

The relation between the distance  $a$  and the radius  $r$  for the same amount of energy  $E$  contained in the exposed portion of the blur spot is approximated by

$$r \cong 1.25a E^{-0.477} \quad (9.1)$$

For example, for  $E = 80\%$ ,  $r = 1.25a 0.80^{-0.477} = 1.39a$ . In other words, if the dimension  $a$  obtained with the knife measurement was 0.2 mm, the radius of the circular blur spot would be  $0.2 \times 1.39 = 0.28$  mm. The main point to remember is that one has to be careful with the interpretation of data obtained by one measuring method when applied to another.

### 9.3 Energy Distribution

To properly assess the energy distribution of a blur spot, measurements are required that indicate the change of the encircled energy as a function of the blur spot radius. The setup shown in Fig. 9.1 is suitable for such a measurement. A number of well-centered circular masks of increasing size are inserted successively at the image location and the energy levels are recorded from 0 to 100%. The resultant plot looks typically like the one shown in Fig. 9.4.

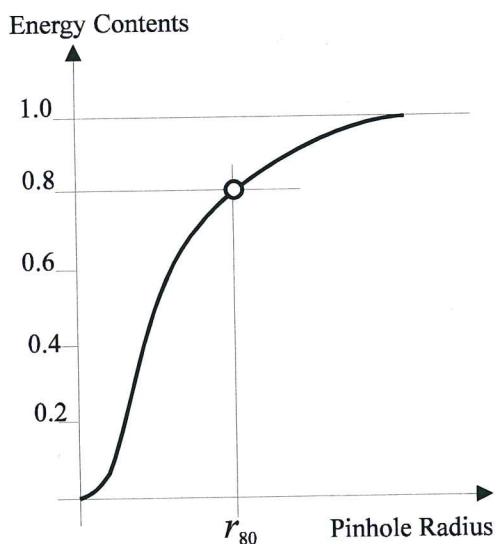


FIG. 9.4 Radial energy distribution.

### 9.4 Modulation Transfer Function

Removing the subjectivity from evaluating lenses by just visually judging their performance was achieved with the introduction of the modulation transfer function. This occurred about a half century ago.<sup>1</sup>

While the subject of MTF is very complex, there are a number of approximations that are especially helpful in the layout stages to assess the performance expectations of a

lens or lens system. Before we discuss these, let us look at some basic principles related to the MTF.

### 9.4.1 Overview

The image quality of an object is a measure of the quality of a lens. With the MTF the image quality is measured as contrast against spatial frequency.

Contrast or modulation is expressed by

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \quad (9.2)$$

For the object,  $I_{O \max}$  and  $I_{O \min}$  are the maximum and minimum intensities of the radiant emittance and for the image,  $I_{I \max}$  and  $I_{I \min}$  relate to the irradiance of the image. This is identified for a typical bar pattern in Fig. 9.5.

The modulation transfer function [MTF( $\nu$ )] is the ratio of the modulation of the image [ $M_i(\nu)$ ] and the modulation of the object [ $M_o$ ].  $\nu$  is the spatial frequency in line pairs (one dark and one light line) per unit length, usually millimeter. (Other dimensions, such as cycles per radian, are used as well). The number of line pairs is increased until the contrast in the image is too low to be detected and the imaged pattern can no longer be resolved. At that "limiting resolution" point, the spatial frequency is called the cutoff frequency  $\nu_0$ . For an aberration-free system,

$$\nu_0 = \frac{1}{\lambda (f/\#)}. \quad (9.3)$$

Fig. 9.6 shows how the modulation reduces as the spatial frequency increases.

