$f(x) = e^{-(x-1)^2}$ trovare i flessi

$f'(x) = -2(x-1)e^{-(x-1)^2}$

$f''(x) = -2e^{-(x-1)^2} - 2(x-1)\cdot(-2(x-1))e^{-(x-1)^2}$

$f''(x) = e^{-(x-1)^2}\left[-2+4(x-1)^2\right]$

I p.ti di flesso sono dati da $4(x-1)^2 - 2 = 0$ $\quad (x-1)^2 = \frac{2}{4} = \frac{1}{2}$

$x - 1 = \pm\frac{1}{\sqrt{2}}$ $\quad x = 1 \pm \frac{1}{\sqrt{2}}$

ESERCIZIO Calcolare il $\lim_{x\to 0} \dfrac{e^{(\cos x - 1)} - 1}{x\sin(2x)} = *$

$$\begin{cases} \sin(\xi) = \xi + o(\xi) \quad \xi \to 0 \\ \sin(2x) = 2x + o(x) \end{cases}$$

$\sin(f(\xi)) \sim f(\xi)$

$x\sin(2x) = x(2x+o(x)) = 2x^2 + o(x^2)$

$$\boxed{\lim_{x\to 0} \frac{e^{(\cos x - 1)} - 1}{2x^2 + o(x^2)}} = \lim_{x\to 0} \frac{-\frac{x^2}{2} + \cancel{o(x^2)}}{2x^2 + \cancel{o(x^2)}} = -\frac{1}{4}$$

$e^{\xi} = 1 + \xi + \frac{\xi^2}{2} + \frac{\xi^3}{3!} \dots$ $\quad \cos(\xi) = 1 - \frac{\xi^2}{2!} + \frac{\xi^4}{4!} \dots$

$e^{(\cos x - 1)} = 1 + (\cos x - 1) + \frac{(\cos x - 1)^2}{2} + \dots$

$$e^{(\cos x - 1)} - 1 = (\cos x - 1) + \frac{(\cos x - 1)^2}{2} + \dots$$

$$\cos x - 1 = \left(-\frac{x^2}{2} + \frac{x^4}{4} \dots\right)$$

$$e^{(\cos x - 1)} - 1 = -\frac{x^2}{2} + o(x^2)$$

Sviluppo in serie geometrica

$$(1-z)(1+z+z^2+z^3+\dots+z^n) = (1+\cancel{z}+\cancel{\dots}+z^n) - (\cancel{z}+\cancel{z^2}+\dots+z^{n+1})$$
$$= 1 - z^{n+1}$$

$$\Rightarrow \frac{1-z^{n+1}}{1-z} = 1 + z + \dots + z^n$$

$$\frac{1}{1-z} = (1+\dots+z^n) + \left(\frac{z^{n+1}}{1-z}\right)$$

Questa quantità è un $o(z^n)$

$$\lim_{z \to 0} \frac{z^{n+1}}{1-z} \cdot \frac{1}{z^n} = 0$$

$\Rightarrow$ ho lo sviluppo di $\left(\frac{1}{1-x}\right)$ $x \to 0$

$$\boxed{\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + o(x^n)}$$ SERIE GEOMETRICA

$$\boxed{\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots (-1)^n x^n + o(x^n)}$$

ESERCIZIO « Determinare lo sviluppo di Maclaurin con resto di Peano della funzione $f(x) = \frac{1}{1-x+x^2}$

fino all'ordine 3 >>

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \frac{f'''(0)}{3!}x^3 + o(x^3)$$

- $$f'(x) = \frac{1-2x}{(1-x+x^2)^2} \qquad \boxed{f'(0) = 1}$$

$$f''(x) = \frac{-2(1-x+x^2)^{\cancel{2}} + 2\cancel{(1-x+x^2)}\cdot(1-2x)^2}{(1-x+x^2)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-2(1-x+x^2) + 2(1-2x)^2}{(1-x+x^2)^3} = \frac{2(4x^2+\cancel{1}-4x) - 2(\cancel{1}-x+x^2)}{(1-x+x^2)^3}$$

- $$f''(x) = \frac{6x^2-6x}{(1-x+x^2)^3} \qquad \boxed{f''(0) = 0}$$

- $$f'''(x) = 6\left[\frac{x^2-x}{(1-x+x^2)^3}\right]' = 6\,\frac{(2x-1)(1-x+x^2)^{\cancel{3}\,1} - 3\cancel{(1-x+x^2)}^2\cdot(2x-1)(x^2-x)}{(1-x+x^2)^{\cancel{6}\,4}}$$

$$f'''(x) = 6\cdot\frac{(2x-1)(1-x+x^2) - 3(2x-1)(x^2-x)}{(1-x+x^2)^4} \qquad \boxed{f'''(0) = -6}$$

$$f(x) = f(0) + f'(0)x + \cancel{f''(0)\frac{x^2}{2}} + f'''(0)\frac{x^3}{3!} + o(x^3)$$

