STELLAR SPECTROSCOPY I.

MECHANICAL STATISTICS

Lezione VI- Fisica delle Galassie
Cap 8-9 Carrol & Ostlie

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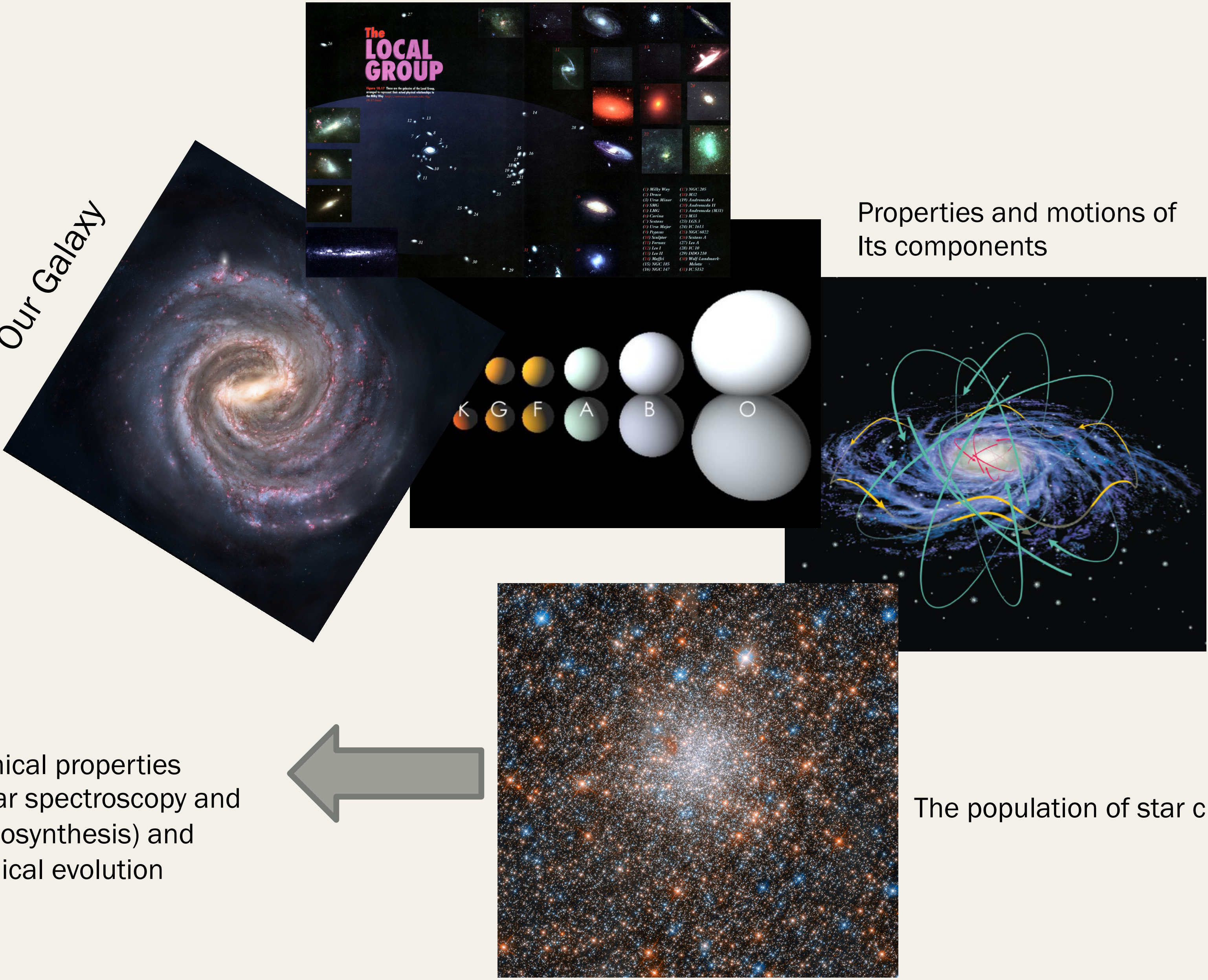
...and the Local Group



Our Galaxy

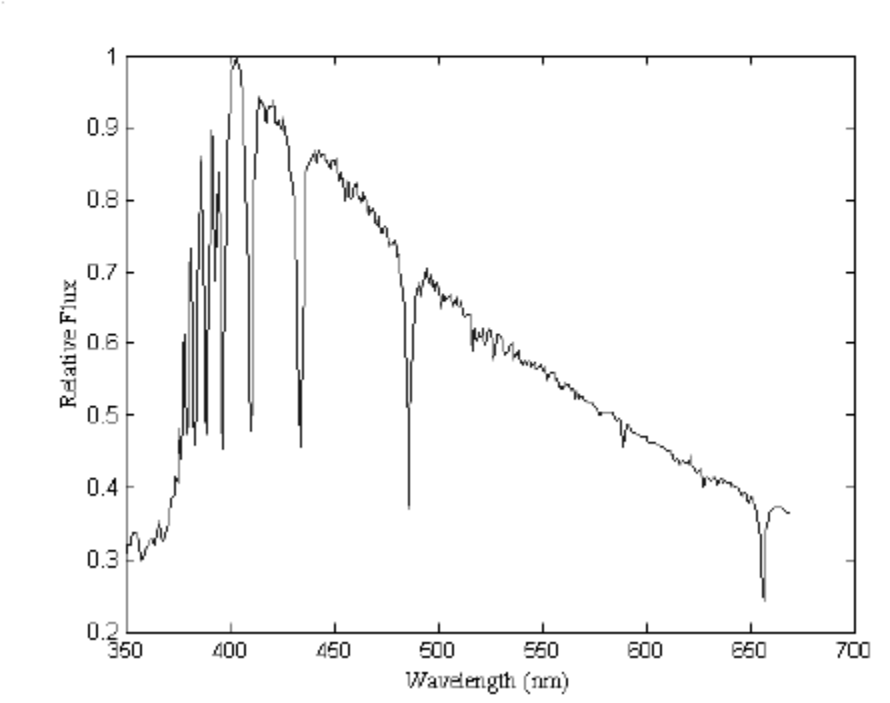
Properties and motions of
Its components

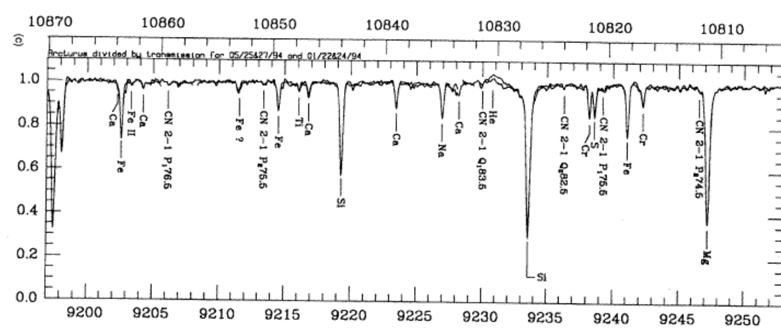
The population of star clusters

Chemical properties (stellar spectroscopy and nucleosynthesis) and chemical evolution

OUTLINE of the lecture:

The aim of this lecture is to learn how we can obtain information about the stellar parameters (temperature, surface gravity, global metallicity and individual element abundances) from a stellar spectrum

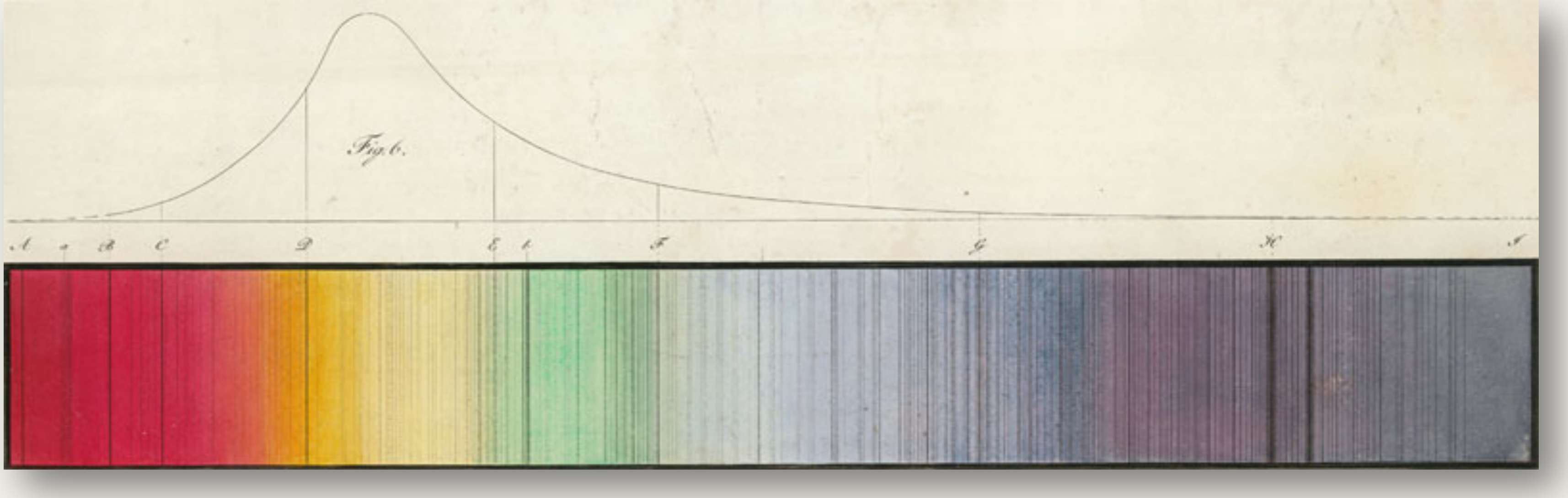




Globally it is a blackbody, but each small portion of the spectrum contains a large quantity of information

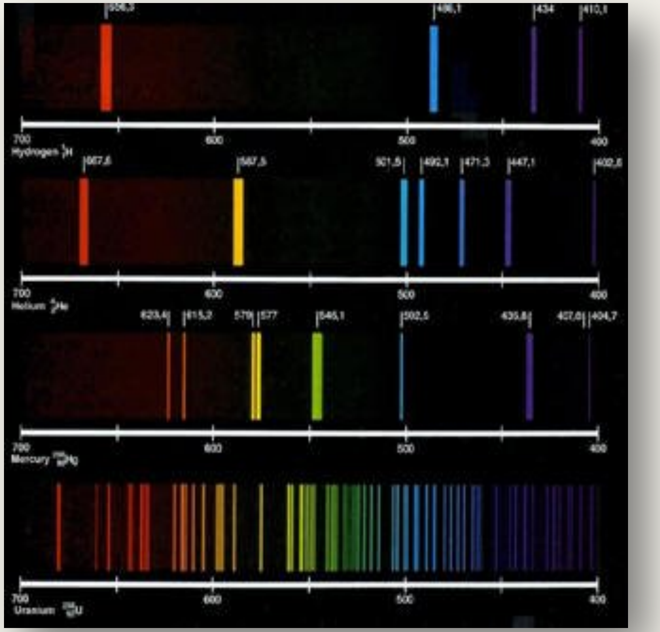
Stellar spectroscopy: a historical overview

- **In 1835,** Auguste Comte, a prominent French philosopher, stated that humans would **never be able to understand the chemical composition of stars.**
- He was soon proved wrong. In the latter half of the 19th century, astronomers began to embrace two new techniques—**spectroscopy** and photography.
- **Joseph Fraunhofer** mounted a prism in front of the objective lens of a small telescope, making for **the first time spectroscopy of the Sun and of bright stars.**
- He found that there were characteristic absorption lines present in the stellar spectra.



