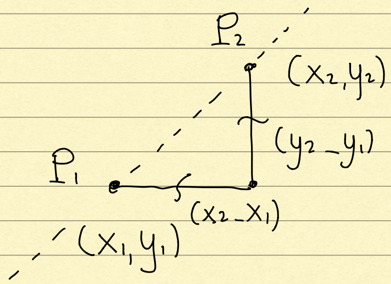
$ax + b = 0 \qquad a \in \mathbb{R} \quad a \neq 0 \qquad$ (Eq.ne 1° grado) (eq1)

$x = -\frac{b}{a}$

la rappresentazione di (eq1) nel piano è una retta

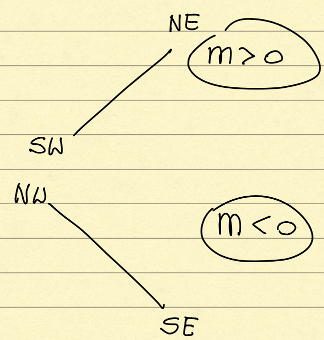
$y = mx + q \qquad$ m, q sono numeri dati

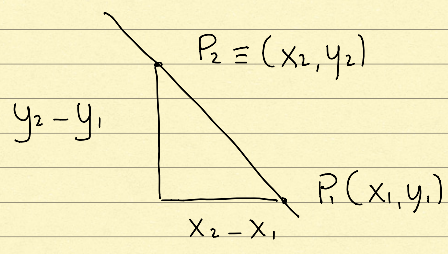


Retta per P_1 e P_2

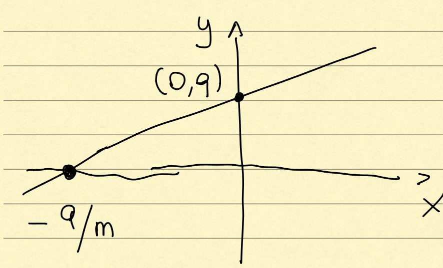
$$\begin{cases} y_1 = m x_1 + q \\ y_2 = m x_2 + q \end{cases}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = m \qquad \text{coefficiente angolare}$$





