$$\boxed{y = \log_a x \quad \text{se} \quad a^y = x} \qquad (\text{Def } 1)$$

$$3 = \log_2 8 \qquad 2^3 = 8$$

Proprietà

i) $\log_a a = 1$

ii) $\log_a 1 = 0$

iii) $\log_a (x_1 \cdot x_2) = \log_a x_1 + \log_a x_2$

iv) $\log_a \left(\dfrac{x_1}{x_2}\right) = \log_a x_1 - \log_a x_2$

v) $\log_a (x^n) = n \log_a x$

vi) $\log_a a^x = x \rightsquigarrow \log_a (a^x) = x$

vii) $\log_a x = \left(\dfrac{\log_b x}{\log_b a}\right)$

Per esercizio dimostro iii) e vii)

Prop. iii)

$\rightarrow y_1 = \log_a x_1 \iff a^{y_1} = x_1$

$\rightarrow y_2 = \log_a x_2 \iff a^{y_2} = x_2$

$$a^{y_1} \cdot a^{y_2} = x_1 \cdot x_2$$

$$a^{y_1 + y_2} = x_1 \cdot x_2 \iff (\text{Def } 1) \quad \log_a (x_1 \cdot x_2) = y_1 + y_2$$

$$\log_a (x_1 \cdot x_2) = \log_a x_1 + \log_a x_2 \qquad \text{C.V.D.}$$

Proprietà vii)

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}} \implies y = \frac{t}{z}$$

$\log_2 2^3 = 8$ $\qquad\qquad$ $\log_3 2^3$

$\log_3 8 = \dfrac{\log_2 8}{\log_2 3}$ $\qquad\qquad$ $\log_a x = y$ se $a^y = x$

DIM.

$y = \log_a x \qquad t = \log_b x \qquad z = \log_b a$

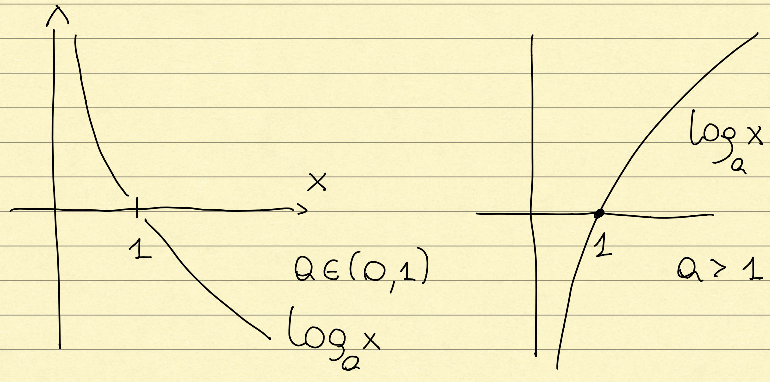
$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$

$a^y = x \qquad b^t = x \qquad b^z = a$

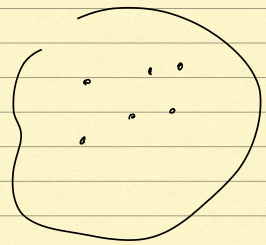
$(b^z)^y = x = b^t \iff b^{zy} = b^t \Rightarrow zy = t$

$\Rightarrow y = \dfrac{t}{z} \Rightarrow \log_a x = \dfrac{\log_b x}{\log_b a}$ C.V.D.

$f(x) = \log_a x = y \qquad a \in (0,1) \qquad a > 1$



Volume $= \log_{10} \dfrac{P}{P_0}$ $\qquad P = 0$



H^+

$c = \dfrac{mol}{lt}$ $\qquad$ Se $c_0 = 10^{-7}\ mol/_{lt}$

$$Ph = -\log_{10} \frac{c}{1\ mol/lt}$$

$$c = c_0 \quad \Rightarrow -\log_{10} \frac{c_0}{1\ mol/lt} = \quad -\log_{10}(10^{-7}) = 7$$

Se $\quad c = 10^{-6}\ mol/_{lt}$ $\qquad Ph = 6$

Equi e disequi logaritmiche

Es. $\qquad 2\log_2 x + \log_{1/2}(3 - x^2) - \log_2\left(\dfrac{1}{x^2+1}\right) = 0$

C.E. $\begin{cases} x > 0 & x > 0 \\ (3 - x^2) > 0 & x^2 < 3 \qquad x \in (-\sqrt{3}, \sqrt{3}) \\ \dfrac{1}{1+x^2} > 0 & \text{Sempre sodd.} \qquad x \in \mathbb{R} \end{cases}$

