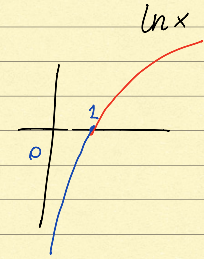
$f(x) = |x \ln x| \qquad g(x) = x \ln x \qquad dom(f) = \{x > 0\}$

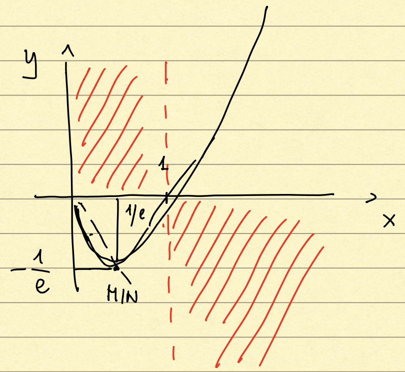
Segno di $g(x)$ $\begin{cases} x > 0 \\ \ln x \gtrless 0 \end{cases}$

$g(x) > 0$ per $x > 1$

$g(x) < 0$ per $x \in (0,1)$

$g(x) = 0$ per $x = 1$




$$\lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} x \ln x = +\infty$$

$$\lim_{x \to 0^+} x \ln x = \left( \begin{array}{l} x = \frac{1}{t} \\ x \to 0^+ \\ t \to +\infty \end{array} \right) = \lim_{t \to +\infty} \frac{1}{t} \ln\left(\frac{1}{t}\right)$$

$$= \lim_{t \to +\infty} -\frac{\ln t}{t} = 0^-$$

Osservo che necessariamente ci deve essere un minimo fra 0 e 1

Derivata 1°

$g(x) = x \ln x$

$g(e^{-1}) = \dots$

$g(x) = x \ln x \qquad \boxed{g'(x) = \ln x + x \cdot \frac{1}{x} = 1 + \ln x}$

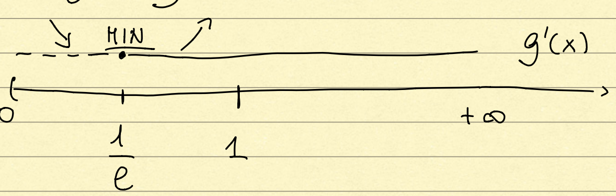
$g' = 0 \quad 1 + \ln x = 0$

$\ln x = -1$

$e^{\ln x} = e^{-1} = x$

$e = 2.71\dots$

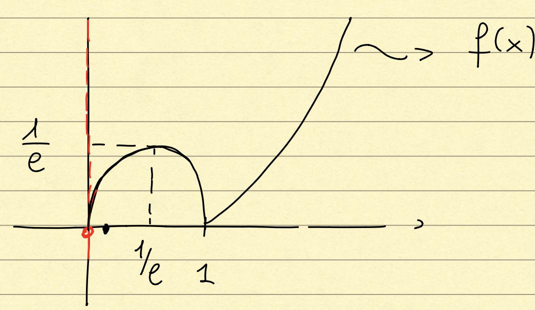
Studio segno $g'(x)$




$$g''(x) = \frac{1}{x} > 0$$

Siccome $f(x) = |g(x)|$

$$\lim_{x \to 0^+} g'(x) = -\infty$$



$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} |g'(x)| = +\infty$$

---

$$f(x) = \frac{x}{2 + \ln x}$$

$$\boxed{\operatorname{dom} f = \{x > 0\} \setminus \{e^{-2}\} = (0, e^{-2}) \cup (e^{-2}, +\infty)}$$

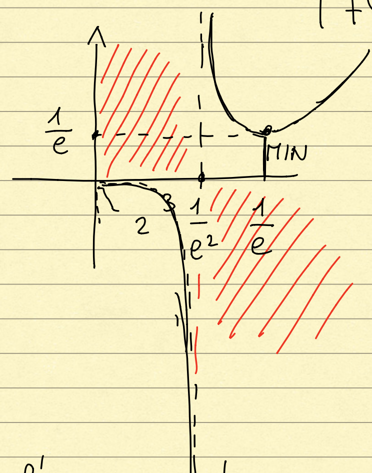
$\ln x + 2 = 0$

$\ln x = -2$

$e^{\ln x} = e^{-2} = x$

SEGNO di f

$$\begin{cases} f(x) > 0 & \text{se } 2 + \ln x > 0 \quad x > e^{-2} \\ f(x) < 0 & \text{se } 2 + \ln x < 0 \quad x < e^{-2} \end{cases}$$