q intercetta ed è l'ordinata dell'int.ne con asse delle y

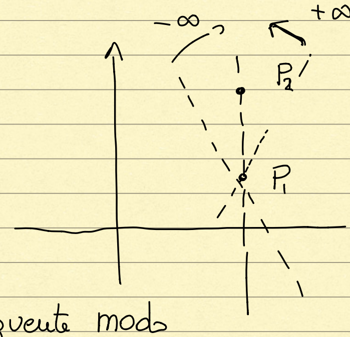


$m \in \mathbb{R} \qquad y = \underset{\sim}{m}x + \underset{\smile}{q}$

$0 = mx + q \qquad x = -\frac{q}{m}$

$m = 0$ $\qquad y = q$

$m = \pm\infty$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$x_1 = x_2$

$\Rightarrow m = \pm\infty$

Posso definire una retta anche nel seguente modo

$$ax + by + c = 0 \qquad y = mx + q$$

$$by = -ax - c \Rightarrow y = -\frac{a}{b}x - \frac{c}{b}$$

$$m = -\frac{a}{b}$$

$$q = -\frac{c}{b}$$

$b = 0$ ho rette verticali

$x = -\frac{c}{a} \Rightarrow x$ è costante

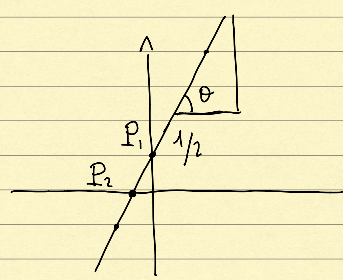
Se $a = 0$ ho rette orizzontali

Esempio

$\boxed{3x - 2y + 1 = 0}$ $\qquad m = 3/2 \qquad q = 1/2$

$a = 3$

$b = -2$



$P_1 = (0, 1/2)$

$P_2 = (-1/3, 0)$

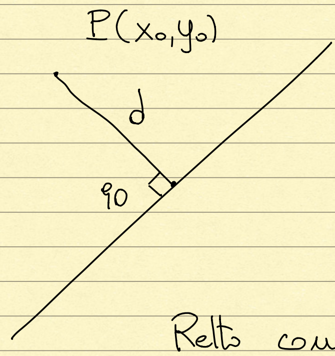
$$m = \frac{\sin\theta}{\cos\theta}$$

Date 2 rette $\quad y = m_1 x + q_1$ $\quad //$ se $m_1 = m_2$ (parallele)

$\qquad y = m_2 x + q_2$ $\quad \perp$ se $m_1 \cdot m_2 = -1$ (perp.)

$a_1 x + b_1 y + c_1 = 0$ $\quad // \quad -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$ $\quad \boxed{a_1 b_2 - a_2 b_1 = 0}$

$a_2 x + b_2 y + c_2 = 0$ $\quad \perp \quad \left(-\frac{a_1}{b_1}\right)\cdot\left(-\frac{a_2}{b_2}\right) = -1$ $\quad \boxed{a_1 a_2 + b_1 b_2 = 0}$

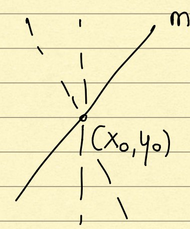


$r: \ ax+by+c=0$

$$d(P,r) = \text{distanza} = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

Retta con coefficiente m passante per $P(x_0,y_0)$

$$\boxed{y-y_0 = m(x-x_0)}$$



Eq.ne di 2° grado

$$ax^2+bx+c=0 \qquad a,b,c \in \mathbb{R} \quad a \neq 0 \qquad (eq2)$$

(eq2) ha
- 2 sol. dist. reali se $\boxed{\Delta = b^2-4ac} > 0$ (1)
- 1 " reale se $\Delta = 0$ (2)
- $\nexists$ sol. reali $\Delta < 0$ (3)

(1) $x_1 = \dfrac{-b-\sqrt{\Delta}}{2a}$, $x_2 = \dfrac{-b+\sqrt{\Delta}}{2a}$ $\qquad \Delta = b^2-4ac$

(2) $x_1 = x_2 = -b/2a$

$(z_1-z_2)\cdot(z_1+z_2) =$

(3) $\nexists$

Mettiamoci nel caso (1) $\qquad x_1+x_2 = \dfrac{-b-b}{2a} = -\dfrac{\cancel{2}b}{\cancel{2}a} = -b/a$

$$x_1 \cdot x_2 = \frac{(-b-\sqrt{\Delta})(-b+\sqrt{\Delta})}{4a^2} = \frac{(-b)^2-(\sqrt{\Delta})^2}{4a^2}$$

$$x_1 \cdot x_2 = \frac{b^2-\Delta}{4a^2} = \frac{\cancel{b^2}-(\cancel{b^2}-4ac)}{4a^2} = \frac{\cancel{4a}c}{\cancel{4}a^{\cancel{2}}} = \frac{c}{a}$$

$x_1 + x_2 = -b/a$

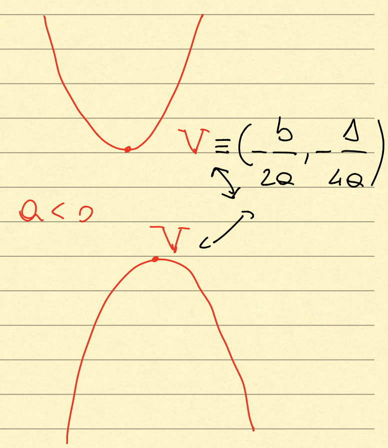
$x_1 \cdot x_2 = c/a$

$$\boxed{ax^2+bx+c=0} \qquad \Delta > 0$$

$$\cancel{a}\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=0$$

GRAFICO

$a>0$ V vertice



$$x^2-(x_1+x_2)x+x_1 \cdot x_2=0$$

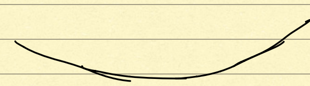
$$x^2-x_1 \cdot x - x_2 x + x_1 \cdot x_2 = 0$$