Spectra of stars: a historical overview

- A major advance was made in 1859 by Gustav Kirchhoff and Robert Bunsen
- In 1859, Bunsen reported to a colleague that Kirchhoff had made "a totally unexpected discovery." He had identified the cause of the dark lines seen in the solar spectra by Fraunhofer.
- When certain chemicals were heated characteristic bright lines appeared.
- In some cases these were at exactly the same positions in the spectrum as Fraunhofer's dark lines.
- The bright lines came from a hot gas, whereas the dark lines showed absorption of light in the cooler gas above the Sun's surface.



Spectra of stars: a historical overview

- The two scientists found that every chemical element produces a unique spectrum.

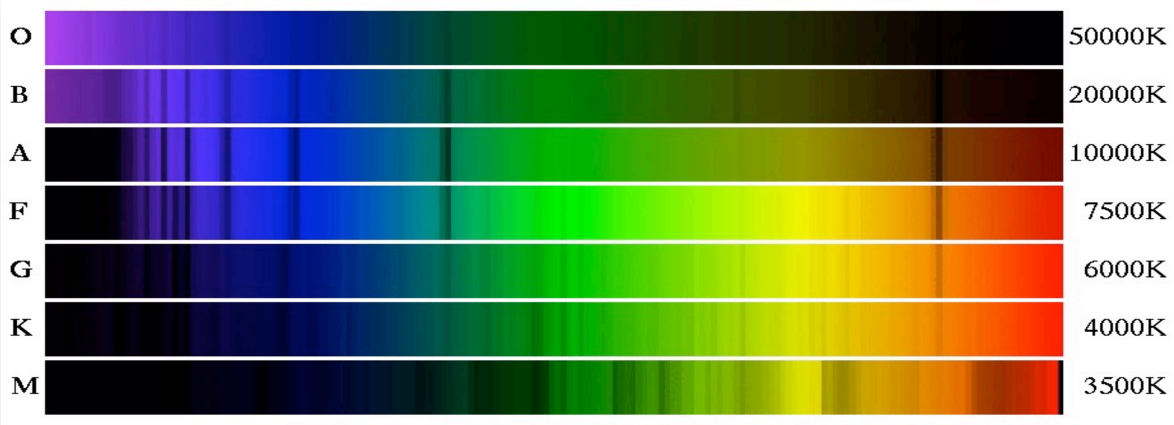
- **Kirchhoff and Bunsen recognized that this could be a powerful tool for "the determination of the chemical composition of the Sun and the fixed stars"**

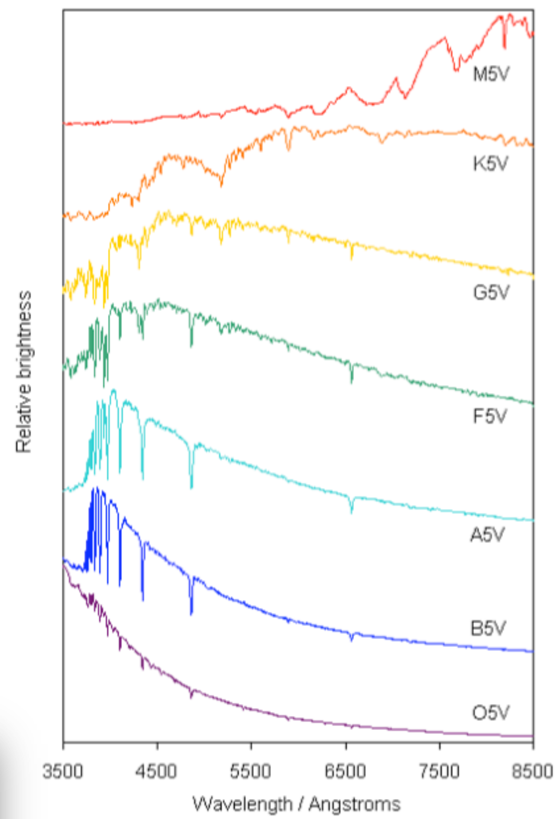
- Kirchhoff identified some **16 different chemical elements among the hundreds of lines observed in the Solar spectrum.**

Spectra of stars: an historical overview

The basis of the modern spectral classifications are funded on the work of Annie J. Cannon

- Cannon expanded the catalog to nine volumes with ~250,000 stars by 1924
- She developed a system of ten spectral types - **O, B, A, F, G, K, M, R, N, S** - that astronomers accepted in 1922.





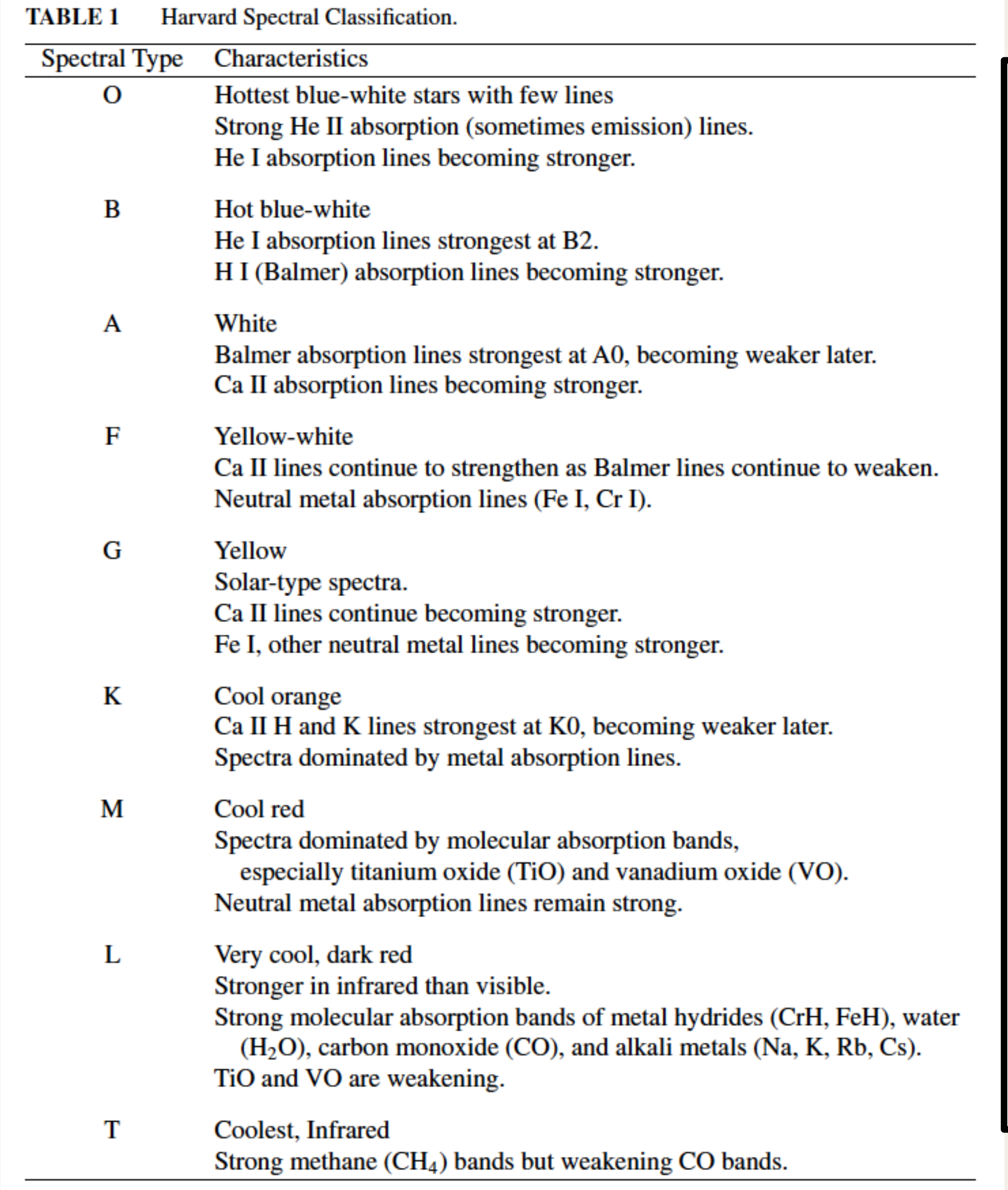


TABLE 1 Harvard Spectral Classification.

Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.

- Are the differences due to different composition?
- Or are they driven by different conditions of line formation?

Statistical mechanics:

Statistical properties of a system composed of many members.

For example, a gas that contains a huge number of particles with a large range of speeds and energies, as the gas that composes the stellar photospheres, can be studied through its statistical properties.

SPECTRA OF STARS: A HISTORICAL OVERVIEW

The composition of the Sun

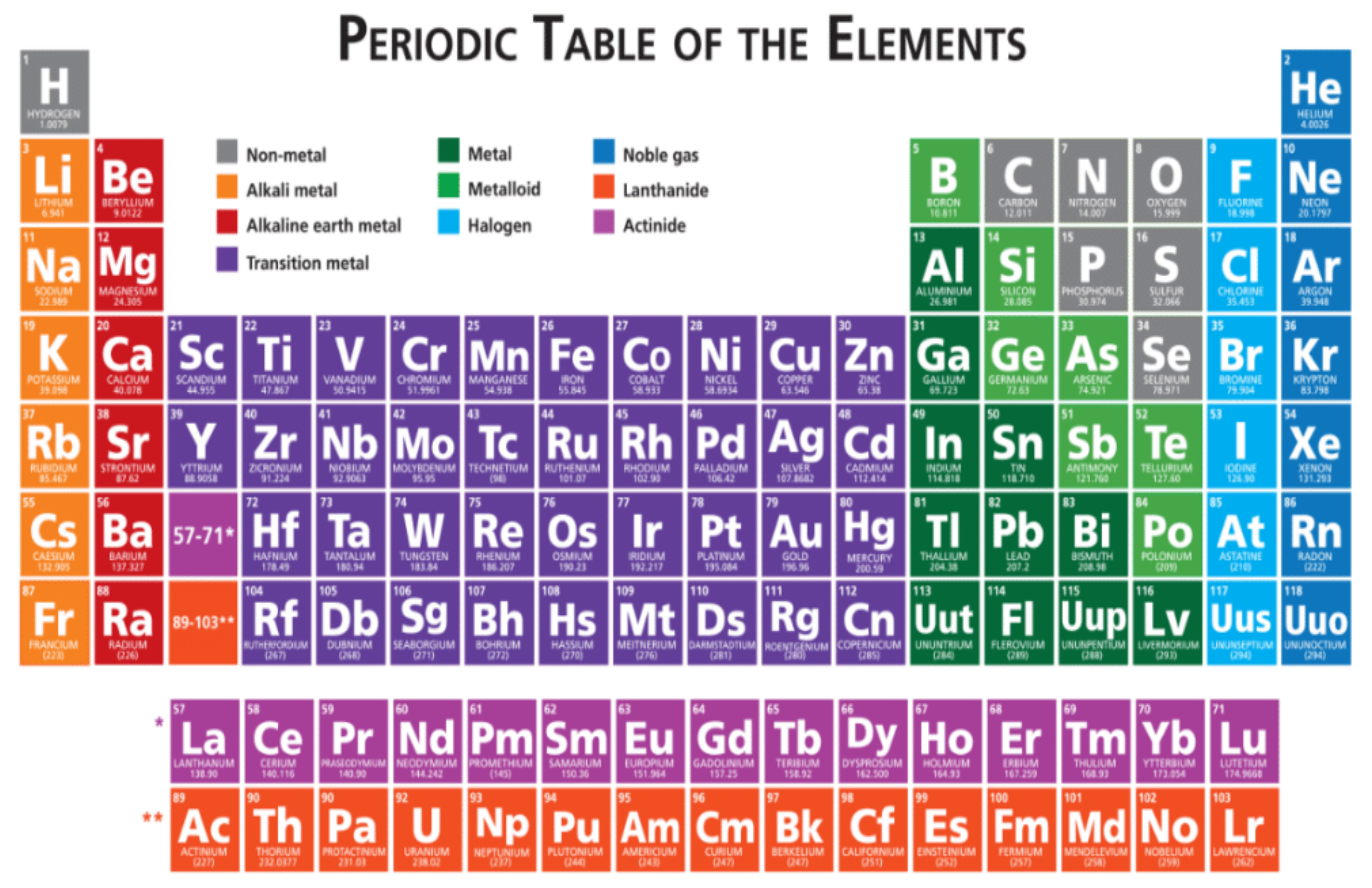
Cecilia Payne-Gaposchkin wrote her PhD Thesis in 1925.