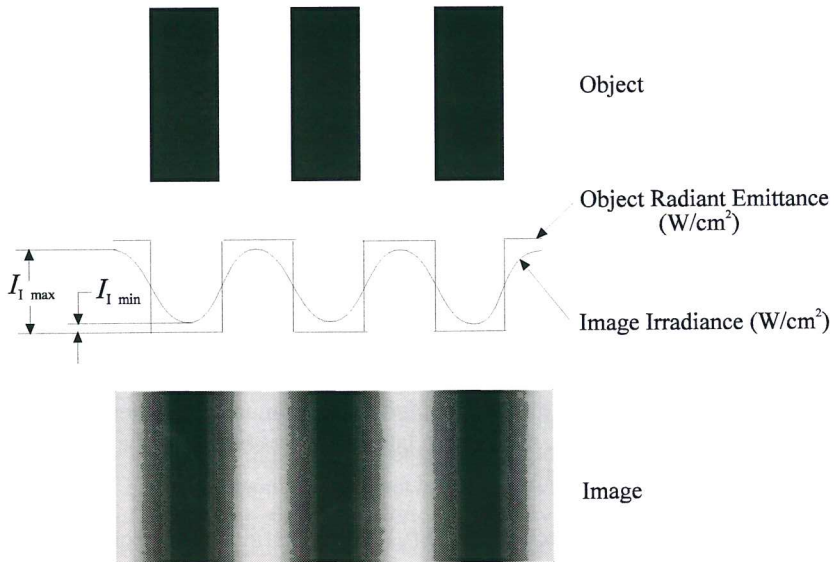


FIG. 9.5 Imagery of a bar pattern with  $I_{O \max} = 1$  and  $I_{O \min} = 0$  ( $M_O = 1$ ).

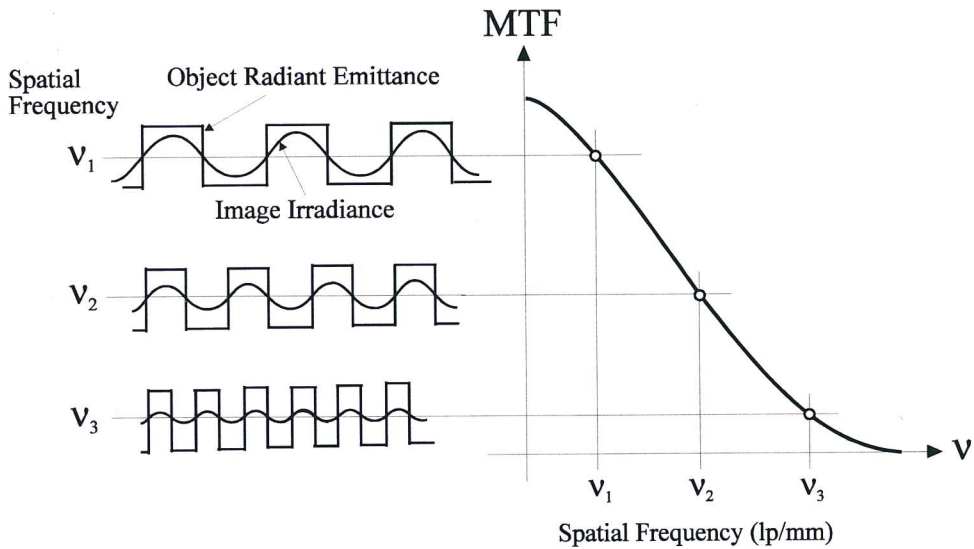


FIG. 9.6 The modulation transfer function.

MTF analyzers are being offered in a variety of configurations. Whether slits, knife edges, sinusoidal, or bar pattern targets are being used, the basic principle is always the same:

*The modulation of the image is compared with the modulation of the object at various spatial frequencies.*

### 9.4.2 Contrast and resolving power

As mentioned earlier, contrast is the relationship between two shades of gray which, at the limit, becomes black and white. Resolution is the limit of discernibleness of two fine structures, such as the imaged lines of the MTF analyzer's bar pattern target. With these two definitions, it can be understood that neither contrast nor resolution alone describe the full performance of an optical system. This is indicated in Fig. 9.7 where the modulation transfer functions of two lenses are compared. Lens A has higher modulation (contrast) at lower frequencies than Lens B. On the other hand, Lens B has a higher cutoff frequency (resolution limit) than Lens A. The question is not which lens is better, it is which lens is more suitable for the intended application. The curves provide the answer.

