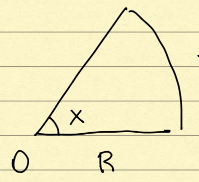


$x = \frac{l}{R}$ Se $l = 2\pi R$ lungh. circonf.

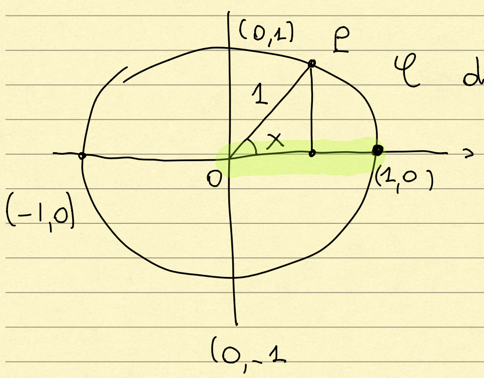
$x = \frac{2\pi \cancel{R}}{\cancel{R}}$

$2\pi \longrightarrow 360°$ x è in radianti

y è in gradi $\frac{x}{y} = \frac{2\pi}{360°} = \frac{\pi}{180}$

rad $\longrightarrow$ gradi gradi $\longrightarrow$ rad

$y = \frac{180° x}{\pi}$ $x = \frac{\pi y}{180°}$



$\mathcal{C}$ di raggio 1

$P = (\cos x, \sin x)$

$\cos 0 = 1$ $\sin 0 = 0$

$\cos\left(\frac{\pi}{2}\right) = 0$ $\sin\left(\frac{\pi}{2}\right) = 1$

$\vdots$ $\vdots$

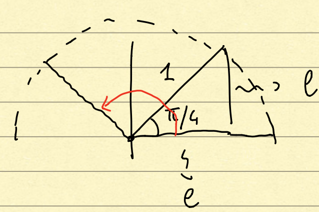
Si vede banalmente che

$\sin^2 x + \cos^2 x = 1$

Es. $\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$

Inoltre $0 \leq |\sin x| \leq 1$

$0 \leq |\cos x| \leq 1$

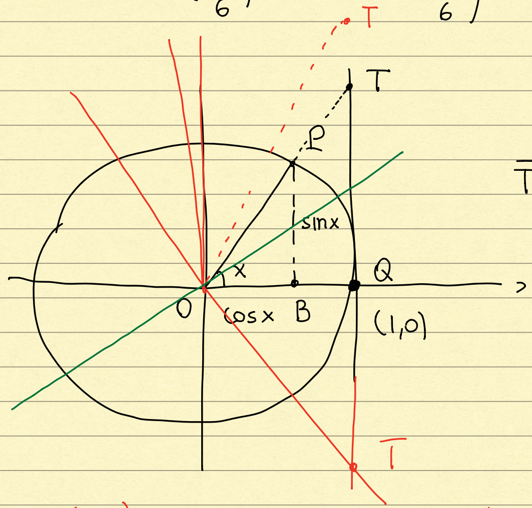


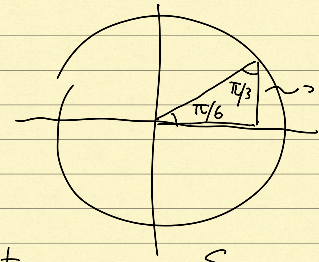
$2l^2 = 1$ $l = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$l = \sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\theta = \frac{3}{4}\pi$ $\cos\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2}$ $\sin\left(\frac{3}{4}\pi\right) = \frac{\sqrt{2}}{2}$

Es. $\cos\left(\frac{\pi}{6}\right) = \quad \sin\left(\frac{\pi}{6}\right) =$





$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\overline{TQ} = \tan x$ Siccome $P\hat{O}B \sim T\hat{O}Q$

$$\frac{\overline{TQ}}{\overline{OQ}} = \frac{\overline{PB}}{\overline{OB}} = \tan x = \frac{\sin x}{\cos x}$$

$\overline{OQ} \to 1$

$\tan\left(\frac{\pi}{2}\right) = +\infty$

$\tan(x) < 0$

$x \in \left(\frac{\pi}{2}, \pi\right)$

$\sin x$ e $\cos x$ sono f.ni periodiche di periodo $T = 2\pi$

2° $\sin x \geq 0$, $\cos x \leq 0$, $\tan x \leq 0$

1° $\sin x \geq 0$, $\cos x \geq 0$, $\tan x \geq 0$

3° $\sin x \leq 0$, $\cos x \leq 0$, $\tan x \geq 0$

4° $\sin x \leq 0$, $\cos x \geq 0$, $\tan x \leq 0$

$$\boxed{\begin{aligned}\cos(x+2\pi) &= \cos x\\ \sin(x+2\pi) &= \sin x\end{aligned}}$$

$$\boxed{\tan(x+\pi) = \tan x}$$

Formule di addizione

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

Formule di duplicazione

$$\cos(2x) = \cos^2 x - \sin^2 x = \begin{cases} 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Formule di bisezione