C.E. $\qquad (0, \sqrt{3})$

Osservo che $\log_{\left(\frac{1}{a}\right)} x = \dfrac{\log_a x}{\log_a\left(\frac{1}{a}\right)}$

$\log_a\left(\frac{1}{a}\right) = \cancel{\log_a 1} - \log_a a$

$\boxed{\log_{\frac{1}{a}} x = -\log_a x}$

$$\log_2 x^2 - \log_2(3-x^2) + \log_2(1+x^2) = 0$$

$$\log_2 \frac{x^2}{(3-x^2)} + \log_2(1+x^2) = 0$$

$$\log_2\left[\frac{x^2(1+x^2)}{(3-x^2)}\right] = 0 \iff \frac{x^2(1+x^2)}{(3-x^2)} = 1$$

$$x^4 + x^2 = 3 - x^2 \qquad x^4 + 2x^2 - 3 = 0$$

$$t = x^2 \qquad t^2 + 2t - 3 = 0 \qquad (t+3)(t-1) = 0$$

$t = x^2 = \cancel{-3}$

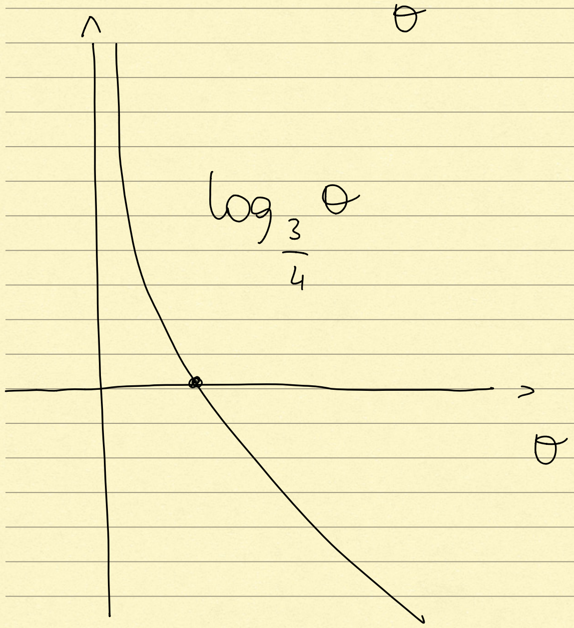
$t = x^2 = 1 \rightsquigarrow x = \pm 1$

Mi va bene solo $x = 1$ perché C.E $(0, \sqrt{3})$

DISEQUAZIONE LOGARITMICA

Es. $\log_{3/4}(\underbrace{6+5x}_{\theta}) \geq 0$

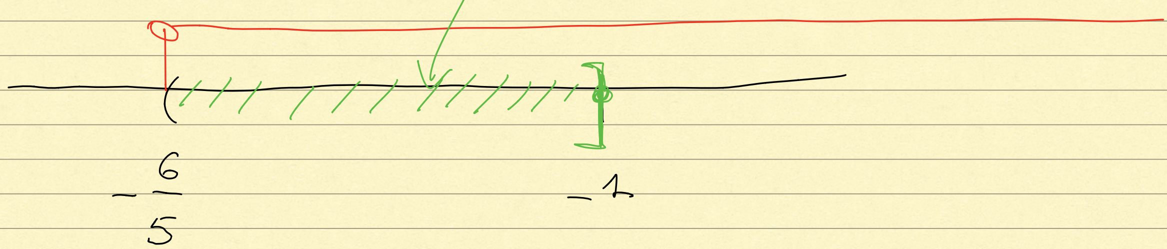
C.E $\begin{cases} 6+5x > 0 \\ x > -6/5 \end{cases}$