$$x(x-x_1)-x_2(x-x_1)=0$$

$$\boxed{(x-x_1)(x-x_2)=0}$$

Parabola $y=ax^2+bx+c$

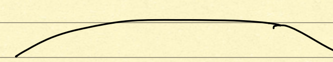
$a>0$ ma a molto piccolo $a \ll 1$



" a " grande $a \gg 1$

$a<0$

$|a| \ll 1$



$|a| \gg 1$

b mi da una misura dello scostamento del vertice rispetto a $x=0$ (asse delle y)

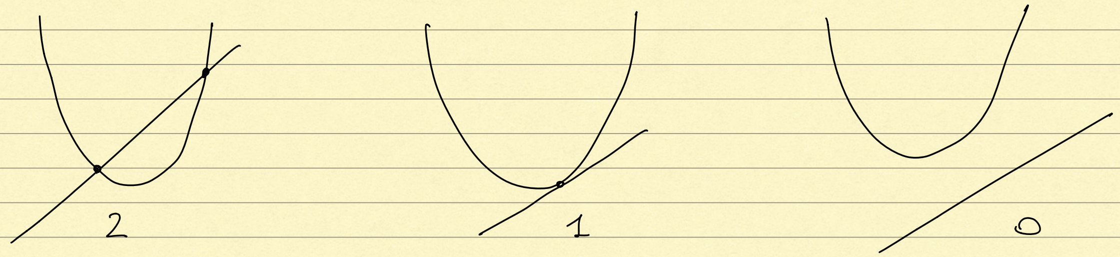
C è l'intersezione con asse y $\qquad y = ax^2 + bx + c$

Se $b = 0$ ho una parabola simmetrica rispetto a $x = 0$

Se $c = 0$ passa per l'origine

Se $b = c = 0$ ha vertice nell'origine

INTERSEZIONI RETTA - PARABOLA



$$\begin{cases} r: \ y = mx + q \\ P: \ y = ax^2 + bx + c \end{cases} \Rightarrow \quad ax^2 + bx + c = mx + q$$

$$\underbrace{a}x^2 + x\underbrace{(b - m)} + \underbrace{(c - q)} = 0$$

2 int. ci sono se $\Delta = (b-m)^2 - 4(c-q)a > 0$

1 int. se $\Delta =$ " " $= 0$

0 int $\Delta < 0$

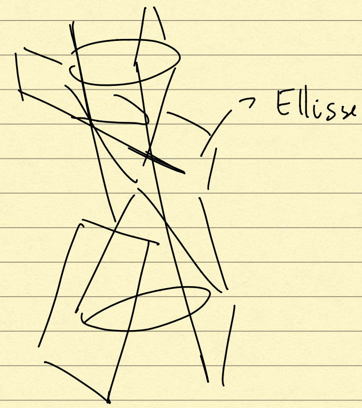
ESER. Determinare le eventuali intersezioni tra

$y = x^2$ e $4x - y - 4 = 0 \rightarrow y = 4x - 4$

$$\begin{cases} y = x^2 \\ y = 4x - 4 \end{cases} \Rightarrow \quad x^2 - 4x + 4 = 0 \qquad \Delta = 16 - 4 \cdot 4 \cdot 1 = 0$$