Her major findings are:

- definitively establishing that the spectral sequence did correspond to quantifiable stellar temperatures.
- discovery that stars are made mainly of hydrogen and helium, contrary to the Earth composition

She was dissuaded from this conclusion by astronomer Henry Russel, who thought that stars would have the same composition as Earth. Russell conceded in 1929 that Payne was correct.

How is composed the Universe?



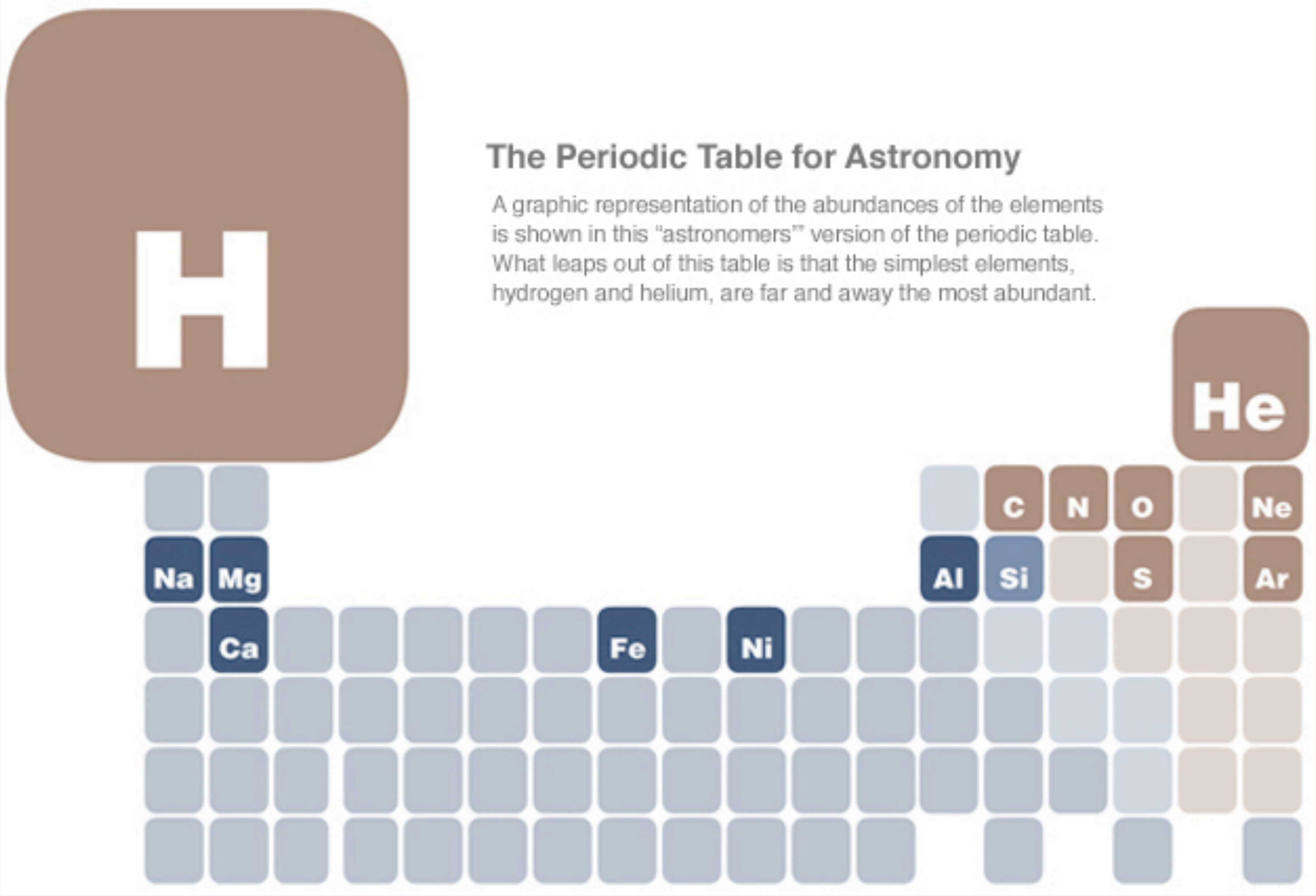
PERIODIC TABLE OF THE ELEMENTS

- ▨ Non-metal
- ▨ Alkali metal
- ▨ Alkaline earth metal
- ▨ Transition metal
- ▨ Metal
- ▨ Metalloid
- ▨ Halogen
- ▨ Noble gas
- ▨ Lanthanide
- ▨ Actinide

1 H HYDROGEN 1.0079																	2 He HELIUM 4.0026
3 Li LITHIUM 6.941	4 Be BERYLLIUM 9.0122											5 B BORON 10.811	6 C CARBON 12.011	7 N NITROGEN 14.007	8 O OXYGEN 15.999	9 F FLUORINE 18.998	10 Ne NEON 20.1797
11 Na SODIUM 22.989	12 Mg MAGNESIUM 24.305											13 Al ALUMINIUM 26.981	14 Si SILICON 28.085	15 P PHOSPHORUS 30.974	16 S SULFUR 32.066	17 Cl CHLORINE 35.453	18 Ar ARGON 39.948
19 K POTASSIUM 39.098	20 Ca CALCIUM 40.078	21 Sc SCANDIUM 44.955	22 Ti TITANIUM 47.867	23 V VANADIUM 50.9415	24 Cr CHROMIUM 51.9961	25 Mn MANGANESE 54.938	26 Fe IRON 55.845	27 Co COBALT 58.933	28 Ni NICKEL 58.6934	29 Cu COPPER 63.546	30 Zn ZINC 65.38	31 Ga GALLIUM 69.723	32 Ge GERMANIUM 72.63	33 As ARSENIC 74.921	34 Se SELENIUM 78.971	35 Br BROMINE 79.904	36 Kr KRYPTON 83.798
37 Rb RUBIDIUM 85.467	38 Sr STRONTIUM 87.62	39 Y YTTRIUM 88.9058	40 Zr ZICRONIUM 91.224	41 Nb NIOBIUM 92.9063	42 Mo MOLYBDENUM 95.95	43 Tc TECHNETIUM (98)	44 Ru RUTHENIUM 101.07	45 Rh RHODIUM 102.90	46 Pd PALLADIUM 106.42	47 Ag SILVER 107.8682	48 Cd CADMIUM 112.414	49 In INDIUM 114.818	50 Sn TIN 118.710	51 Sb ANTIMONY 121.760	52 Te TELLURIUM 127.60	53 I IODINE 126.90	54 Xe XENON 131.293
55 Cs CAESIUM 132.905	56 Ba BARIUM 137.327	57-71*	72 Hf HAFNIUM 178.49	73 Ta TANTALUM 180.94	74 W TUNGSTEN 183.84	75 Re RHENIUM 186.207	76 Os OSMIUM 190.23	77 Ir IRIDIUM 192.217	78 Pt PLATINUM 195.084	79 Au GOLD 196.96	80 Hg MERCURY 200.59	81 Tl THALLIUM 204.38	82 Pb LEAD 207.2	83 Bi BISMUTH 208.98	84 Po POLONIUM (209)	85 At ASTATINE (210)	86 Rn RADON (222)
87 Fr FRANCIUM (223)	88 Ra RADIUM (226)	89-103**	104 Rf RUTHERFORDIUM (267)	105 Db DUBNIUM (268)	106 Sg SEABORGIUM (271)	107 Bh BOHRIUM (272)	108 Hs HASSIUM (270)	109 Mt MEITNERIUM (276)	110 Ds DARMSTADTIUM (281)	111 Rg ROENTGENIUM (280)	112 Cn COPERNICIUM (285)	113 Uut UNUNTRIUM (284)	114 Fl FLEROVIUM (289)	115 Uup UNUNPENTIUM (288)	116 Lv LIVERMORIUM (293)	117 Uus UNUNSEPTIUM (294)	118 Uuo UNUNOCTIUM (294)

*	57 La LANTHANUM 138.90	58 Ce CERIUM 140.116	59 Pr PRASEODYMIUM 140.90	60 Nd NEODYMIUM 144.242	61 Pm PROMETHIUM (145)	62 Sm SAMARIUM 150.36	63 Eu EUROPIUM 151.964	64 Gd GADOLINIUM 157.25	65 Tb TERIBIUM 158.92	66 Dy DYSPROSIUM 162.500	67 Ho HOLMIUM 164.93	68 Er ERBIUM 167.259	69 Tm THULIUM 168.93	70 Yb YTTERBIUM 173.054	71 Lu LUTETIUM 174.9668
**	89 Ac ACTINIUM (227)	90 Th THORIUM 232.0377	90 Pa PROTACTINIUM 231.03	92 U URANIUM 238.02	93 Np NEPTUNIUM (237)	94 Pu PLUTONIUM (244)	95 Am AMERICIUM (243)	96 Cm CURIUM (247)	97 Bk BERKELIUM (247)	98 Cf CALIFORNIUM (251)	99 Es EINSTEINIUM (252)	100 Fm FERMIUM (257)	101 Md MENDELEVIUM (258)	102 No NOBELIUM (259)	103 Lr LAWRENCIUM (262)

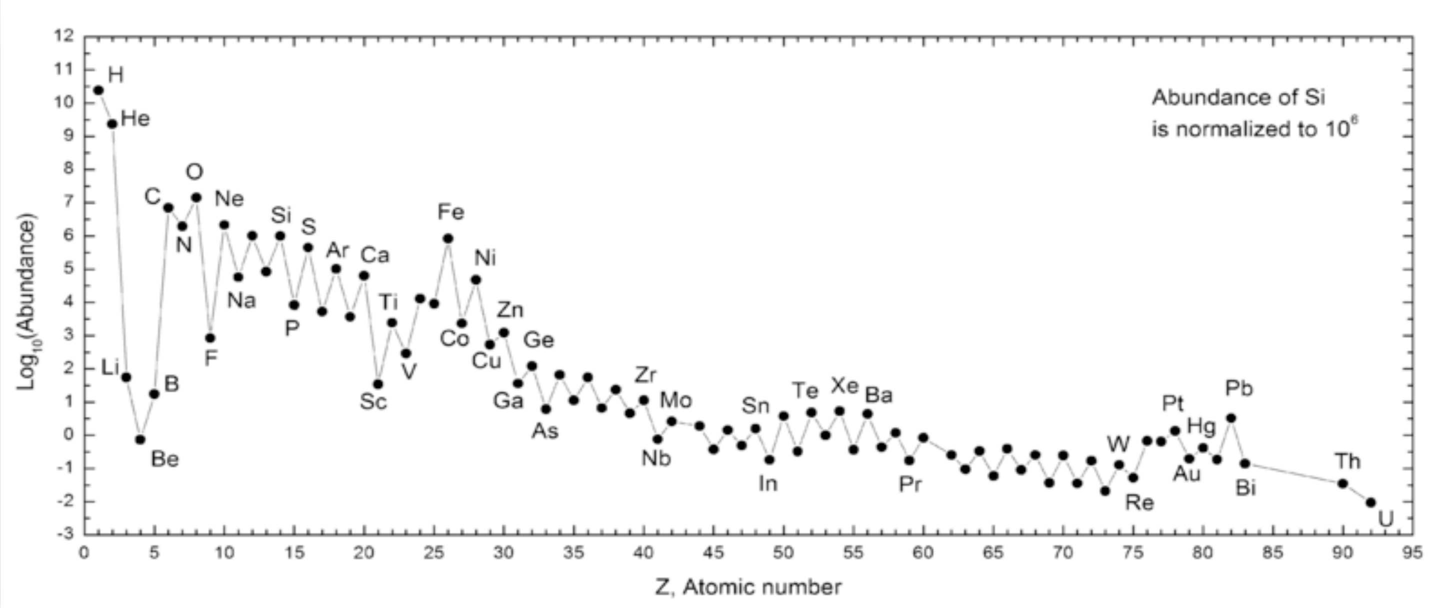
How is composed the Universe?