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos x = \pm\sqrt{\frac{1+\cos 2x}{2}}$$

$$x = \frac{\alpha}{2} \Rightarrow \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan(x \pm y) = \frac{\sin(x\pm y)}{\cos(x\pm y)} = \frac{\sin x\cos y \pm \sin y\cos x}{\cos x\cos y \mp \sin x\sin y} =$$

$$= \frac{\cos x\,\cancel{\sin y}\left[\frac{\sin x}{\cos x}\cdot\frac{\cos y}{\sin y} \pm 1\right]}{\cos x\,\cancel{\sin y}\left[\frac{\cos y}{\sin y} \mp \frac{\sin x}{\cos x}\right]} = \frac{\frac{\tan x}{\tan y} \pm 1}{\left[\frac{1}{\tan y} \mp \tan x\right]}\cdot\frac{\tan y}{\tan y} =$$

$$= \boxed{\frac{\tan x \pm \tan y}{1 \mp \tan x\cdot\tan y} = \tan(x\pm y)} \;*$$

Usando * posso far vedere che $\tan(x+\pi) = \tan x$

$$\tan(x+\pi) = \frac{\tan x + \underbrace{\cancel{\tan(\pi)}}_{=0}}{1 - \tan x\cdot\underbrace{\cancel{\tan(\pi)}}_{=0}} = \tan x$$

ESERCIZIO Trovare tutte le soluzioni di

$$\sin 2x\cdot\left(\frac{1}{\tan x} - \tan x\right) = 2(\sin x + \cos x)$$

$\sin(2x) = 2\sin x\cos x$ $\qquad$ $\frac{1}{\tan x} = \frac{\cos x}{\sin x} = \text{cotan}\, x$

$$2\sin x\cos x\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right) = 2(\sin x + \cos x)$$

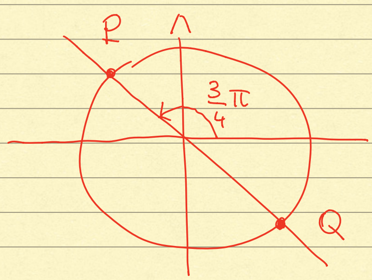
$$(\cancel{2}\cancel{\sin x}\cancel{\cos x})\cdot\frac{(\cos^2 x - \sin^2 x)}{(\cancel{\sin x}\cdot\cancel{\cos x})} = \cancel{2}(\sin x + \cos x)$$

$\sin x\cos x \neq 0$ $\qquad$ $x \neq k\frac{\pi}{2}$ $\quad k \in \mathbb{Z}$ $\qquad$ C.E.

$$\cos^2 x - \sin^2 x = \sin x + \cos x$$

$$(\cos x - \sin x)\underbrace{\cancel{(\cos x + \sin x)}} = \underbrace{\cancel{(\cos x + \sin x)}}$$

1° $\underbrace{\cos x + \sin x = 0}$ è soluzione dell'equa.



P e Q sono tali che $\sin x = -\cos x$

$\sin\left(\frac{3}{4}\pi\right) = \frac{\sqrt{2}}{2}$ $\qquad$ $\cos\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2}$

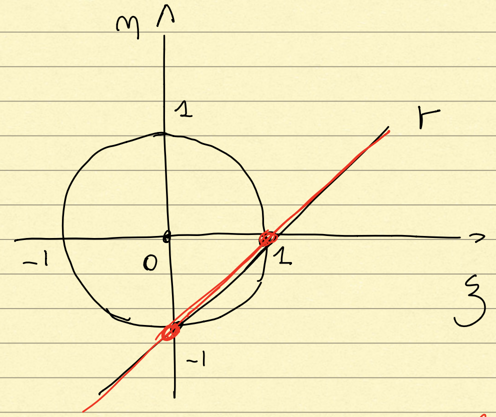
Allora le soluzioni sono $x = \frac{3}{4}\pi + k\pi$ $\quad k \in \mathbb{Z}$

⤳ queste sol. vanno bene rispetto a C.E.

Mi rimane $\cos x - \sin x = 1$

Per risolvere

$$\begin{cases} \xi = \cos x \\ \eta = \sin x \end{cases} \qquad \begin{cases} \xi - \eta = 1 \\ \xi^2 + \eta^2 = 1 \end{cases}$$



retta r: $\eta = \xi - 1$

Le sol. sono $(\xi,\eta) = \begin{cases}(1,0)\\(0,-1)\end{cases}$

$\begin{cases}1 = \cos x\\0 = \sin x\end{cases} \Rightarrow x = 2k\pi \quad k \in \mathbb{Z}$

Queste soluz. non vanno bene perché $\notin$ C.E.

