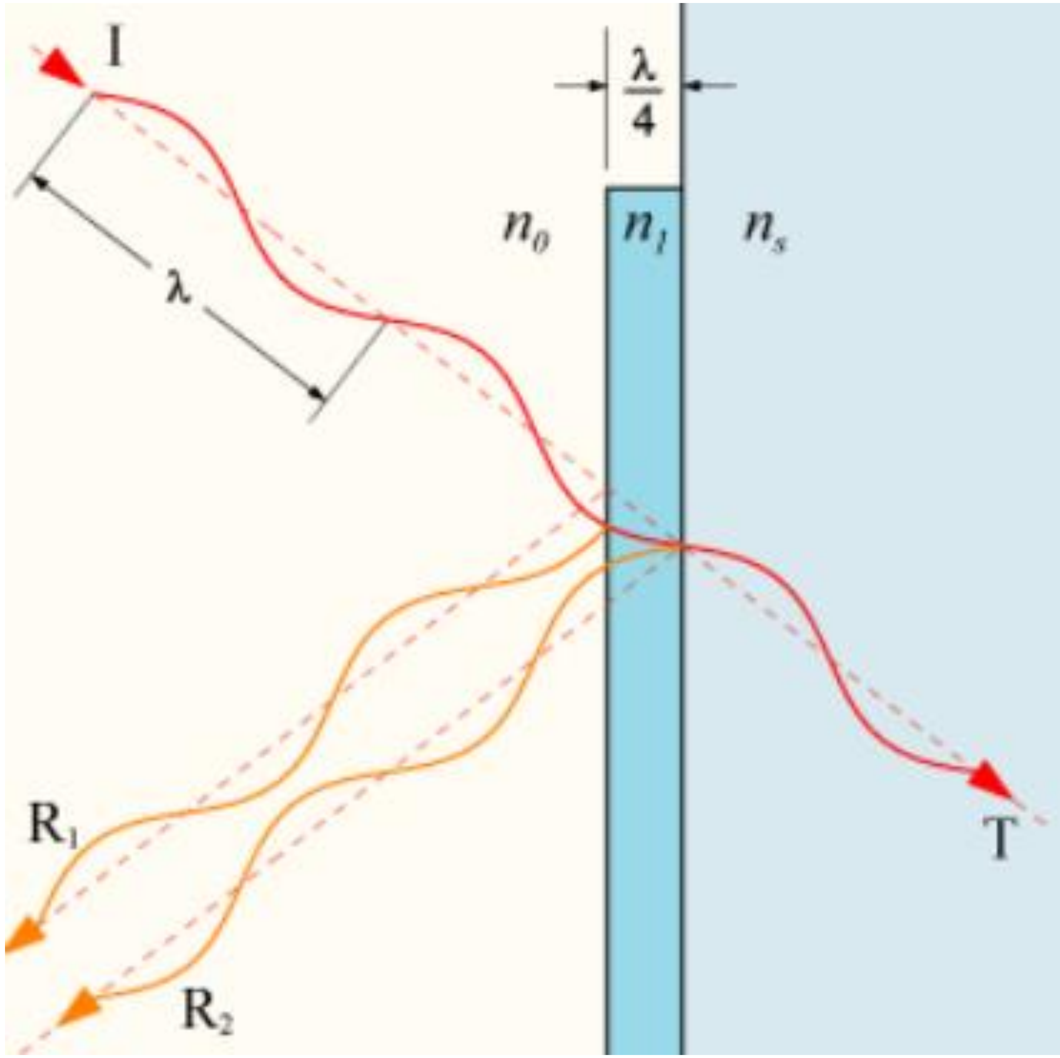
$n = 1.38$

Trattamenti AR

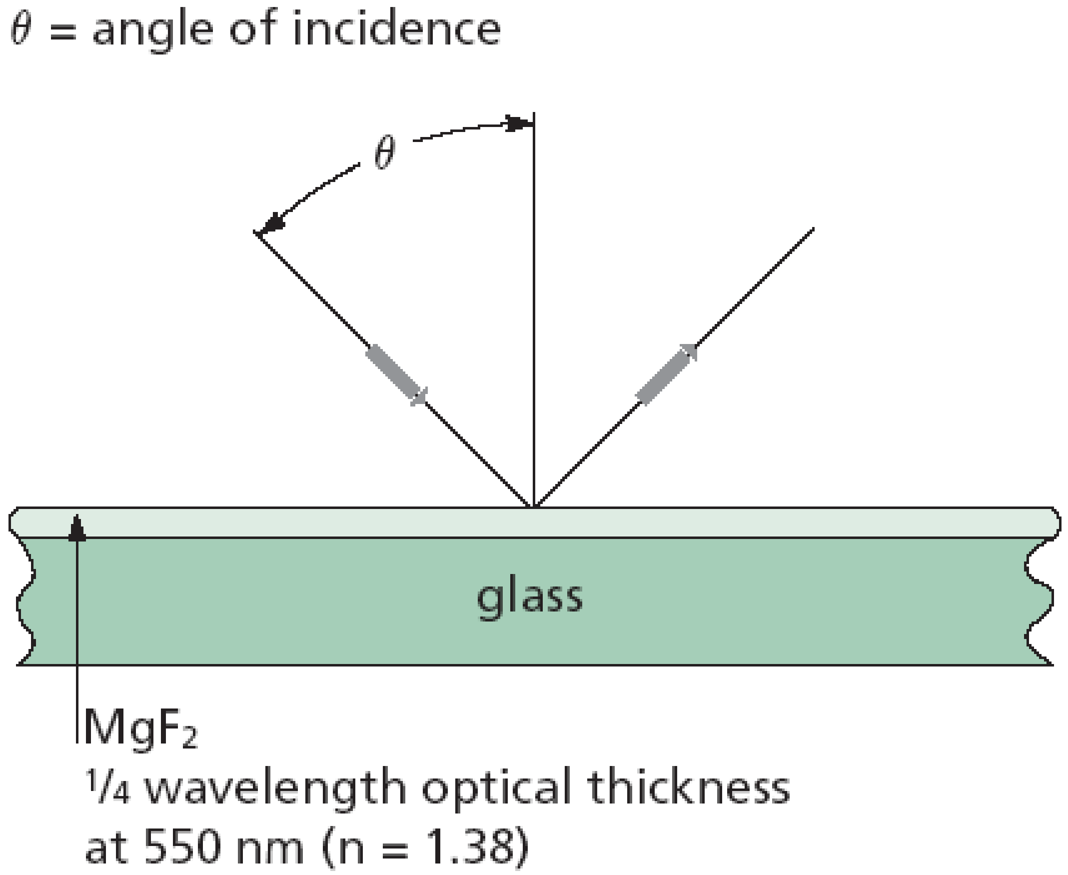
TABLE 4A.1 Thin-Film Materials

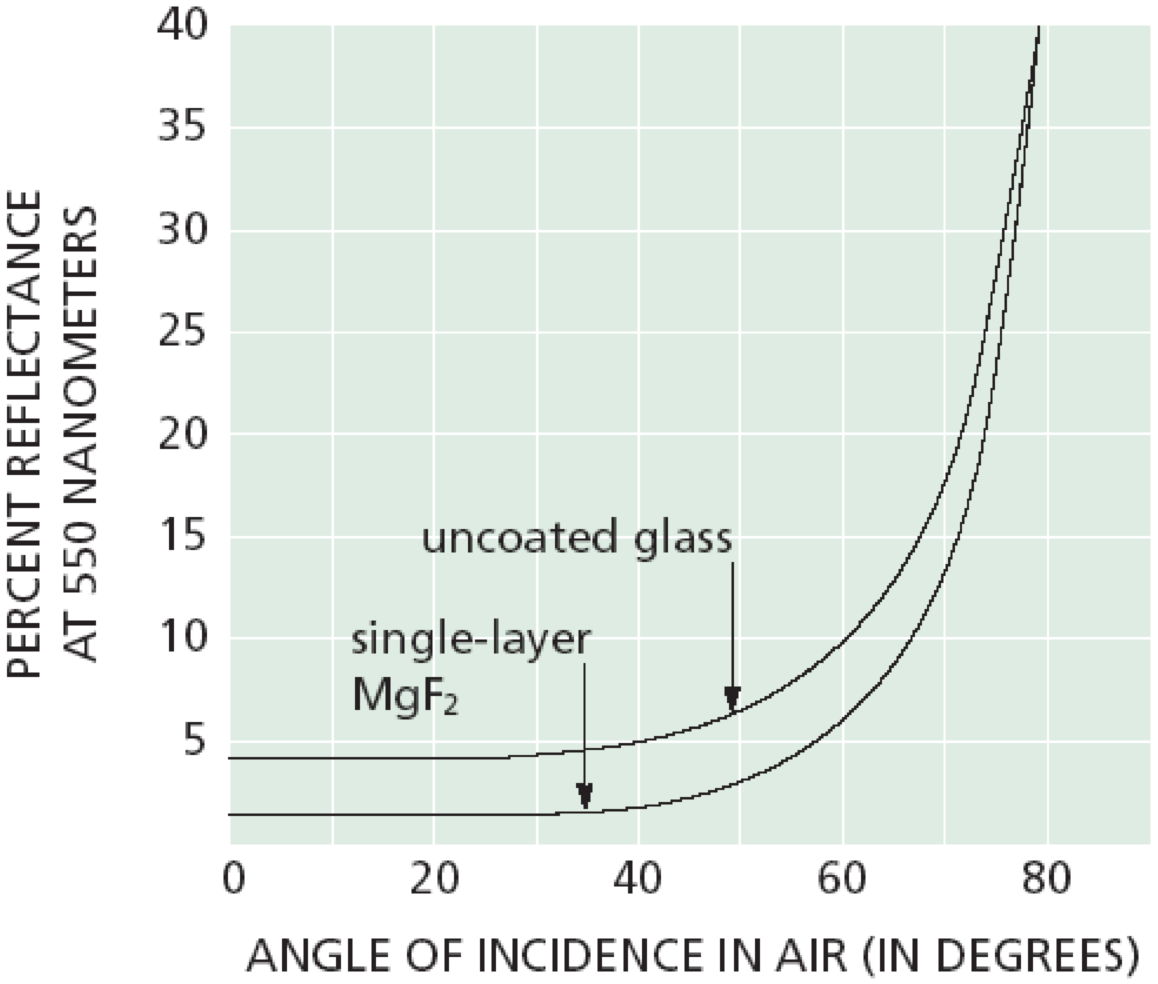
Material	Index of Refraction	Wavelength Range, μm
Cryolite(Na_3AlF_6)	1.35	0.15–14
Magnesium fluoride (MgF_2)	1.38	0.12–8
Silicon dioxide (SiO_2)	1.46	0.17–8
Thorium fluoride (ThF_4)	1.52	0.15–13
Aluminum oxide (Al_2O_3)	1.62	0.15–6
Silicon monoxide (SiO)	1.9	0.5–8
Zirconium dioxide (ZrO_2)	2.00	0.3–7
Cerium dioxide (CeO_2)	2.2	0.4–16
Titanium dioxide (TiO_2)	2.3	0.4–12
Zinc sulfide (ZnS)	2.3	0.4–12
Zinc selenide (ZnSe)	2.44	0.5–20
Cadmium telluride (CdTe)	2.69	1.0–30
Silicon (Si)	3.5	1.1–10
Germanium (Ge)	4.05	1.5–20
Lead telluride (PbTe)	5.1	3.9–20 +