The Periodic Table for Astronomy

A graphic representation of the abundances of the elements is shown in this "astronomers'" version of the periodic table. What leaps out of this table is that the simplest elements, hydrogen and helium, are far and away the most abundant.

How is composed the Universe?

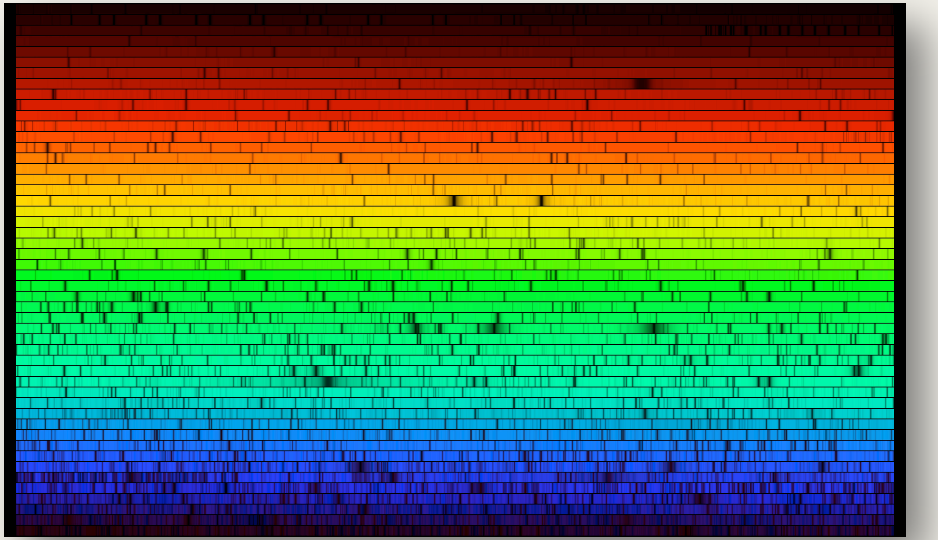


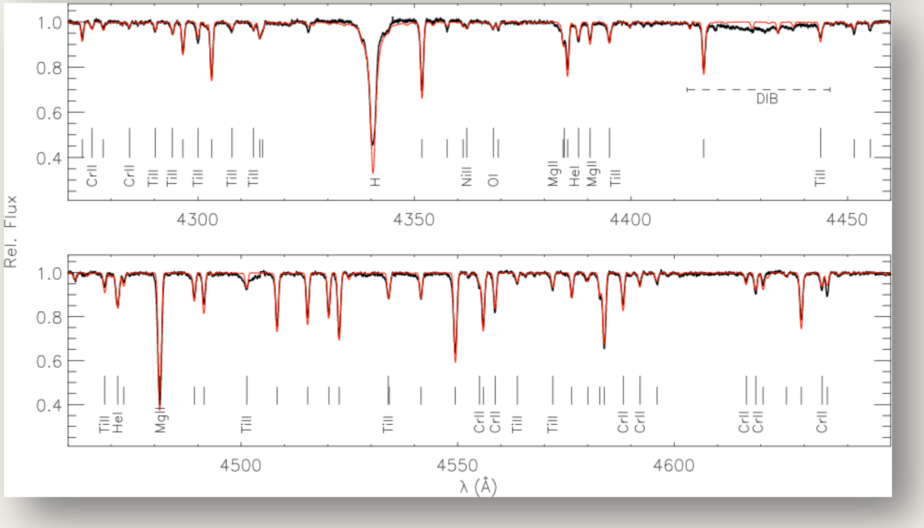
Elements in the cosmos:

- → Decline in abundances with atomic number
- → "odd-even effect” → elements that are even multiples of a He nucleus are enhanced → result of synthesis by alpha particle capture
- → Drop in abundance for the light nuclei Li, Be, and B → instability of nuclei of mass 5, making the early creation of these elements in the Universe rare, as well as the easy destruction of these elements in stars,
- → Elements around iron (V, Cr, Mn, Fe, Co, Ni) show enhanced abundance, forming an "iron peak". These elements have the highest binding energy, which is the energy required to remove a nucleon.

From ASTR 5610 (Majewski) Lecture Notes

Stellar parameters and abundances





The abundances of most of these elements are present in stellar photospheres.

From the analysis of the stellar spectrum, we can derive:

- Effective temperature
- Surface gravity (pressure)
- Chemical composition of the stellar atmosphere

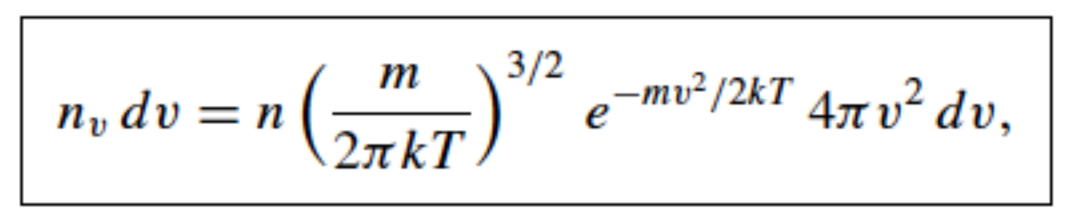
For details Gray (chapters 5–14) or Carroll & Ostlie (1996, chapters 9– 10)

Statistical Mechanics

This branch of physics studies the statistical properties of a system composed of many members. For example, a gas can contain a huge number of particles with a large range of speeds and energies.

Although in practice it would be impossible to calculate the detailed behavior of any single particle, the gas as a whole does have certain well-defined properties, such as its temperature, pressure, and density.

Maxwell–Boltzmann velocity distribution function



$$\boxed{n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\,4\pi v^2\,dv,}$$

→ Gives the number of gas particles per unit volume having speeds between v and v + dv

→ Valid when the system of particles is assumed to have reached thermodynamical equilibrium

Shape of the distribution and most probable speed

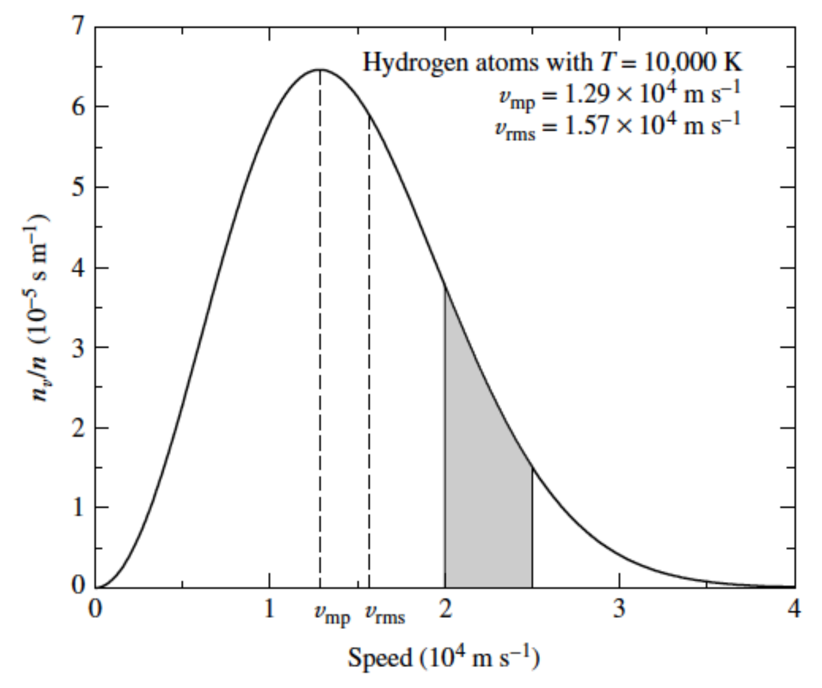
The exponent of the distribution function is the ratio of a gas particle's kinetic energy, ½ m v^2 , to the characteristic thermal energy, kT .

$$v_{\rm mp} = \sqrt{\frac{2kT}{m}}.$$

the distribution peaks when these energies are equal

Maxwell–Boltzmann distribution function for hydrogen atoms at a temperature of 10,000 K.

→ Integrating the distribution in a velocity interval we have the number of atoms as a function of the temperature of the system



The Boltzmann equation

The atoms of a gas gain and lose energy as they collide.

The distribution in the speeds of the impacting atoms produces a definite distribution of the electrons among the atomic orbitals.

This distribution of electrons is governed by a fundamental result of statistical mechanics:

Orbitals of higher energy are less likely occupied by electrons.

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b-E_a)/kT}$$

→ P probability of the system to have energy E, and quantum numbers s (n, l, m_l, m_s).

→ $e^{-E/kT}$ is called the Boltzmann factor

→ Energy are expressed in eV

The Boltzmann factor

The Boltzmann factor plays such a fundamental role in the study of statistical mechanics

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b-E_a)/kT}$$

→ If $E_b > E_a$ and $T \rightarrow 0$

→ $-(E_b - E_a)/kT \rightarrow -\infty$

→ $P(s_b)/P(s_a) \rightarrow 0$

→ If we lower the temperature, there isn't any thermal energy available to raise the energy of an atom to a higher level.

→ If $E_b > E_a$ and $T \rightarrow \infty$

→ $-(E_b - E_a)/kT \rightarrow 0$

→ $P(s_b)/P(s_a) \rightarrow 1$

→ If we increase the temperature, any energy level can be reached → but we do not under-populate the lower energy level

Degenerate states and statistical weights

Energy levels of the system may be degenerate, with more than one quantum state having the same energy.