LIMITI

- $\lim_{x \to 0^+} f(x) = \frac{0^+}{-\infty} = 0^-$
- $\lim_{x \to (e^{-2})^-} f(x) = \frac{e^{-2}}{0^-} = -\infty$
- $\lim_{x \to (e^{-2})^+} f(x) = \frac{e^{-2}}{0^+} = +\infty$
- $\lim_{x \to +\infty} f(x) = +\infty$

Derivata 1°

$$f'(x) = \frac{1 \cdot (2 + \ln x) - \frac{1}{x} \cdot x}{(2 + \ln x)^2}$$

$$\boxed{f'(x) = \frac{1 + \ln x}{(2 + \ln x)^2}}$$

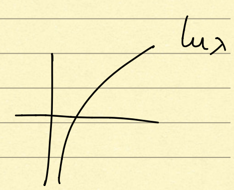
$f' = 0$ se $\ln x = -1$

ossia se $x = e^{-1}$

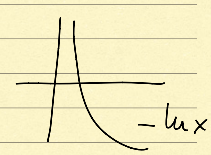
$$f'(x) \ (\text{per } x \to 0^+) \approx \frac{\ln x}{(\ln x)^2} = \frac{1}{\ln x}$$

Derivata 2°

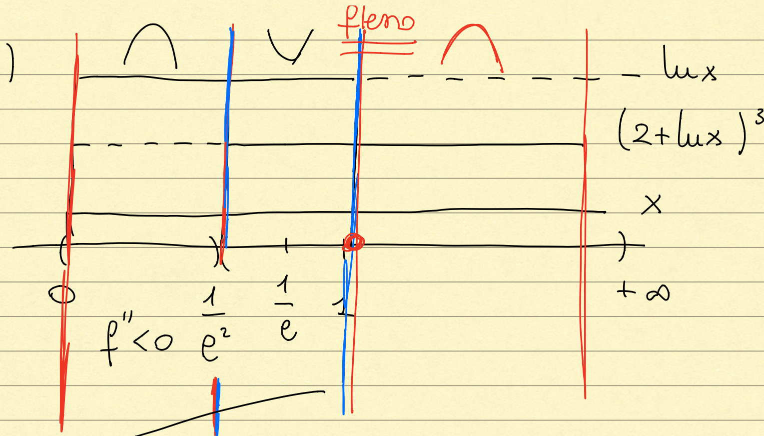
$$f''(x) = \frac{\frac{1}{x} \cdot (2 + \ln x)^{\cancel{2}} - 2\cancel{(2 + \ln x)}\frac{1}{x} \cdot (1 + \ln x)}{(2 + \ln x)^{\cancel{4}\,3}}$$




$$f''(x) = \frac{\frac{1}{x}\left[\cancel{2} + \ln x - 2(\cancel{1} + \ln x)\right]}{(2 + \ln x)^3} = \frac{-\ln x}{x\,(2 + \ln x)^3}$$



Studio il segno di $f''(x)$




$x = 1$ flesso

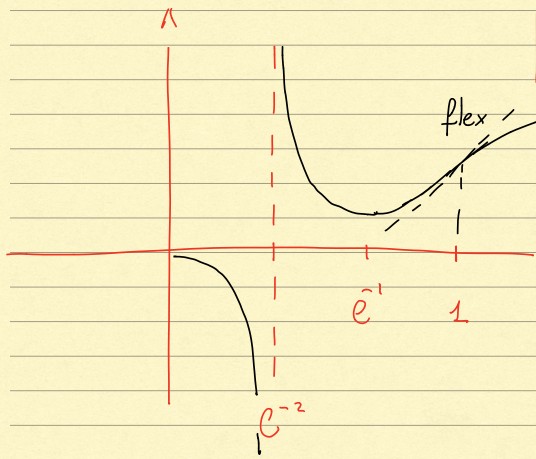
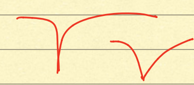



GRAFICO QUALITATIVO

Di $f(x) = \frac{x}{2+\ln x}$

$$\frac{f(x)}{x} = \frac{1}{2+\ln x} \longrightarrow 0 \quad \text{se } x \to +\infty$$