Trattamenti AR



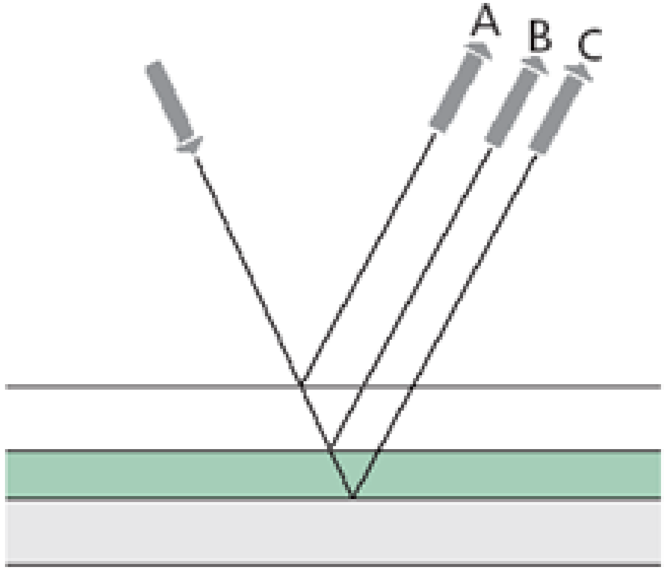
Trattamenti AR

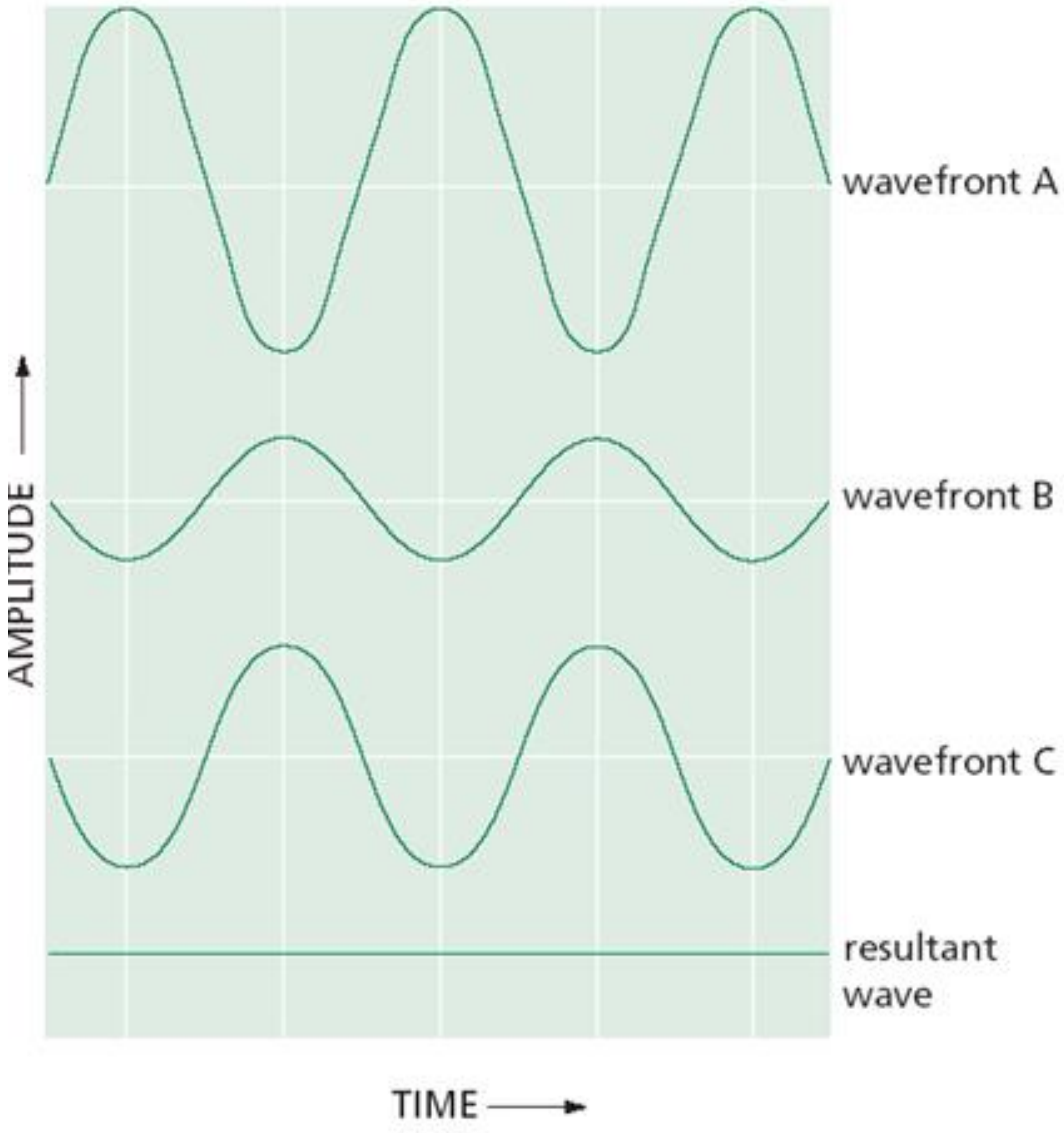




Trattamenti AR

Trattamenti multistrato



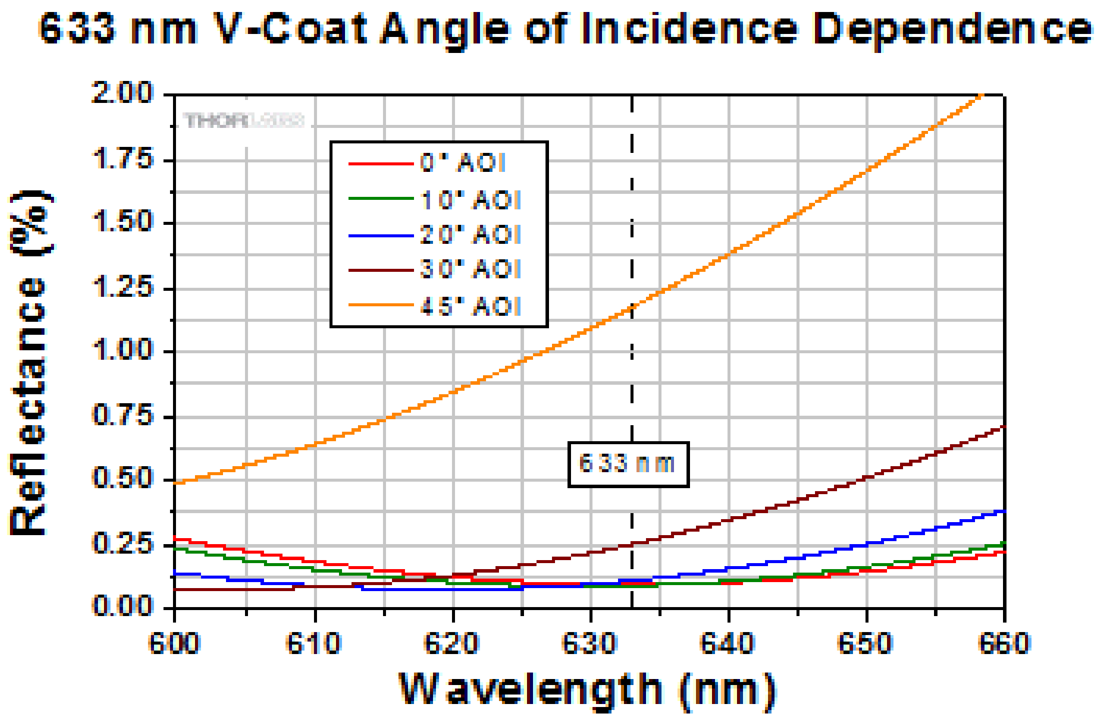


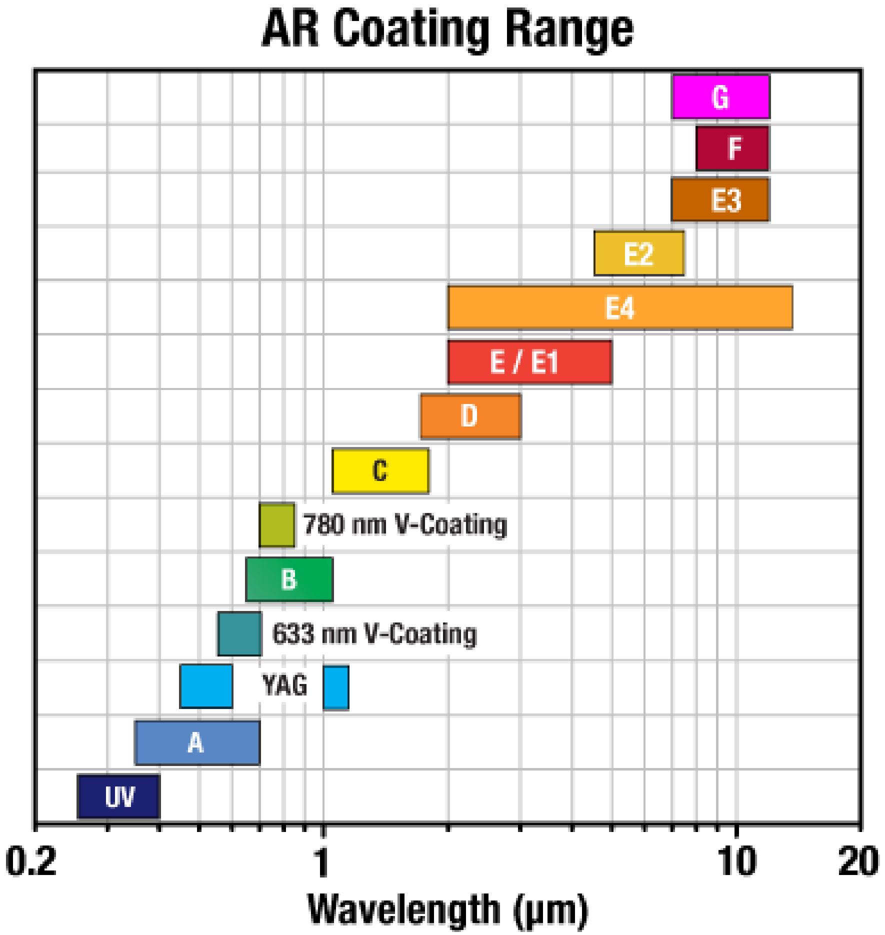