To account properly for the number of states that have a given energy, define **g to be the number of states with energy E**

→ g is called statistical weight

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b - E_a)/kT}$$

The ratio of the probability $P(E_b)$ that the system will be found in any of the g_b degenerate states with energy E_b to the probability $P(E_a)$ that the system is in any of the g_a degenerate states with energy E_a

The Boltzmann equation

Stellar atmospheres contain a vast number of atoms, so the ratio of probabilities is indistinguishable from the ratio of the number of atoms.

$$\boxed{\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}.}$$

The equation of Boltzmann gives us the ratio between the numbers **of atom of a given element** in a given state of ionization (neutral, single ionized, etc.) **in two specific energy levels E_a and E_b, as a function of the system temperature.**

The Boltzmann equation: all levels

- Boltzmann's Law **[EXCITED STATES]:**

This equation tells us **the probability of an atom to be in a given excited state.**

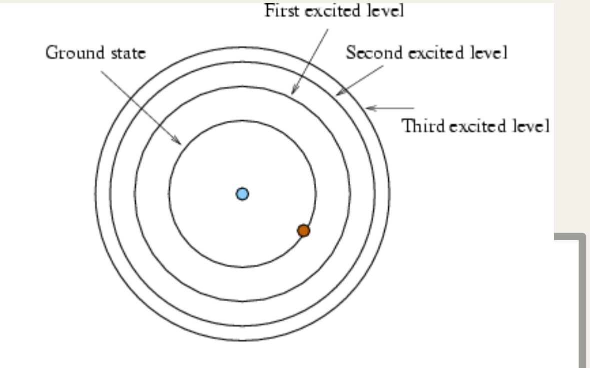
The relative population of excited states in a gas in thermodynamic equilibrium is given by the Boltzmann Excitation Distribution. The number of atoms of energy level n per unit volume N_n is proportional to the total number of atoms (N) of the same species:

$$\frac{N_n}{N} = \frac{g_n}{\mathcal{U}_n(T)} \exp\left(-\frac{E_n}{kT}\right)$$

where g_n is the statistical weight of the nth level, χ_n is the excitation potential of the nth level and $U_n(T)$ is the partition function of the particle in a gas of temperature T

Example: Balmer lines

For a gas of neutral hydrogen atoms, at what temperature will equal numbers of atoms have electrons in the ground state (n = 1) and in the first excited state (n = 2)?



Hydrogen atom, with the four lowest energy levels marked

$N_1/N_2=1$ → same number

$g_n=2\,n^2$ → $g_1=2$ for n=1 and $g_2=8$ for n=2 → number of degenerate states for Hydrogen

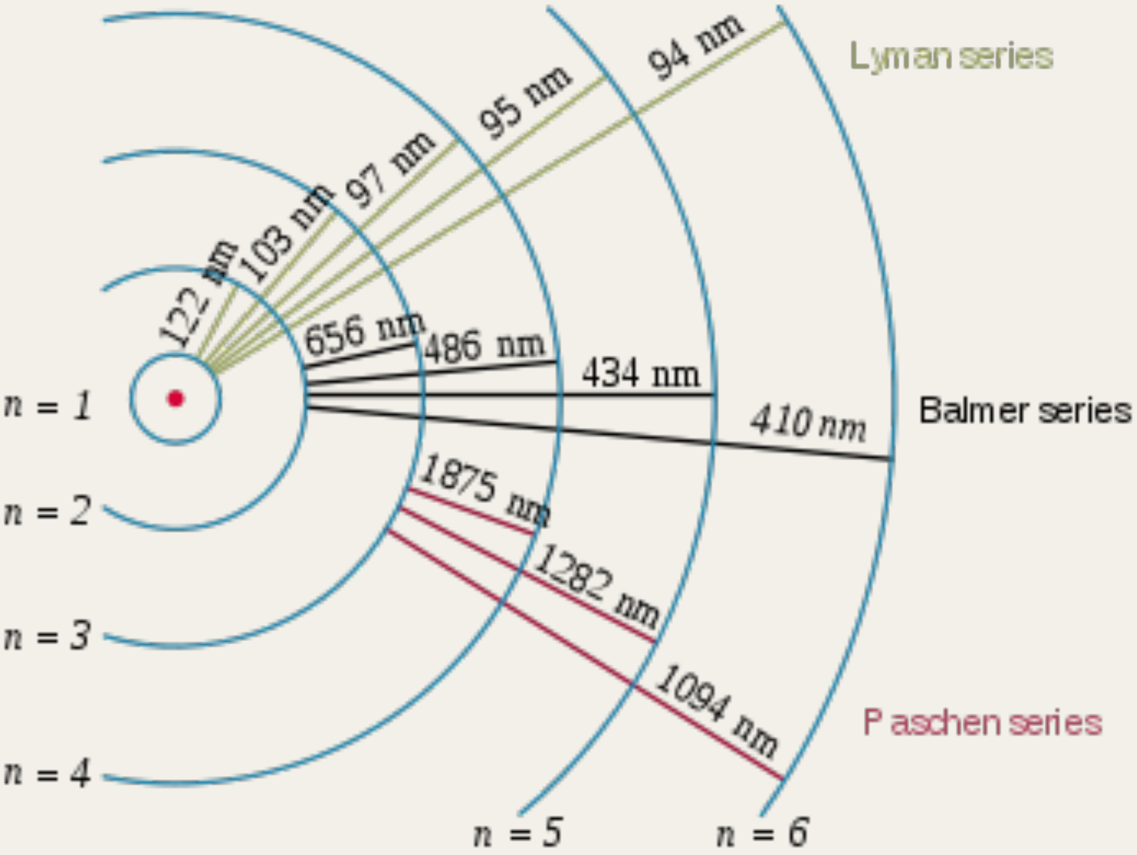
$1=N_1/N_2=4 \times e^{-[(-13.6\text{ eV}/4)-(-13.6\text{ eV}/1))]/kT}$ → Boltzmann's equation

(considering the energy levels of the Hydrogen atom $E = -E_0/n^2$, where $E_0 = 13.6$ eV)

$T= 10.2\text{ eV}/(k\ln(4)) = 8.5 \times 10^4$ K (where $k = 8.617\text{x}10{-}5\text{ eV K}^{-1}$).

Very high temperatures are required for a significant number of hydrogen atoms to have electrons in the first excited state.

Example: Balmer lines



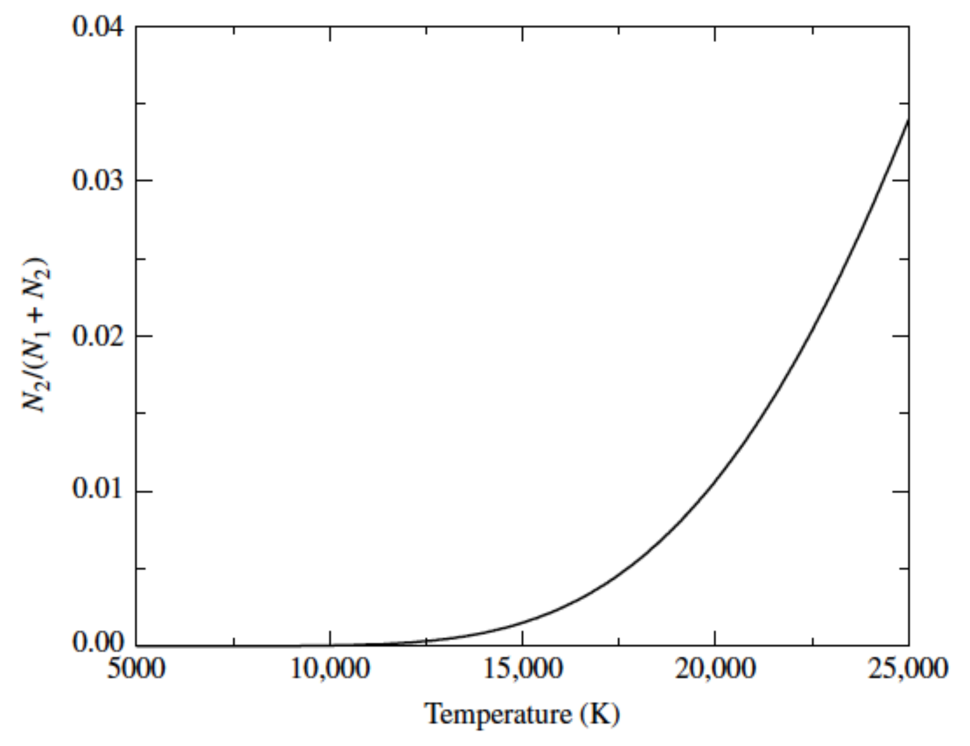
Example: Balmer lines

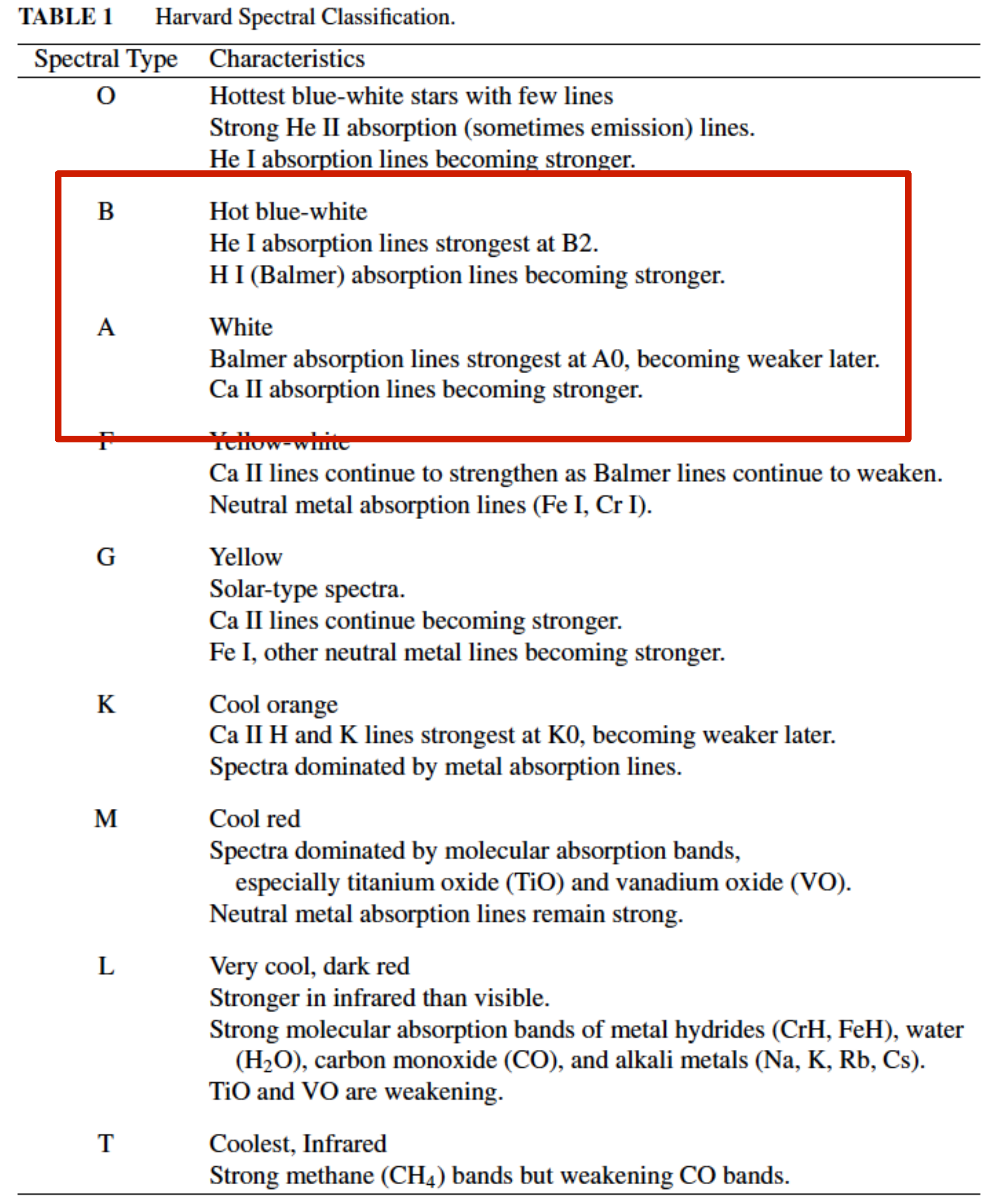
The Balmer absorption lines are produced by electrons in hydrogen atoms making an upward transition from the n= 2 orbital.