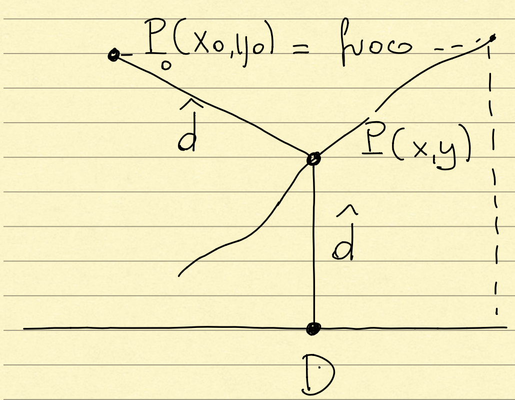
$$(x - 2)^2 = 0$$

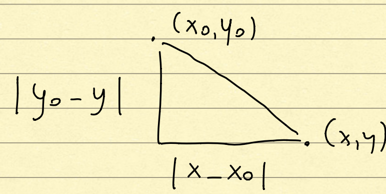
la retta è tangente



La parabola può anche essere definita nel seguente modo geometrico

«Luogo di p.ti equidistanti da un p.to $P \equiv (x_0, y_0)$ (detto fuoco) e da una retta r data»





$y = d$ (retta)

La retta per semplicità la prendiamo // asse delle x

dimostriamolo

$$\overline{P_0P}^2 = \text{(Pitagora)} = (x-x_0)^2 + (y-y_0)^2 = \hat{d}$$

$$\overline{PD}^2 = (y-d)^2 = \hat{d}$$

$$\overline{P_0P}^2 = \overline{PD}^2 \quad \Rightarrow \quad (y-d)^2 = (x-x_0)^2 + (y-y_0)^2$$

$$\cancel{y^2} + d^2 - 2dy = x^2 + \cancel{y^2} - 2x_0x - 2y_0y + x_0^2 + y_0^2$$

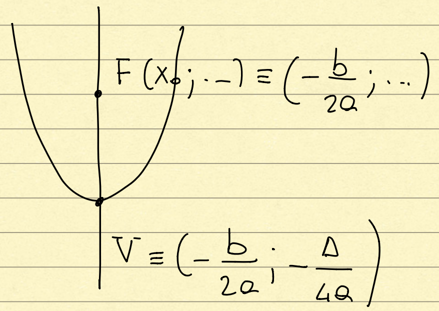
$$2y_0y - 2dy = x^2 - 2xx_0 + (x_0^2 + y_0^2 - d^2)$$

$$2(y_0-d)y = x^2 - 2xx_0 + (x_0^2+y_0^2-d^2)$$

$$y = \underbrace{\left(\frac{1}{2(y_0-d)}\right)}_{a} x^2 + \underbrace{\left(-\frac{\cancel{2}x_0}{\cancel{2}(y_0-d)}\right)}_{b} x + \underbrace{\left(\frac{x_0^2+y_0^2-d^2}{2(y_0-d)}\right)}_{c}$$

$$\frac{a}{b} = \frac{1}{2\cancel{(y_0-d)}}\cdot\left(-\frac{\cancel{(y_0-d)}}{x_0}\right) = -\frac{1}{2x_0} \Rightarrow x_0 = -\frac{b}{2a}$$

coord. x del vertice



$$\Delta = b^2 - 4ac = \frac{x_0^2}{(y_0-d)^2} - \cancel{4}\cdot\frac{1}{\cancel{2}(y_0-d)}\cdot\frac{(x_0^2+y_0^2-d^2)}{\cancel{2}(y_0-d)}$$

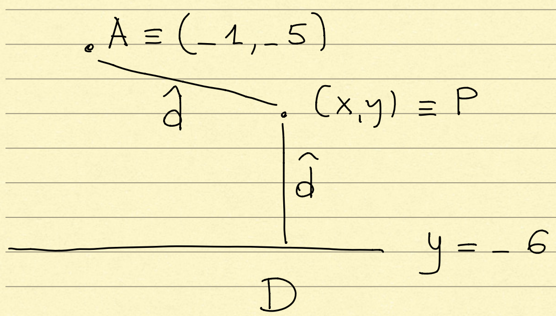
$$\Delta = \frac{\cancel{x_0^2} - (\cancel{x_0^2}+y_0^2-d^2)}{(y_0-d)^2} = \frac{d^2-y_0^2}{(d-y_0)^2} = \frac{(d-y_0)(d+y_0)}{(d-y_0)^2} = \frac{d+y_0}{d-y_0}$$