$$\mathcal{M} = \mathcal{M}_{\mathrm{I}}\mathcal{M}_{\mathrm{II}}$$

$$R_2 = \left[\frac{n_2^2 n_0 - n_s n_1^2}{n_2^2 n_0 + n_s n_1^2}\right]^2 \Rightarrow \left(\frac{n_2}{n_1}\right)^2 = \frac{n_s}{n_0}$$

Trattamenti AR

- V- Coating R< 0.25% $\Delta\lambda \sim 10$ nm
- Broadband Coating R< 1-3% $\Delta\lambda > 300$ nm



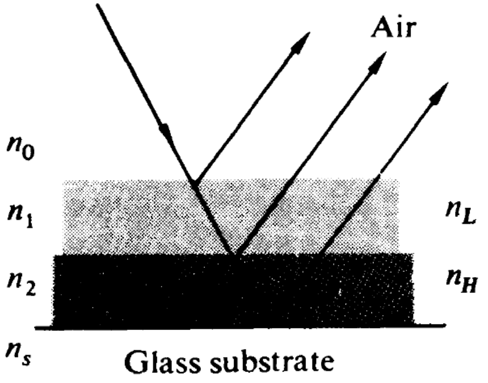


- Attenzione all'**angolo di incidenza** ed alla **polarizzazione**!