However, why do the Balmer lines reach their maximum intensity at a much lower temperature of ~9500 K and they are not present in the hottest stars?

Occupancy of the ground and first excited states as a function of temperature

→ Increasing the temperature does not increase the intensity of the Balmer lines





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The Saha equation: the partition function

The answer is related to the relative number of atoms in **different stages of ionization**

→ **If we increase the temperature, we give to the atom enough energy to ionize and to lose its electron**

→ For instance, the energy necessary to ionize H is 13.6 eV (from the ground state)

If the atom and ion are not in the ground state, we have to take an average over the orbital energies to allow for the possible partitioning of the atom's electrons among its orbitals.

E_J excited states

E_1 Ground state

$$Z = \sum_{j=1}^{\infty} g_j \, e^{-(E_j - E_1)/kT} .$$

Degenercy

The **partition function is the weighted sum of the number of ways the atom can arrange its electrons with the same energy**, with more energetic (and therefore less likely) configurations receiving less weight from the Boltzmann factor when the sum is taken.

The Saha equation:

Considering the partition function of the atom Z_i and of the ion Z_{i+1}, we have the Saha equation:

$$\boxed{\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.}$$

- n_e: electron density → higher densities give lower N_{i+1} because there is a higher probability of recombination
- 2: takes into account of the spin values of the free electron
- $(2\,\pi\,m_e\,k\,T\,/h^2)$: related to the free electron
- $\boldsymbol{x}_{\mathrm{I}}$: the ionization energy needed to remove an electron from an atom

The Saha equation: electron pressure

→ We can express the electron number density as a function of electron pressure using the law of ideal gas

$$P_e = n_e k T .$$

If we use the partition functions Zi and Zi+1 for the atom in its initial and final stages of ionization, the **ratio of the number of atoms in stage (i + 1) to the number of atoms in stage i** is

$$\boxed{\frac{N_{i+1}}{N_i} = \frac{2kT Z_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e k T}{h^2} \right)^{3/2} e^{-\chi_i / kT} .}$$

The electron pressure P_e term in that equation explains why stellar spectra are sensitive to pressure.

Example: fraction of ionised H

Consider the degree of ionization in a stellar atmosphere that is assumed to be composed of pure hydrogen.

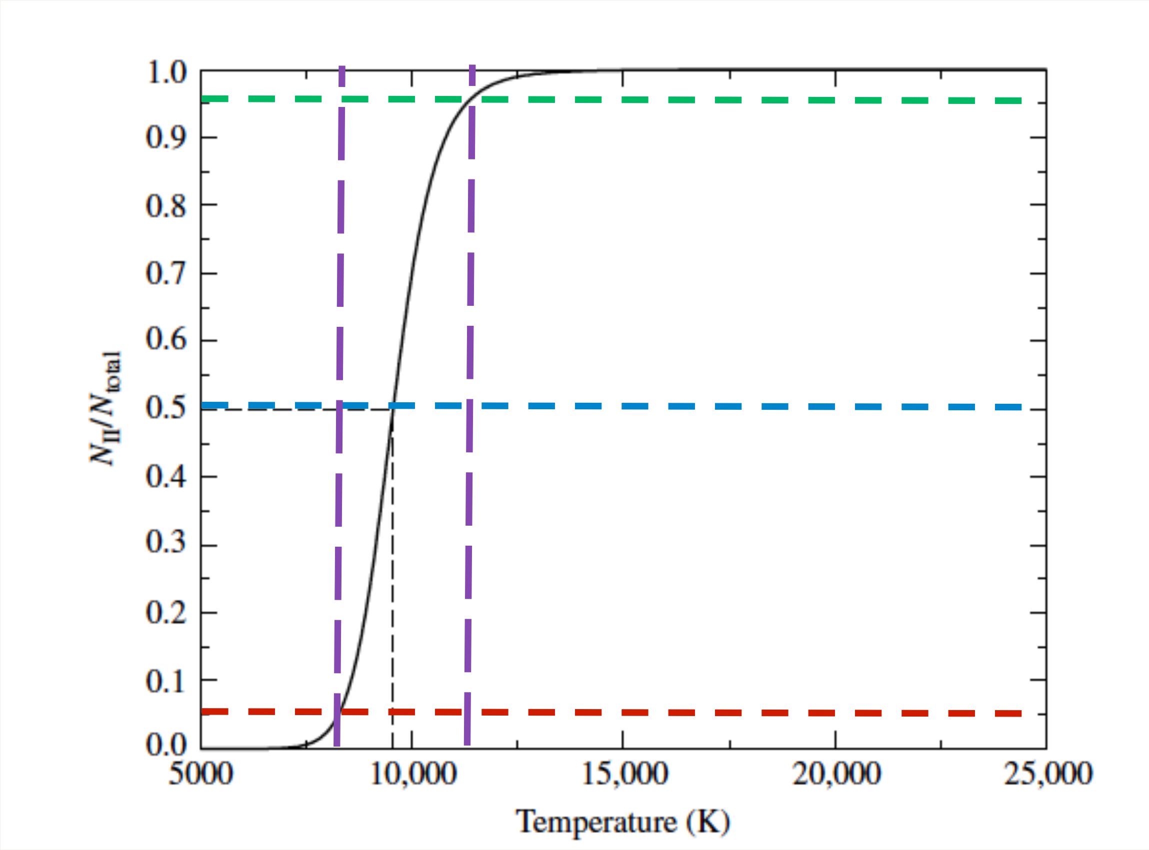
Assume for simplicity that the electron pressure is a constant $P_e = 20 \text{ N m}^{-2}$.

Using the Saha equation we can calculate the fraction of ionized atoms as a function of T (range between 5000 K and 25000 K)

- → Derive the partition functions:
  - → $Z_{II}=1$ (it is just a proton) -> ion
  - → $Z_I=2$ (since almost all the H atoms are in the ground state)→ neutral atom $g=n^2$
- → Solve the Saha equation:

$$\frac{N_{II}}{N_{\text{total}}} = \frac{N_{II}}{N_I + N_{II}} = \frac{N_{II}/N_I}{1 + N_{II}/N_I}$$

Example: fraction of ionised H



- At about 8300 K, 5% of the atoms have become ionized.
- 50% is ionized at a temperature of 9600 K
- At T ~11,300 K, 95% of the hydrogen is in the form of H II.
- Thus the ionization of hydrogen takes place within a temperature interval of approximately 3000 K.

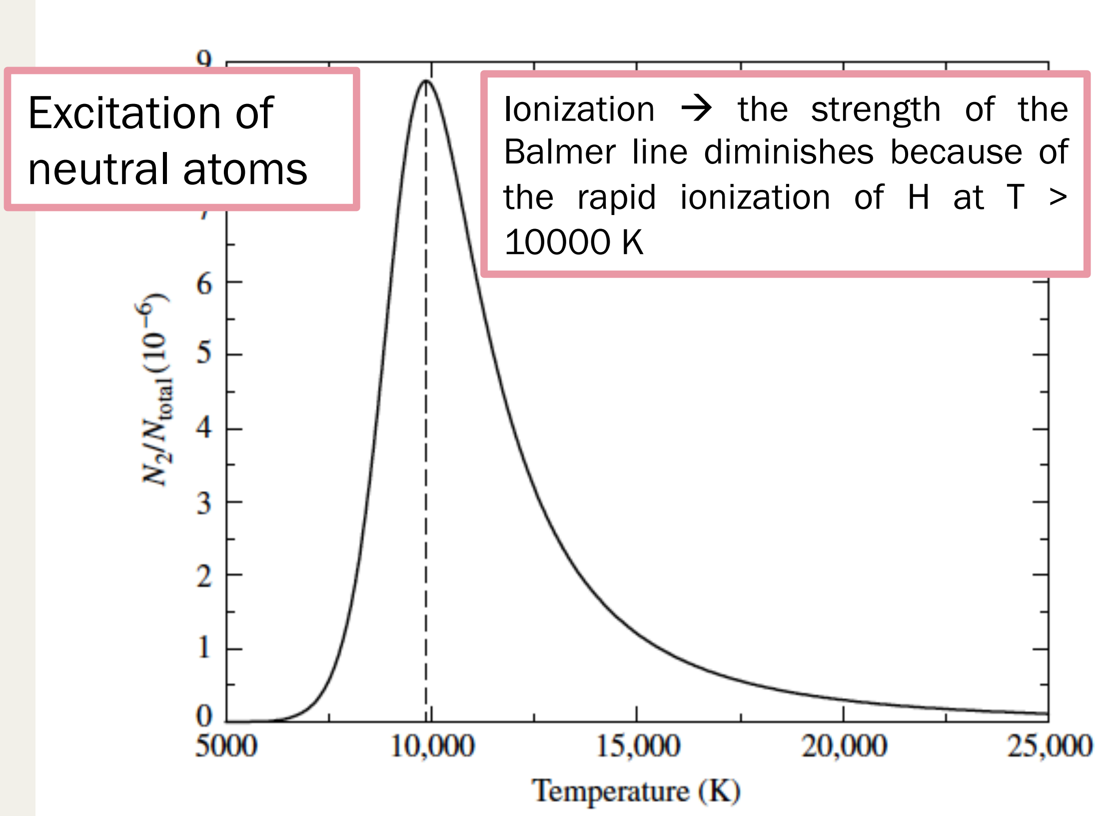
Example: maximum Balmer line intensity

$$\frac{N_2}{N_{\text{total}}} = \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_{\text{I}}}{N_{\text{total}}}\right) = \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_{\text{II}}/N_{\text{I}}}\right).$$

Boltzmann

Saha

- In the approximation $N_1+N_2 \sim N_{\text{I}}$ (all atoms are in the ground state or first excited state)

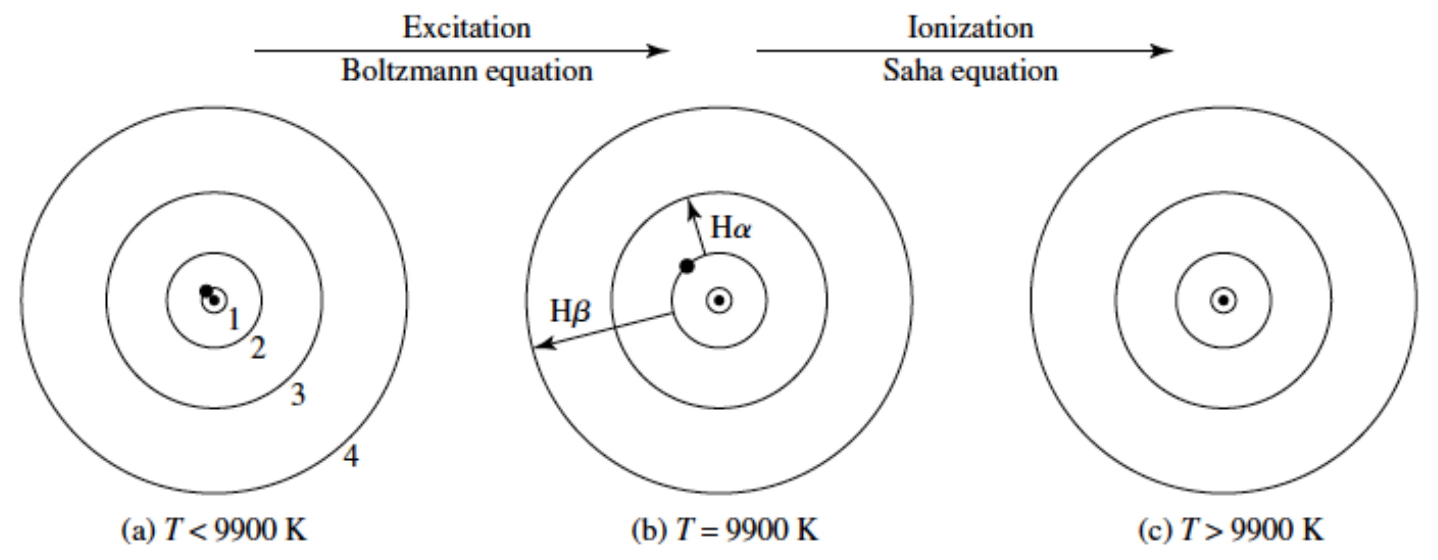


Excitation of neutral atoms

Ionization → the strength of the Balmer line diminishes because of the rapid ionization of H at T > 10000 K

- Combining the Boltzmann and Saha equations for the n = 2 energy level of hydrogen
- The strength of the Balmer lines depends on N2/Ntotal , the fraction of all hydrogen atoms that are in the first excited state.