$$1-\Delta = 1 - \frac{d+y_0}{d-y_0} = \frac{\cancel{d}-y_0-\cancel{d}-y_0}{d-y_0} = -\frac{2y_0}{d-y_0}$$

$$a = \frac{1}{2(y_0 - d)} \Rightarrow d - y_0 = -\frac{1}{2a}$$

$$1 - \Delta = +\frac{2y_0}{\left(+\frac{1}{2a}\right)} \Rightarrow \boxed{\begin{array}{l} y_0 = \left(\dfrac{1-\Delta}{4a}\right) \\[2ex] x_0 = -\dfrac{b}{2a} \end{array}}$$

ESERCIZI « Scrivere l'equazione del luogo dei p.ti equidistanti dalla retta $y + 6 = 0$ e dal p.to $A = (-1, -5)$



$$\overline{PA}^2 = (x+1)^2 + (y+5)^2$$

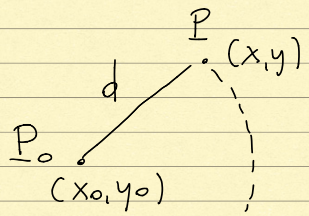
$$\overline{PD}^2 = (y+6)^2$$

$$\Rightarrow (y+6)^2 = (x+1)^2 + (y+5)^2$$

$$\cancel{y^2} + 36 + 12y = x^2 + 2x + 1 + \cancel{y^2} + 25 + 10y$$

$$2y = x^2 + 2x + 26 - 36 \qquad y = \left(\frac{x^2}{2} + x - 5\right)$$

Eq. Cerchio " Luogo (x,y) di p.ti equidist. da un p.to dato



$$\overline{PP_0} = R = \sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$R^2 = (x-x_0)^2+(y-y_0)^2$$

$$x^2+y^2 \underbrace{-2x_0}_{=A} \cdot x \underbrace{-2y_0}_{B} y + \underbrace{(x_0^2+y_0^2-R^2)}_{C} = 0$$

L'eq.ne di una circonf. è $\boxed{X^2+Y^2+AX+BY+C=0}$

$$C = x_0^2+y_0^2-R^2 \quad \Rightarrow \quad R^2 = x_0^2+y_0^2-C > 0$$

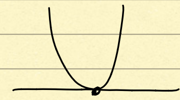
$$x_0 = -A/2 \qquad y_0 = -B/2$$

$\Rightarrow$ condiz. di $\exists$ del cerchio è $R^2 = \dfrac{A^2}{4}+\dfrac{B^2}{4}-C > 0$

ESERCIZIO « Det. l'eq.ne della parabola con asse // asse y passante per $A \equiv (3;4)$ ed avente vertice in $V \equiv (1;0)$
Trovare poi il raggio R della circonferenza avente il centro nel 1° quadrante $(x_0,y_0) \equiv$ Cent. $x_0, y_0 > 0$ tangente agli assi ed alla retta tangente alla parabola nel

suo p.to di ascissa 0. »

$y = ax^2 + bx + c$



Se $V \equiv (1;0) \Rightarrow \Delta = 0$

$V \equiv \left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right) \Rightarrow -\frac{b}{2a} = 1$ $\quad b = -2a$

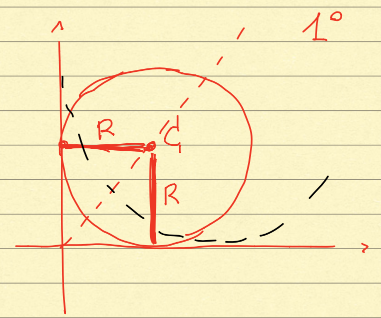
$\Delta = b^2 - 4ac = 4a^2 - 4ac = 0 = 4a(a-c) = 0 \Rightarrow a = c$