Trattamenti HR

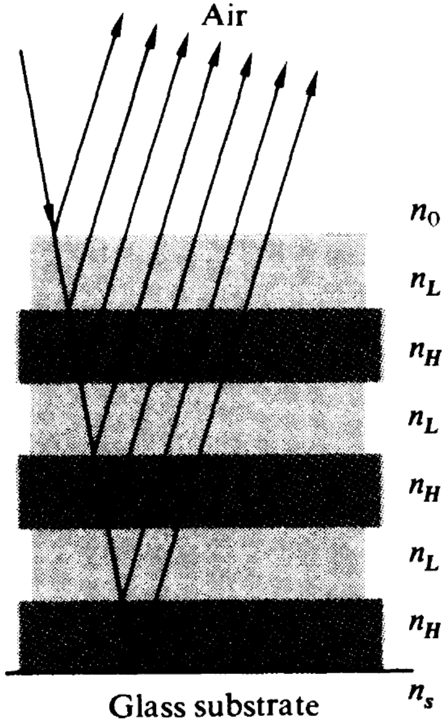
$$R = \frac{(1 - p)}{(1 + p)}$$

$$p = \left(\frac{n_H}{n_L}\right)^{N-1} \times \frac{n_H^2}{n_S}$$



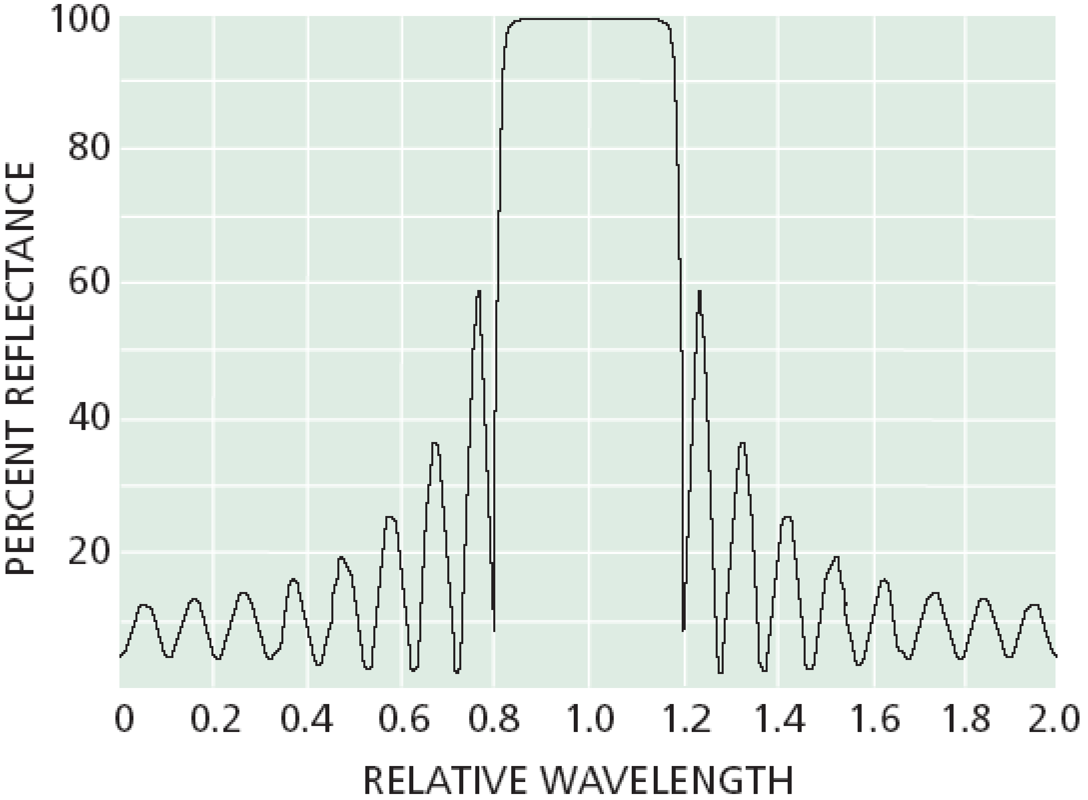
$g\,HL\,a$

Double-quarter

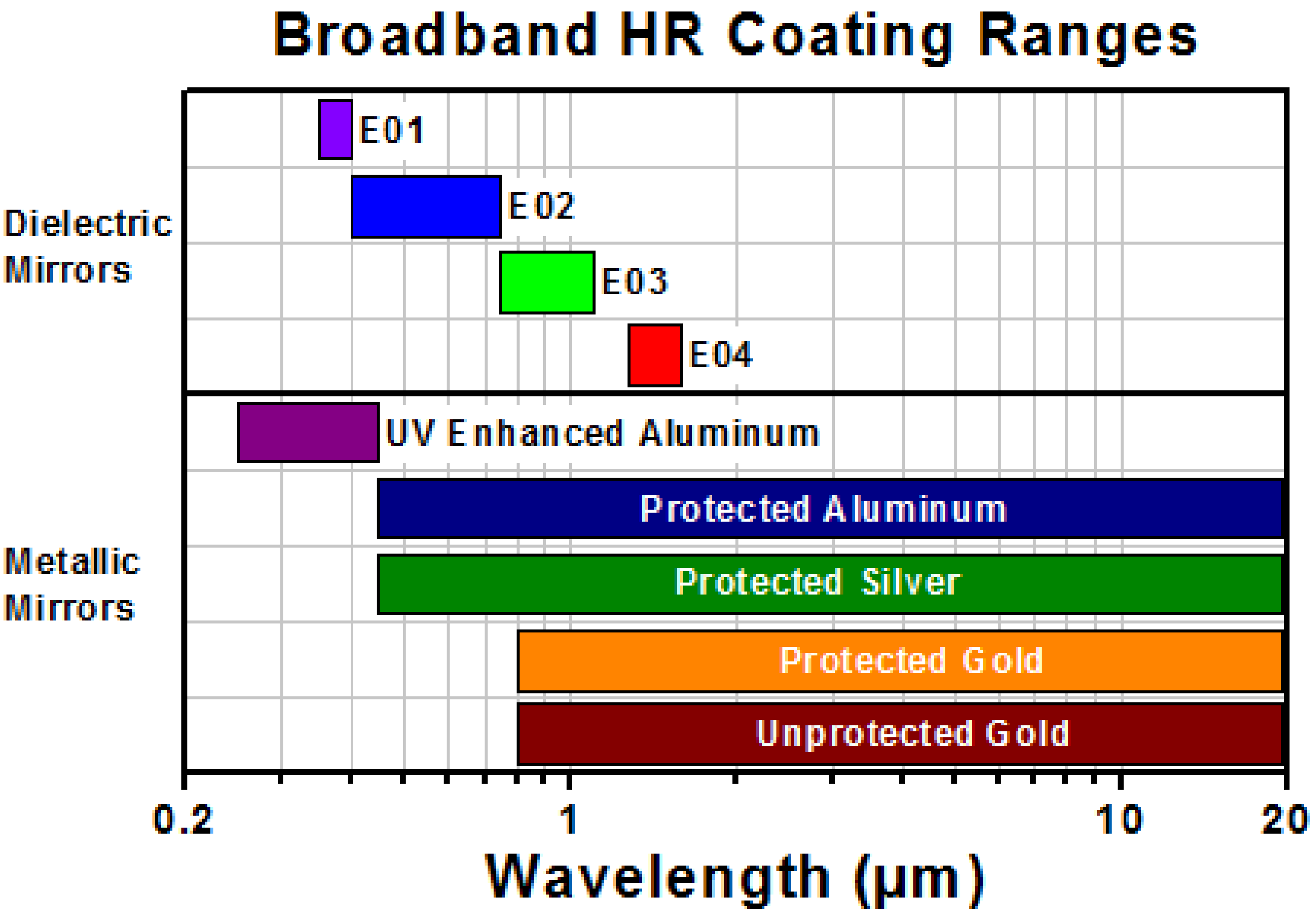