$0 < \theta \leq 1$

$0 < 6+5x \leq 1$

$-\frac{6}{5} < x \leq -1$



Es. Risolvere

$\log_{\sqrt{2x^2-7x+6}}\left(\frac{x}{3}\right) > 0$

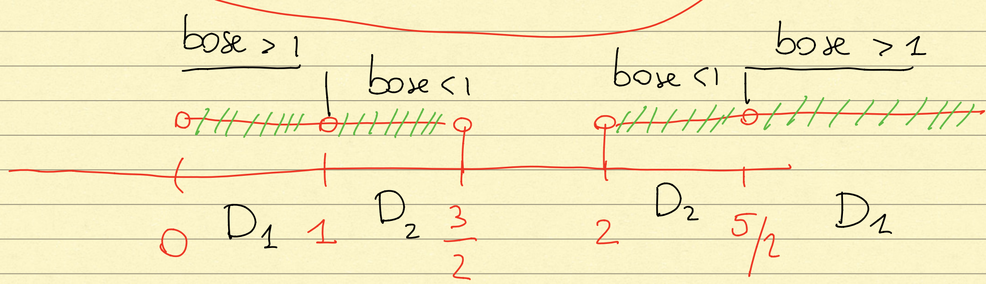
C.E $\begin{cases} \frac{x}{3} > 0 \\ 2x^2-7x+6 > 0 \\ \qquad\qquad \neq 1 \end{cases}$

$$
\text{C.E.}\quad \begin{cases} x > 0 \\ 2x^2 - 7x + 6 > 0 \\ x > 2 \ \text{e} \ x < 3/2 \end{cases}
\qquad \Delta = 49 - 48 = 1 \qquad x_{1,2} = \frac{7 \pm 1}{4} \nearrow 2, \ \searrow 3/2
$$

$$2x^2 - 7x + 6 \neq 1$$

$$2x^2 - 7x + 5 \neq 0 \qquad x_{1,2} = \frac{7 \pm \sqrt{9}}{4} = \nearrow 5/2, \ \searrow 1$$

$$x \neq 1 \quad \text{e} \quad x \neq 5/2$$



$$\text{C.E.} \qquad (0,1) \cup (1, 3/2) \cup (2, 5/2) \cup \left(\frac{5}{2}, +\infty\right)$$

Devo vedere dove $\sqrt{\ldots\ldots} > 1 \leadsto$

dove $\sqrt{\ldots\ldots} \in (0,1) \leadsto$

$$\underbrace{\sqrt{2x^2 - 7x + 6}}_{\text{base}} > 1 \qquad \underbrace{2x^2 - 7x + 5 > 0}_{x = 1 \quad x = 5/2}$$

$\boxed{D_1 = (0,1) \cup (5/2, +\infty)}$ base > 1

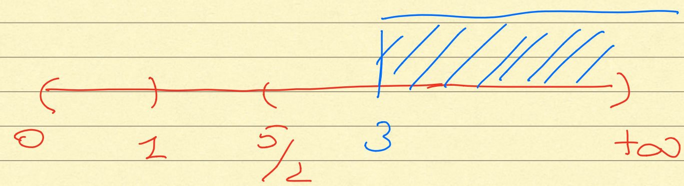
$\boxed{D_2 = \left(1, \frac{3}{2}\right) \cup \left(2, 5/2\right)}$ base $\in (0,1)$

D_1 $\quad \log_{\sqrt{\dots}} \left(\frac{x}{3}\right) > 0$ $\quad$ Sol. $\left(\frac{x}{3}\right) > 1$

$\sqrt{\dots} > 1$ $\qquad\qquad$ $x > 3$

In D_1 le sol. sono $x > 3$ $\qquad$ Ok!

$\{x > 3\} \subset D_1$

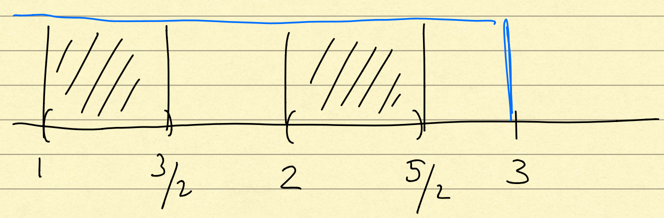


Adesso controllo se ci sono sol. in D_2

In questo caso $\sqrt{\dots} < 1 \Rightarrow \log_{\sqrt{\dots}} \left(\frac{x}{3}\right) > 0$

$\Rightarrow \quad 0 < \frac{x}{3} < 1 \qquad x < 3$

In questo caso le sol. sono esattamente date da D_2



Le sol. alla fine sono $D2 \cup \{x > 3\}$

Numeri complessi :

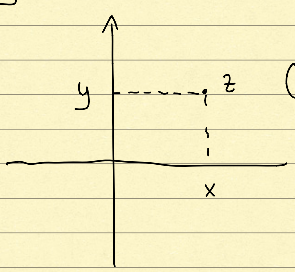
Def (Unità immaginaria) $\quad i = \sqrt{-1} \qquad i^2 = -1$

Definisco $z \in \mathbb{C}$ se $z = x + iy \qquad x, y \in \mathbb{R}$

($\mathbb{C}$: insieme dei numeri complessi)