$$\boxed{f(x) = 1 + x - x^3 + o(x^3)} \quad ++$$

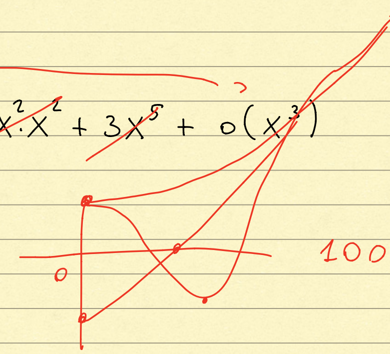
Usiamo lo sviluppo $\frac{1}{1-z} = 1 + z + z^2 + z^3 + o(z^3)$

$$f(x) = \frac{1}{1-\underbrace{(x-x^2)}_{=z}} = 1 + (x-x^2) + (x-x^2)^2 + (x-x^2)^3 + (\dots)^4$$

$$f(x) = 1 + x - x^2 + x^2 - 2x^3 + x^3 + o(x^3)$$

$$f(x) = 1 + x - \cancel{x^2} + \cancel{x^2} + \cancel{x^4} - 2x^3 + x^3 - \cancel{x^6} - \cancel{3x^2 \cdot x^2} + \cancel{3x^5} + o(x^3)$$

$$\boxed{f(x) = 1 + x - x^3 + o(x^3)} \quad ++$$



Esercizio

Verificare che la disuguaglianza $\boxed{e^{x^2} - e^{-x} + 100 \geq 100 > 0}$

$\forall x \in [0, +\infty)$

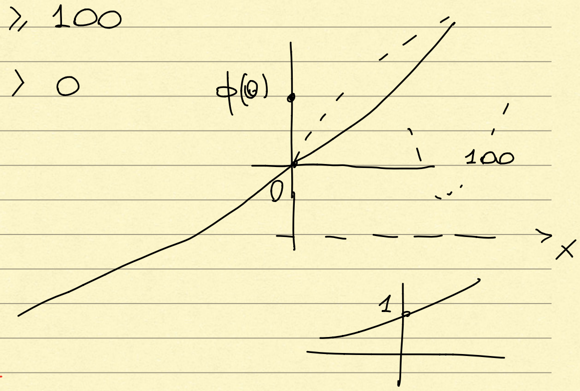
$\phi(x) = e^{x^2} - e^{-x} + 100$

$\phi(0) \geq 100$

$\phi'(x) > 0$

$\phi(0) = 100$

$\phi'(x) = 2xe^{x^2} + e^{-x} > 0 \quad \forall x \in \mathbb{R}$



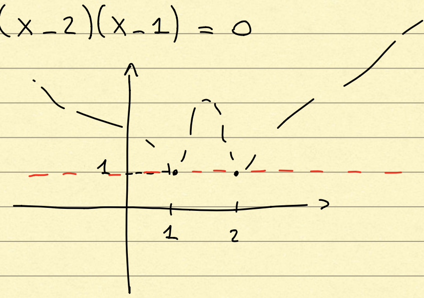
STUDIO DI FUNZIONE

Studiare la funzione $f(x) = e^{|x^2-3x+2|}$

$$\begin{cases} \mathrm{dom}(f) = \mathbb{R} \\ f(x) \geq 1 \end{cases}$$

$f(x) = 0 \iff x^2 - 3x + 2 = 0 \qquad (x-2)(x-1) = 0$

$f(x) = 0 \iff x = 1$ o $x = 2$



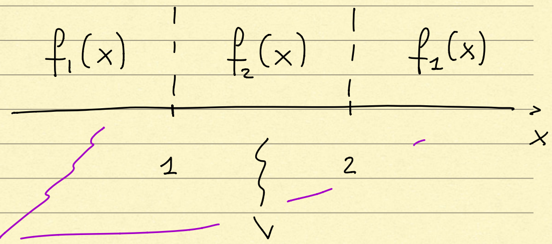
$$|x^2 - 3x + 2| = \begin{cases} x^2 - 3x + 2 & \text{se } \boxed{x^2 - 3x + 2 \geq 0} \\ -x^2 + 3x - 2 & \text{se } \boxed{x^2 - 3x + 2 \leq 0} \end{cases}$$

$P(x) = x^2 - 3x + 2$

per $x \geq 2$ o $x \leq 1$ $\quad x^2 - 3x + 2 \geq 0$

" $\quad x \in [1,2] \quad x^2 - 3x + 2 \leq 0$

$$|x^2 - 3x + 2| = \begin{cases} x^2 - 3x + 2 & \text{se } x \geq 2, \; x \leq 1 \\ -x^2 + 3x - 2 & \text{se } x \in [1,2] \end{cases}$$