$\begin{cases}0 = \cos x\\-1 = \sin x\end{cases} \Rightarrow x = \frac{3}{2}\pi + 2k\pi$

$\Rightarrow$ le uniche sol. accettabili sono $x = \frac{3}{4}\pi + k\pi$

ESERCIZIO

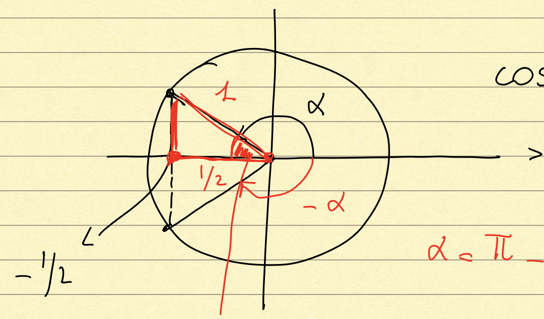
$$\cos\left(3x - \pi/3\right) > -\frac{1}{2} \quad (*)$$

$\left\{ f(x) = \ln\left[\cos\left(3x - \frac{\pi}{3}\right) + \frac{1}{2}\right]\right.$

C.E. o dominio è la soluz. di $*$

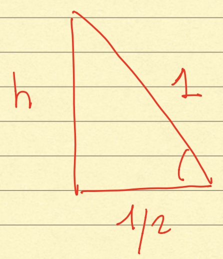
$\theta = 3x - \pi/3$

$$\cos(\theta) > -\frac{1}{2}$$



$\cos(\alpha) = -1/2$

$\alpha = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$



$\pi/3$ $\qquad -\alpha = -\frac{2}{3}\pi$

$\cos\left(\frac{2}{3}\pi\right) = -1/2$

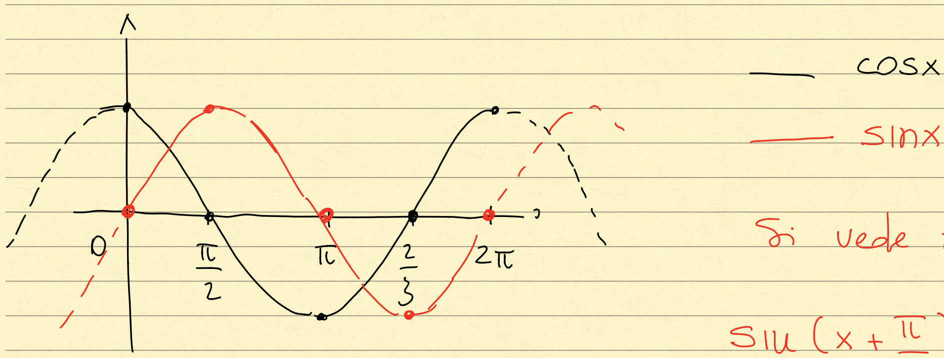
$\cos\left(-\frac{2}{3}\pi\right) = -1/2$

$\Rightarrow$ se $\theta \in \left(-\frac{2}{3}\pi, \frac{2}{3}\pi\right) + 2k\pi$ sicuramente $\cos\theta > -1/2$

$$-\frac{2}{3}\pi + 2k\pi < \theta < \frac{2}{3}\pi + 2k\pi$$

$$-\frac{2}{3}\pi + 2k\pi < 3x - \frac{\pi}{3} < \frac{2}{3}\pi + 2k\pi$$

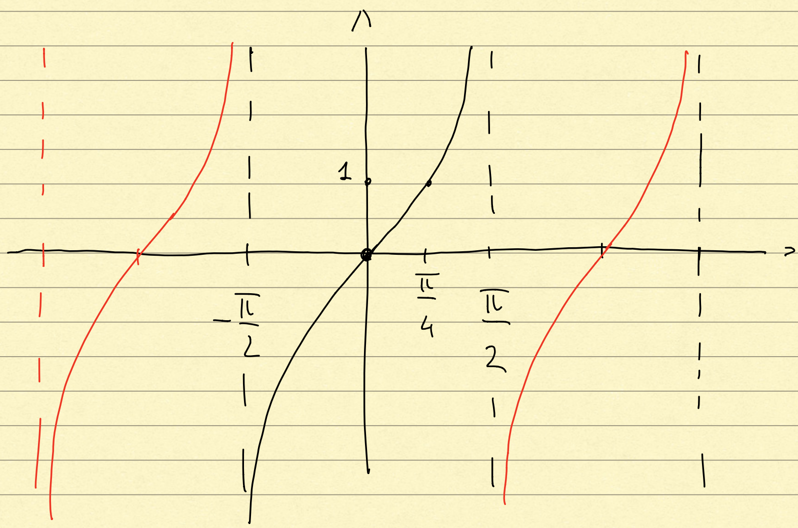
$$-\frac{\pi}{9} + \frac{2}{3}k\pi < x < \frac{\pi}{9} + \frac{2}{3}k\pi$$



Si vede subito che

$\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$$\sin\left(x+\frac{\pi}{2}\right) = \cancel{\sin x \cos\frac{\pi}{2}} + \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}\cos x = \cos x$$



MOODLE