Example: maximum Balmer line intensity



The position of the electron in the hydrogen atom at different temperatures.

(a) At low temperature the electron is in the ground state → no Balmer lines because there are no electron in the first excited stare

(b) Balmer absorption lines are produced because the electron is excited and it is in orbitals with higher energy. It moves from them to the first excited state, producing Balmer lines

(c) The atom has been ionized: the electron leaves the atom. No more Balmer lines.

Stellar atmospheres

→ The stellar light comes from the atmosphere of the star, i.e. the layers of gas overlying the opaque interior.

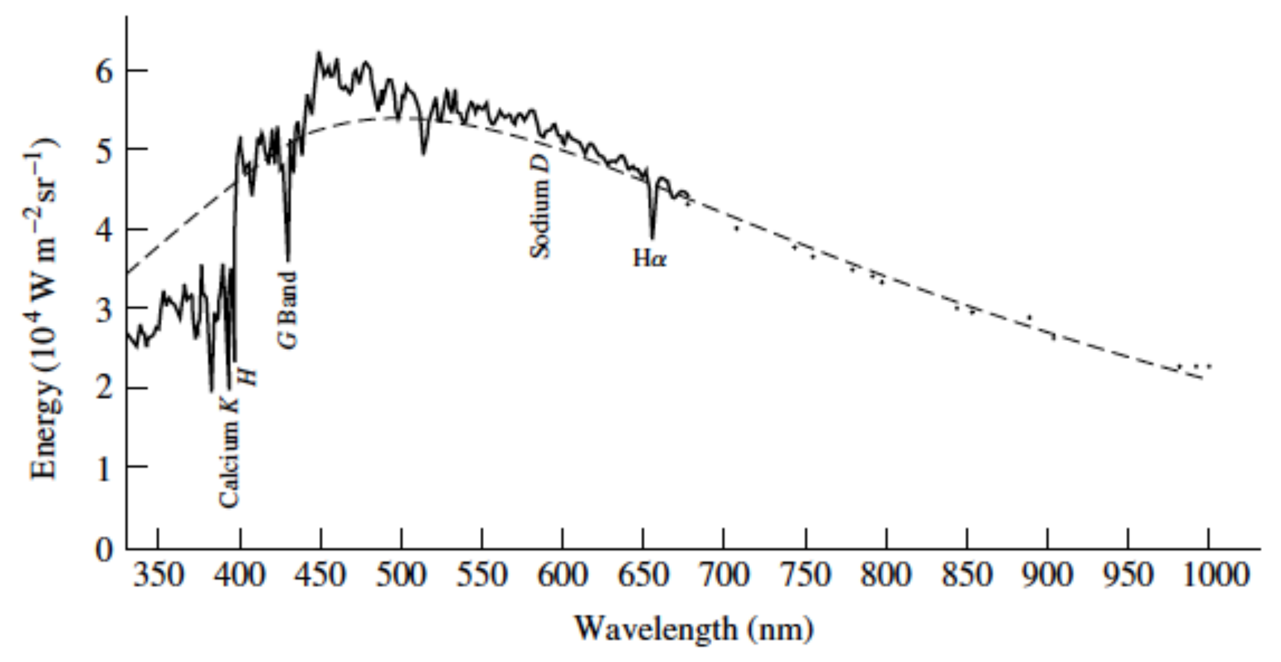
→ The observed photos release the energy produced by the thermonuclear reactions, gravitational contraction, and cooling in the central parts of the star.

→ The **temperature, density, and composition of the atmospheric layers** from which these photons escape determine the features of the star’s spectrum.

To interpret the observed spectral lines properly, we must describe how light travels through the gas composing a star.

As a first approximation….

→ Stars are black-bodies…



The Solar spectrum deviates from the blackbody Planck function because of the absorption of several lines of the so-called metals, which produce the line blanketing

Thermodynamic Equilibrium:

what do we mean when we talk about temperature?

→ Stars are black-bodies…with a characteristic temperature….

→ The effective temperature in the Stefan-Boltzmann law

Global temperature

→ The excitation temperature in the Boltzmann equation.

→ The ionization temperature in the Saha equation

→ The kinetic temperature in the Maxwell–Boltzmann distribution

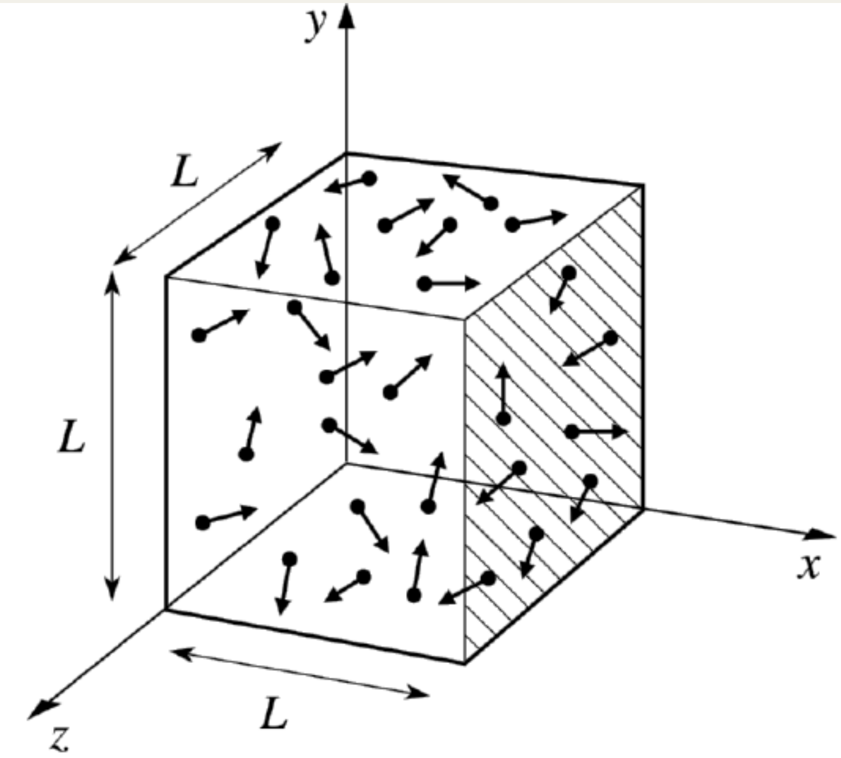
Local temperatures

- → Vary with gas conditions
- → Location in the star

Thermodynamic equilibrium:

When the confined gas particles and blackbody radiation reach an equilibrium, individually and with each other, and they can be described by a single well-defined temperature.

Thermodynamic Equilibrium:
what do we mean when we talk about temperature?



All local temperatures

→ Different temperatures are all the same for a gas confined in an 'ideal' box

→ The blackbody radiation and the particles of the gas are in equilibrium

→ They can be described with a single temperature

→ Every process has the same rate of the opposite process

Thermodynamic Equilibrium:
what do we mean when we talk about temperature?

Stars are not in global thermodynamic equilibrium

→ Net outward flow of energy occurs through the star

→ The temperature of the star varies with the location

→ Exchange of gas and photons from different regions of the star

→ Collisions between particles and interaction with radiation field

It is not easy to define a single temperature to describe the different process regulated, i.e., by the Boltzmann and Saha equations…..

Temperature and Local Thermodynamic Equilibrium

The idealized case of a **single temperature** can still be employed **if the distance over which the temperature changes significantly is large compared with the distances traveled by the particles and photons between collisions** (their mean free paths).

Local Thermodynamic Equilibrium (LTE)

The particles and the photons cannot escape the local environment and so are effectively confined to a limited volume of nearly constant temperature.

When the Local Thermodynamic Equilibrium is valid?

The Solar photosphere is the 'surface' layer of the Sun where the photons can escape.

To understand which are the conditions of LTE for the Sun, we can compare:

→ The characteristics distance for the variation of the temperature, the scale height, $H_T = T/(dT/dr)$

→ The free path of the main component of the Sun, the hydrogen atom (mean distance between two collisions)

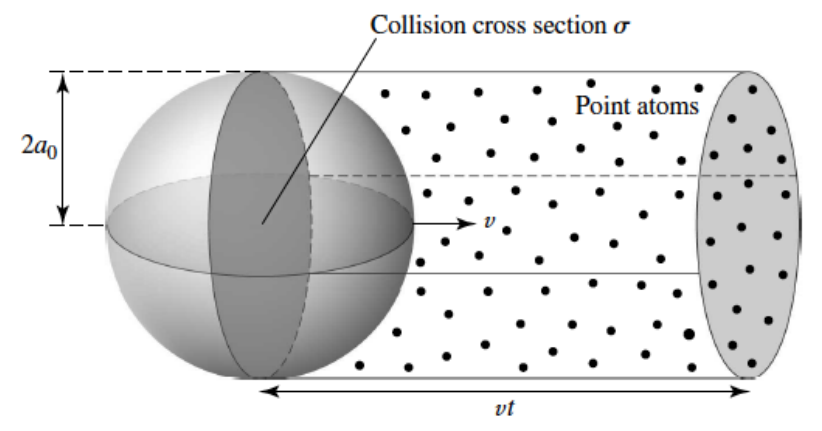


FIGURE 6 Mean free path, ℓ, of a hydrogen atom.

$H_T \sim 700$ km Scale of T variation in the Sun

$L \sim 2\ 10^{-4}$ m *Free path of hydrogen atom*

- The atoms in the gas see constant kinetic temperature between collisions.
- They are effectively confined within a limited volume of space in the photosphere.

→ Not valid for photons L ~ 160 km (in the photosphere)

Stellar atmosphere: Radiative transfer

How energy is transported in the stellar interior?

→ Consider a bean of parallel light rays travelling in the stellar interior, through gas.