$x = Re(z) \qquad z = Re(z) + i\,Im(z)$

$y = Im(z) \qquad$ I numeri complessi possono essere visualizzati nel piano $\mathbb{R}^2$



(e' il p.to individuato dalla coppia (x,y))

SOMMA $\quad z = x + iy \qquad w = \xi + i\eta$

$$z + w = (x + \xi) + i(y + \eta)$$

PRODOTTO di w e z

$$z \cdot w = (x + iy)(\xi + i\eta) = (x\xi + i^2\eta y) + i(x\eta + \xi y)$$

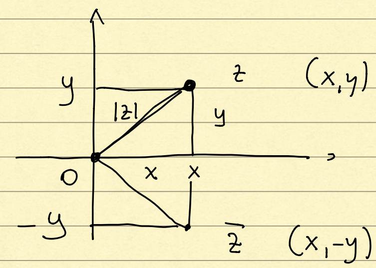
$$z \cdot w = (x\xi - \eta y) + i(x\eta + \xi y)$$

Quoziente di w, z

$$\frac{z}{w} = \frac{x + iy}{\xi + i\eta} = \frac{(x + iy)}{(\xi + i\eta)} \cdot \frac{(\xi - i\eta)}{(\xi - i\eta)} = \frac{(x + iy)(\xi - i\eta)}{\xi^2 - i^2\eta^2} = \frac{(x\xi + y\eta) + i(y\xi - x\eta)}{\xi^2 + \eta^2}$$

$$\mathrm{Re}\left(\frac{z}{w}\right) = \frac{(x\xi + y\eta)}{\xi^2 + \eta^2} \qquad \mathrm{Im}\left(\frac{z}{w}\right) = \frac{(y\xi - x\eta)}{\xi^2 + y^2}$$

CONIUGATO DI UN NUMERO $z \in \mathbb{C}$



$$\begin{cases} z = x + iy \\ \bar{z} = x - iy \quad \text{(Coniugato di } z\text{)} \end{cases}$$

$$\mathrm{Re}(z) = \frac{z + \bar{z}}{2} = x$$

$$\mathrm{Im}(z) = \frac{z - \bar{z}}{2} = y$$

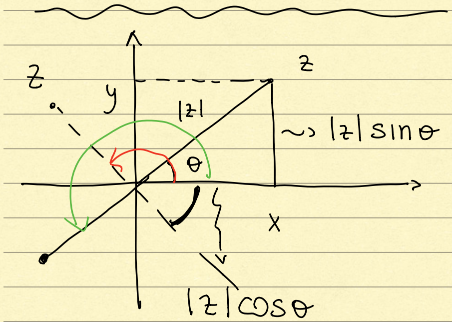
MODULO DI UN NUMERO $z \in \mathbb{C}$

$$|z| = \sqrt{x^2 + y^2} \qquad |\bar{z}| = \sqrt{x^2 + y^2}$$

- Osservo che $z \cdot \bar{z} = (x+iy)(x-iy) = x^2 - (iy)^2 = x^2 + y^2 = |z|^2$

$$z \cdot \bar{z} = |z|^2$$

ARGOMENTO DI UN NUMERO COMPLESSO



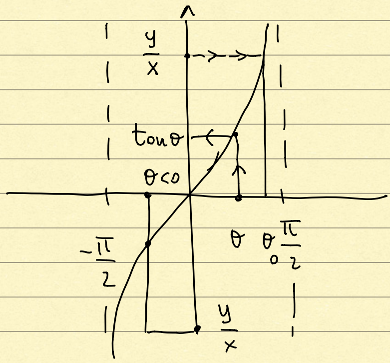
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\tan\theta = \frac{y}{x} \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$$

θ si dice argomento di z

$\theta = \mathrm{Arg}(z)$ vale se $x, y \in$ I° q.