$f_1(x) = e^{x^2 - 3x + 2} \qquad f_2(x) = e^{-x^2 + 3x - 2}$

Limiti

$$\lim_{x \to \pm\infty} f_1(x) = +\infty$$

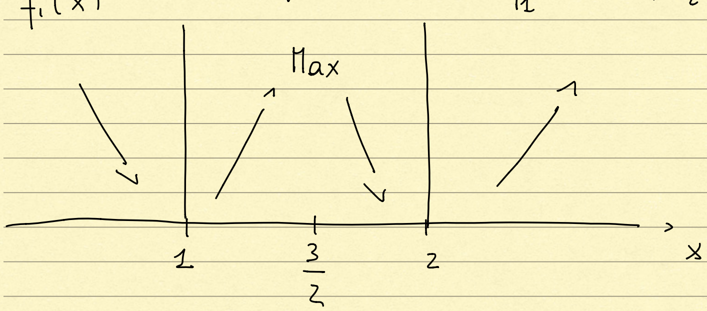
SEGNO

$f_1'(x) = e^{x^2 - 3x + 2} \cdot (2x - 3)$

$\begin{cases} f_1' \geq 0 & \text{per } x \geq 3/2 \\ f_1' \leq 0 & \text{per } x \leq 3/2 \end{cases}$ Vale in $x \geq 2$, $x \leq 1$

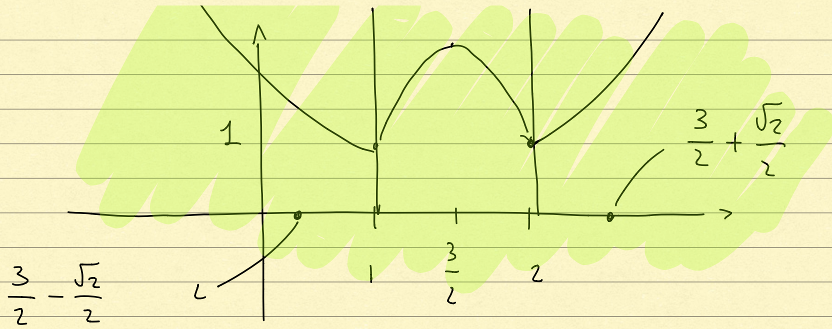
$f_2'(x) = e^{-x^2 + 3x - 2} \cdot (3 - 2x)$

$\Rightarrow \begin{cases} f_2' \geq 0 & \text{per } x \leq 3/2 \\ f_2' \leq 0 & \text{per } x \geq 3/2 \end{cases}$ Vale in $x \in [1,2]$



$f_1'(1^-) = -1 \quad \Big| \quad f_1'(2^+) = 1$

$f_2'(1^+) = +1 \quad \Big| \quad f_2'(2^-) = -1$



Derivate 2ª $\qquad f_1''(x) = e^{x^2-3x+2}\left[(2x-3)^2+2\right] > 0$

$$f_2''(x) = e^{-x^2+3x-2}\cdot\left[(3-2x)^2-2\right]$$

Dove si annulla $f_2''(x)=0$?? $\qquad (3-2x)^2 = 2$

$3-2x = \pm\sqrt{2} \qquad\qquad 2x = 3\pm\sqrt{2}$

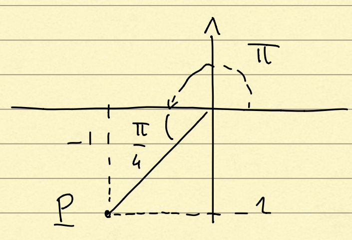
$f_2''(x) \le 0$ se $\qquad\qquad x = \frac{3}{2} \pm \frac{\sqrt{2}}{2}$

$$x \in \left[\frac{3}{2}-\frac{\sqrt{2}}{2} ; \frac{3}{2}+\frac{\sqrt{2}}{2}\right]$$

ESERCIZIO

Risolvere l'equazione complessa $\qquad e^z = \exp(z) = -1-i$

$w = \exp(z) \qquad \Rightarrow \qquad w = -1-i$



$P \equiv (-1,-1) \sim w = -1-i$

ω scritto in forma esponenziale mi da $\omega = \sqrt{2} \cdot e^{i\left(\frac{\pi}{4}+\pi\right)}$

$$\omega = \sqrt{2}\, e^{i\frac{5}{4}\pi} = \underbrace{\sqrt{2}\, e^{i\left(\frac{5}{4}\pi + 2k\pi\right)}} \qquad k \in \mathbb{Z}$$

$$\exp(z) = \sqrt{2}\, e^{i\left(\frac{5}{4}\pi + 2k\pi\right)}$$

$$\ln(e^{z}) = \ln(\exp(z)) = z = \ln\left[\sqrt{2}\, e^{i\left(\frac{5}{4}\pi + 2k\pi\right)}\right]$$

$$z = \ln(\sqrt{2}) + \ln\left(e^{i\left(\frac{5}{4}\pi + 2k\pi\right)}\right)$$

$$\boxed{z = \ln(\sqrt{2}) + i\left(\frac{5}{4}\pi + 2k\pi\right)} \qquad k \in \mathbb{Z}$$