$\begin{cases} b = -2a \\ c = a \end{cases} \Rightarrow y = ax^2 - 2ax + a = a(x^2 - 2x + 1)$

$y = a(x-1)^2$

Impongo il passaggio per $A \equiv (3;4)$ $\quad \cancel{4} = a \cdot \cancel{4} \quad a = 1$

$\begin{cases} a = 1 \\ b = -2 \\ c = 1 \end{cases}$ $\quad y = x^2 - 2x + 1 = (x-1)^2$ $\quad$ Eq.ne parabola cercata $\quad \mathcal{P}$



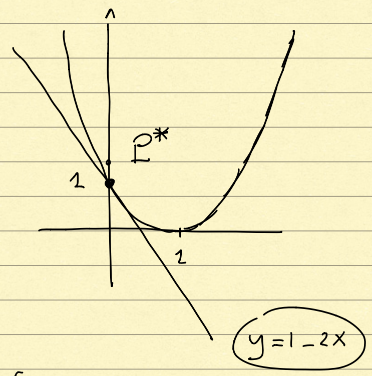
$C \equiv$ centro $\mathcal{C} \equiv (R, R)$

$(x-R)^2 + (y-R)^2 = R^2$

$x^2 + y^2 - 2xR - 2yR + 2R^2 = R^2$

$x^2 + y^2 - 2xR - 2yR + R^2 = 0 \quad \mathcal{C}$

Metto a sistema $\mathcal{C}$ e $\mathcal{P}$



$P^* = (0;1)$ retta per P^* e imporre la tangenza con la parabola.

$$(y - y^*) = m(x - x^*)$$
$$(y - 1) = m(x - 0) \Rightarrow \boxed{y = 1 + mx}$$

$$\begin{cases} y = 1 + mx \\ y = (x-1)^2 \end{cases}$$ e impongo $\Delta = 0$ $\Rightarrow$ $$\begin{cases} (x-1)^2 = 1 + mx \\ x^2 - x(2+m) + \cancel{1} = \cancel{1} \end{cases}$$

$$\Delta = (2+m)^2 - 4 \cdot 0 \cdot 1 = (2+m)^2 = 0 \qquad m = -2$$

Adesso metto a sist. retta e circonf.

$$\begin{cases} x^2 + y^2 - 2xR - 2yR + R^2 = 0 \\ y = 1 - 2x \end{cases} \qquad \begin{cases} (x-R)^2 + (y-R)^2 = R^2 \\ \end{cases}$$

$$(x-R)^2 + ((1-2x)-R)^2 = R^2$$

$$x^2 + \cancel{R^2} - 2xR + (1-2x)^2 + R^2 - 2R(1-2x) = \cancel{R^2}$$

$$x^2 - 2xR + 1 + 4x^2 - 4x + R^2 - 2R + 4Rx = 0$$

$$5x^2 + 2Rx - 4x + (R^2 - 2R + 1) = 0$$

$$5x^2 + x(2R-4) + (R-1)^2 = 0$$

Impongo $\Delta = 0 = \underbrace{(2R-4)}^2 - 20(R-1)^2 = 0$

$$\cancel{4}(R-2)^2 - \overset{5}{\cancel{20}}(R-1)^2 = 0 \Rightarrow \cancel{R^2} + 4 - \cancel{4R} - 5(\cancel{R^2} + 1 - \cancel{2R}) = 0$$

$$-4R^2 + 6R - 1 = 0 \qquad 4R^2 - 6R + 1 = 0$$

$$R = \frac{6 \pm \sqrt{36 - 16}}{8} = \frac{3}{4} \pm \frac{\sqrt{5}}{4} = \left(\frac{3 \pm \sqrt{5}}{4}\right) \begin{cases} R_1 \\ R_2 \end{cases}$$