→ The intensity of the bean will vary because of several processes: absorption and emission

The general mechanism is called radiative transfer

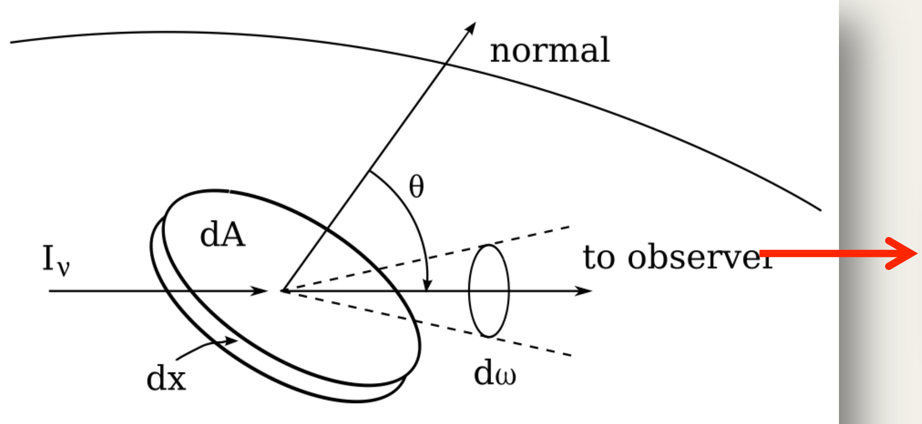


Figure 3.1: Volume element illustrating flux and intensity in a stellar atmosphere. Adapted from Gray (1992), figure 5.1 and 5.3.

The basic equation that describes radiative transfer is the following:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

It is a differential equation describing the flow of radiation through matter.

In an equilibrium, steady-state star, there can be no change in the total energy contained within any layer of the stellar atmosphere or interior

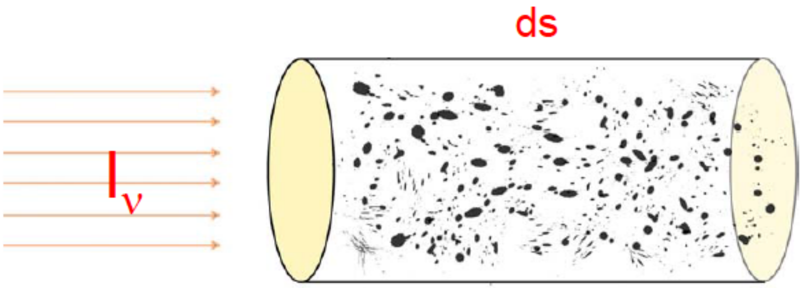
The equations of state in a stellar atmosphere: absorption

The first term indicates how energy is absorbed within a stellar atmosphere due to both scattering or absorption of photons.

The change of intensity of a ray (dI/ds) is proportional to the intensity of the ray, and to an absorption coefficient **(opacity)**

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

K_ν is the cross section for absorbing photons of wavelength λ (frequency ν) per unit mass of stellar material and has units of m^2 kg^{-1} . In general, **the opacity of a gas is a function of its composition, density, and temperature**



$$dI_\nu = -\kappa_\nu I_\nu ds$$

κ_ν : absorption coefficient

$[\kappa_\nu] = \mathrm{cm}^{-1}$

microscopical view: $\kappa_\nu = n\,\sigma_\nu$

loss of intensity in the beam (true absorption/scattering)

The equations of state in a stellar atmosphere: absorption

In general, there are **four primary sources of opacity available for removing stellar photons from a beam**. Each involves a change in the quantum state of an electron, and the terms bound and free are used to describe whether the electron is bound to an atom or ion in its initial and final states.

- **Bound–bound transitions** (excitations and de-excitations) occur when an electron in an atom or ion makes a transition from one orbital to another: **responsible for forming the absorption lines in stellar spectra**
- **Bound-free:** also known as photoionization , occurs when an incident photon has enough energy to ionize an atom.
- **Free–free absorption** is a scattering process: a free electron in the vicinity of an ion absorbs a photon, causing the speed of the electron to increase
- **Electron scattering:** a photon is scattered (not absorbed) by a free electron through the process of Thomson scattering

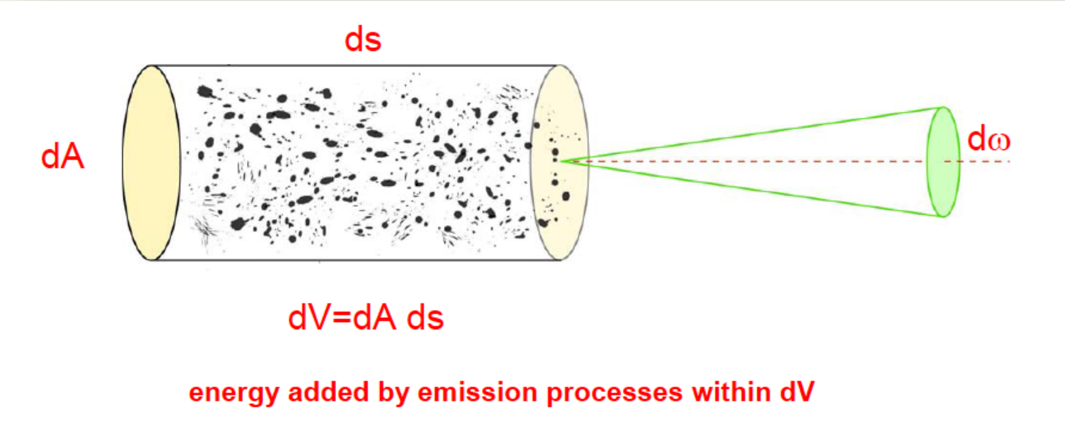
The equations of state in a stellar atmosphere: emission

The second term indicates how energy is emitted in each volume dV.

The change of intensity of a ray (dI/ds) is the emission coefficient, which is a function of wavelength (or frequency).

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu \boxed{+ \epsilon_\nu}$$

Any process that **add light to the beam** can be considered in the emission coefficient → temporal effect → random scattering



The equations of state in a stellar atmosphere: emission

Any process that adds photons to a beam of light will be called emission.

- **scattering of photons into the beam**
- **true emission of photons by electrons making downward atomic transitions.**

Each of the four primary sources of opacity has an inverse emission process: bound–bound and free–bound emission, free–free emission (bremsstrahlung), and electron scattering.

In a star there is not a direct flow of photons going toward the surface.

The individual photons travel only **temporarily** with the beam as they are repeatedly **scattered in random directions** following their encounters with gas particles.

The equations of state in a stellar atmosphere: random walk

The transport of energy through a star by radiation may be extremely inefficient

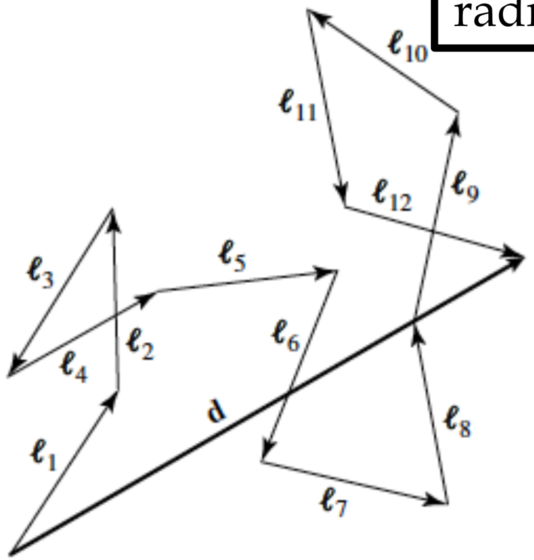


FIGURE 11 Displacement **d** of a random-walking photon.

For a large number of randomly directed steps, the sum of all the cosine terms approaches zero. As a result, for a random walk, the displacement d is related to the size of each step, ℓ , by

$$d = \ell \sqrt{N}.$$

The equations of state in a stellar atmosphere: optical depth

The average number of steps needed for a photon to travel the distance d before leaving the surface is

$$\mathbf{N = \tau_{\lambda}^{2}}$$

for $\tau_{\lambda} \gg 1$ (optical depth)

- when $\tau_{\lambda} \approx 1$, a photon escapes from **that level of the star.**
- **the average level in the atmosphere from which photons of wavelength λ escape is at a characteristic optical depth of about $\tau_{\lambda} = 2/3$.**

τ_{λ}

The optical depth may be thought of as the number of mean free paths from the original position to the surface, as measured along the ray's path.

$\tau_{\lambda} >> 1$: optically thick

$\tau_{\lambda} << 1$: optically thin

The transfer equation

The ratio of the rates of emission and absorption determines how rapidly the intensity of the beam of light changes and describes the tendency of the population of photons in the beam to resemble the local source of photons in the surrounding stellar material.

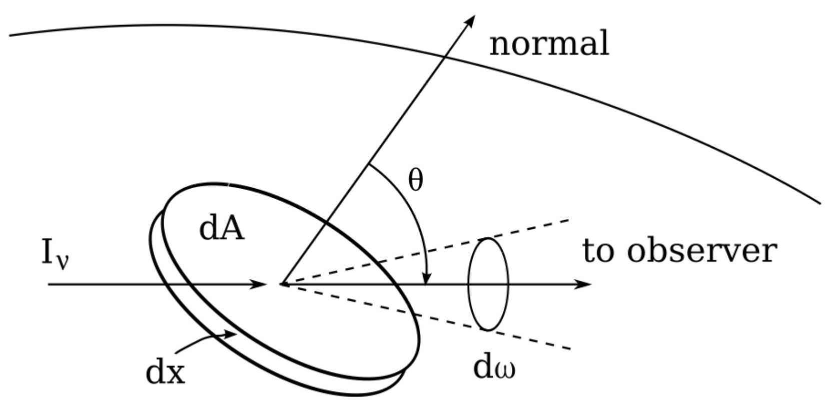


Figure 3.1: Volume element illustrating flux and intensity in a stellar atmosphere. Adapted from Gray (1992), figure 5.1 and 5.3.

The basic equation that describes radiative transfer is the following:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

It is a differential equation describing the flow of radiation through matter.

The source function

If the intensity of the light does not vary (dI/ds=0) then the intensity is equal to the source function, $I_\lambda = S_\lambda$.

The ratio between the emission and absorption coefficients give us the source function, which is of vital importance to solve the radiative transfer equation

$$S_\nu = \frac{\epsilon_\nu}{\kappa_\nu}$$

source function

To solve it, we have to do some approximations, and thus the following considerations will be valid only when and where these approximations are applicable.

The ratio of the rates of emission and absorption determines how rapidly the intensity of the beam of light changes and describes the tendency of the population of photons in the beam to resemble the local source of photons in the surrounding stellar material.

The source function: Blackbody radiation

1. **Hydrostatic equilibrium:** pressure balances gravity → no expansion, no contraction

2. **Thin atmosphere:** the photosphere is small compared to the radius of the star → it is as a superposition of parallel planes with a single (1D, radial) dimension describing the structure.

3. **Local Thermodynamic Equilibrium (LTE):** LTE is a valid approximation for each volume element in the atmosphere, and thus every layer has a unique temperature with **the source function being the Planck function**:

$$S_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$$

The source function: Blackbody radiation

However, a star cannot be in perfect thermodynamic equilibrium

→ there is a net flow of energy from the center to the surface

In the following conditions we can approximate with the source function with the Planck function:

- $\tau_\lambda \gg 1$, a random-walking photon will take at least $\tau_\lambda^{\ 2}$ steps to reach the surface
- the photon mean free path is small compared to the temperature scale height, the photons are effectively confined to a limited volume, a region of nearly constant temperature
- **If the conditions for local thermodynamic equilibrium (LTE) are satisfied the source function is equal to the Planck function, $S_\lambda = B_\lambda$**

Now we know…

- How energy level are populated in atoms (Boltzmann equation)
- How the balance between atoms and ions is regulated (Saha equation)
- How energy is transported in the stellar atmospheres

We have a theoretical arsenal for the analysis of spectral lines.

The shape of an individual spectral line contains a wealth of information about the environment in which it was formed.