ESERCIZIO

$$f(x) = \frac{|2x-1|}{\underbrace{2x^2-x+1}_{\text{sempre} > 0}}$$

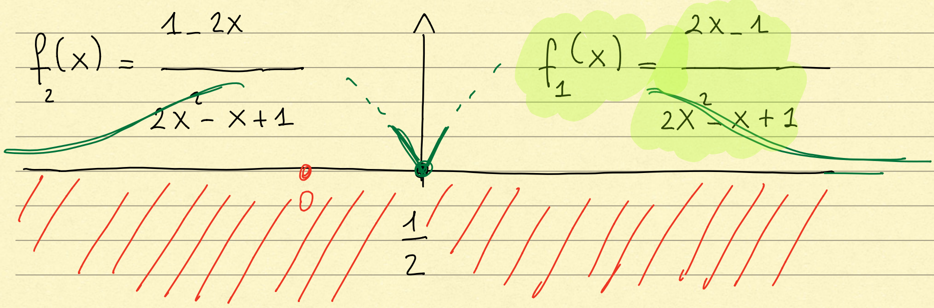
Il dominio di $f$ è tutto $\mathbb{R}$
infatti $2x^2-x+1$ non si annulla mai
poiché $\Delta = 1-8 = -7 < 0$

$$|h(x)| = \begin{cases} h(x) & \text{se } x \text{ è t.c. } h(x) \geq 0 \\ -h(x) & \text{se } x \text{ è t.c. } h(x) \leq 0 \end{cases}$$

$$|2x-1| = \begin{cases} 2x-1 & 2x-1 \geq 0 \quad x \geq \frac{1}{2} \\ 1-2x & \text{"} \quad \text{"} \leq 0 \quad x \leq \frac{1}{2} \end{cases}$$

SEGNO di $f(x)$

$$f_2(x) = \frac{1-2x}{2x^2-x+1} \qquad f_1(x) = \frac{2x-1}{2x^2-x+1}$$




sempre $> 0$
eccetto in $x = 1/2$
dove vale 0.

**LIMITI** $\lim_{x \to +\infty} f_1(x) = 0^+$ $\quad \lim_{x \to -\infty} f_2(x) = 0^+$

**DERIVATA 1ª** Intanto osservo che $f_1(x) = -f_2(x)$

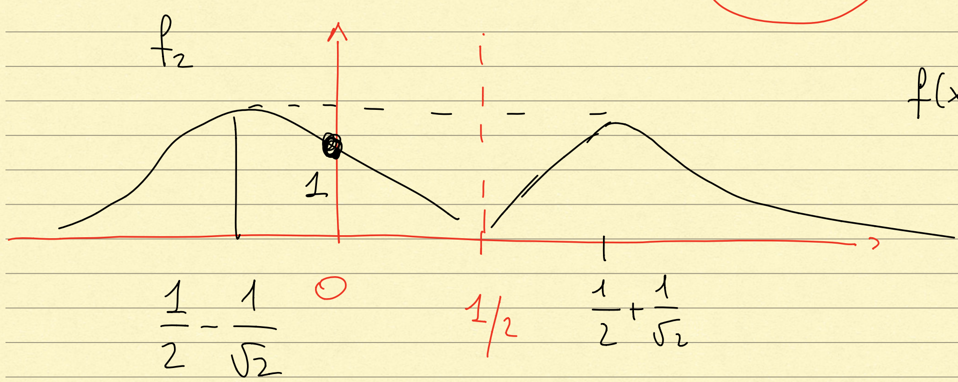
$f_1'(x) = -f_2'(x)$

Calcolo $f_1'(x) = \left[\dfrac{2x-1}{2x^2-x+1}\right]' = \dfrac{2\cdot(2x^2-x+1) - (4x-1)\cdot(2x-1)}{(2x^2-x+1)^2}$

$f_1'(x) = \dfrac{-4x^2+4x+1}{(2x^2-x+1)^2} = -f_2'(x)$ Cerco gli zeri di $f1'(x)$

$-4x^2+4x+1 = 0$ $\quad x_{1,2} = \dfrac{-4 \pm \sqrt{16+16}}{-8}$

$x_{1,2} = \dfrac{4 \pm \sqrt{2\cdot 16}}{8} = \dfrac{1}{2} \pm \dfrac{\sqrt{2}}{2} = \dfrac{1}{2} \pm \dfrac{1}{\sqrt{2}}$ questi valori annullano $f_2'$ e $f_1'$




$f(x) = \dfrac{|2x-1|}{2x^2-x+1}$

$f_2'(0) = -1$

**Derivata 2ª**

$$f_1''(x) = \frac{(-8x+4)(2x^2-x+1)^{\cancel{2}} - 2\cancel{(2x^2-x+1)}(-4x^2+4x+1)\cdot(4x-1)}{(2x^2-x+1)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{16x^3 + 16x^2 - 60x^2 - 12x + 6}{(2x^2 - x + 1)^3}$$