$g\,HL\,HL\,HL\,a$
$g(HL)^3a$

Quarter-wave stack



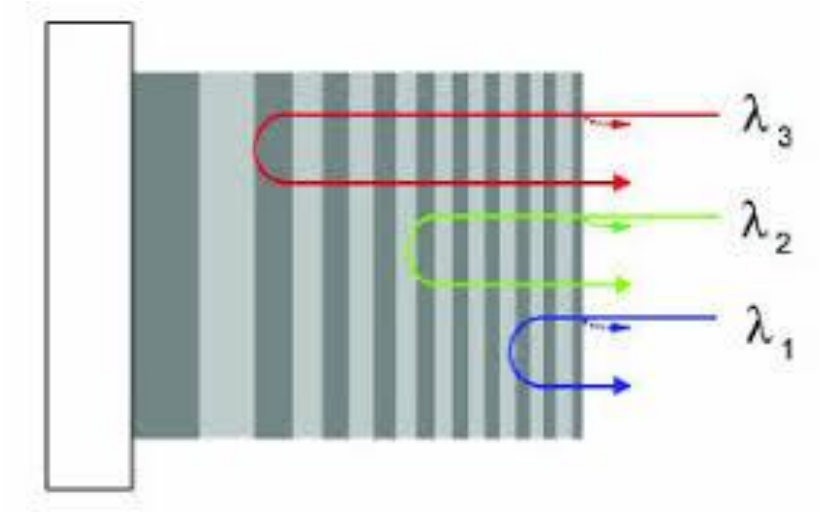
Trattamenti HR



R > 99%

R > 90% - 95%

Specchi «Chirped»



- Servono per compensare la dispersione e/o comprimere un impulso

- Laser a femtosecondi