Convenzionalmente $\theta \in [0, 2\pi)$



$\tan\theta_0 = \frac{y}{x}$

Se $\frac{y}{x} < 0$ ($y, x \in$ II° quadrante)
(θ è negativo
ma io voglio $\theta \in [0, 2\pi)$)

$$\theta = \mathrm{Arg}(z) = \arctan\left(\frac{y}{x}\right) + \pi$$

(Vale se $x, y \in$ II o III quadrante)

Se $x, y \in$ IV quadrante

$$\theta = \mathrm{Arg}(z) = \arctan\left(\frac{y}{x}\right) + 2\pi$$

In conclusione

$$\theta = \mathrm{Arg}(z) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{se } y, x \in \mathrm{I}° \\ \quad '' \qquad '' \quad + \pi & \text{se } y, x \in \mathrm{II}°, \mathrm{III}° \\ \quad '' \qquad '' \quad + 2\pi & \quad '' \quad \in \mathrm{IV}° \end{cases}$$

Un numero complesso è individuato da $|z|$ e θ

$z = x + iy$

$$z = (\underbrace{|z|\cos\theta}_{x}) + i(\underbrace{|z|\sin\theta}_{y})$$

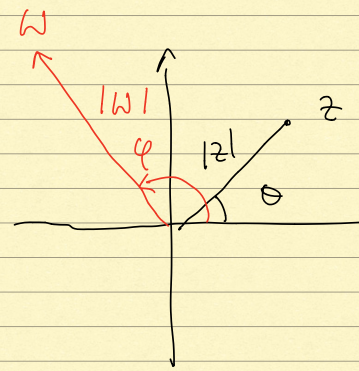
$x = |z| \cos\theta$

$y = |z| \sin\theta$

$$\boxed{z = |z|\,(\cos\theta + i\sin\theta)}$$

Forma trigonometrica

Prodotto $w \cdot z$



$$w = |w|\,(\cos\varphi + i\sin\varphi)$$

$$z = |z|\,(\cos\theta + i\sin\theta)$$

$$w \cdot z = |w||z|\,(\cos\varphi + i\sin\varphi)(\cos\theta + i\sin\theta)$$

$$w \cdot z = |w||z|\left[\underbrace{(\cos\varphi\cos\theta - \sin\varphi\sin\theta)}_{\cos(\theta+\varphi)} + i\,\underbrace{(\sin\varphi\cos\theta + \cos\varphi\sin\theta)}_{\sin(\theta+\varphi)}\right]$$

$$w \cdot z = |z| \cdot |w|\,\left[\cos(\theta+\varphi) + i\sin(\theta+\varphi)\right]$$

FORMA esponenziale di un numero complesso

Def. $$e^{i\theta} = (\cos\theta + i\sin\theta) \qquad (Eq\ 1)$$

$$e^{i(\theta+\varphi)} = (\cos(\theta+\varphi) + i\sin(\theta+\varphi))$$

Se $z = |z| e^{i\theta}$

$w = |w| e^{i\varphi}$

$$z \cdot w = |z| \cdot |w| \, e^{i\theta} \cdot e^{i\varphi} = |z| \cdot |w| e^{i(\theta+\varphi)} = |z||w| \left(\cos(\theta+\varphi) + i \sin(\theta+\varphi) \right)$$

$$z = |z| e^{i\theta} \qquad \left\{ \begin{array}{l} |e^{i\theta}| = 1 \quad \forall \, \theta \\ \\ z = |z| e^{i\theta} \\ |z| = |z||e^{i\theta}| \Rightarrow |z| = |z| \end{array} \right.$$

$$\bar{z} = |z| e^{-i\theta}$$

Con la forma esponenziale posso det. i coniugati di somma prodotto e quoziente

$z = |z| e^{i\theta}$

$w = |w| e^{i\varphi}$

i) $\overline{z + w} = \bar{z} + \bar{w} \qquad \overline{|z| e^{i\theta} + |w| e^{i\varphi}} = |z| e^{-i\theta} + |w| e^{-i\varphi}$

ii) $$\overline{z \cdot w} = \overline{|z| e^{i\theta} |w| e^{i\varphi}} = \overline{|z||w| e^{i(\theta+\varphi)}} = |z| e^{-i(\theta+\varphi)} |w| =$$

$$= |z| e^{-i\theta} |w| e^{-i\varphi} = \bar{z} \cdot \bar{w} \qquad \Rightarrow \qquad \overline{z \cdot w} = \bar{z} \cdot \bar{w}$$

iii) $$\overline{\left(\frac{z}{w} \right)} = \overline{\frac{|z| e^{i\theta}}{|w| e^{i\varphi}}} = \overline{\frac{|z|}{|w|} e^{i(\theta-\varphi)}} = \frac{|z|}{|w|} e^{-i(\theta-\varphi)} =$$

$$= \frac{|z| e^{-i\theta}}{|w| e^{-i\varphi}} = \frac{\bar{z}}{\bar{w}} \qquad \Rightarrow \qquad \overline{\left(\frac{z}{w} \right)} = \frac{\bar{z}}{\bar{w}}$$

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