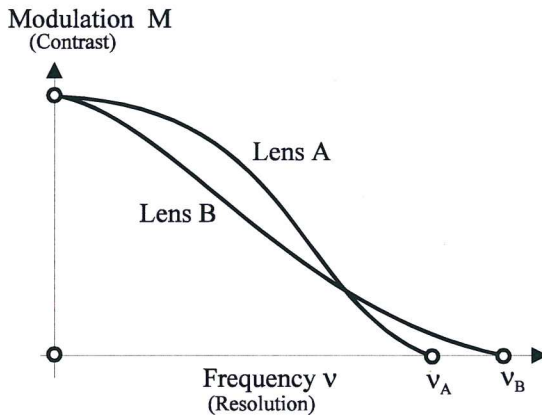


FIG. 9.7 Contrast and resolving power, two general cases.

Erich Heynacher and Fritz Köber, both with Carl Zeiss, Inc. have published an interesting paper on this subject of contrast and resolution.<sup>2</sup> With their kind permission, we show reproductions in Fig. 9.8 of four photographs of the same subject (Emperor Henry II in a stained glass window at the Strassburg cathedral) taken with four different lenses. While there is of course a degradation in the quality of the reproductions presented here, the relation between contrast and resolution is well enough preserved to demonstrate the principle indicated in Fig. 9.7. As the authors of that paper suggest, a very educational experiment is to observe the pictures from a larger viewing distance. As the distance is increased, the resolution limit of the eye becomes the limiting factor and confirms again that evaluating or judging the image quality of an optical system is not just a matter of the resolving power of the lens. All of this can be applied to IR imaging systems where the eye is replaced by a detector.



- 1 • Poor contrast
- Reasonable resolution



- 2 • Better contrast
- Worse resolution



- 3 • Better contrast than 1, but worse than 2
- Best resolution



- 4 • Best contrast
- Same resolution as 1

FIG. 9.8 Contrast and resolving power.

The associated MTF curves shown in Fig. 9.9 confirm clearly the characteristics of the four lenses used to take the pictures shown in Fig. 9.8.

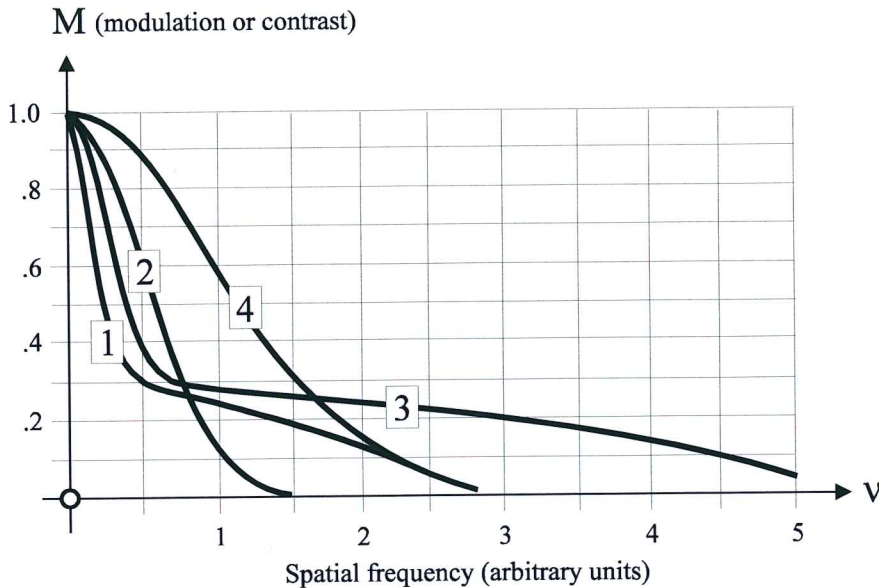


FIG. 9.9 Contrast and resolving power, the transfer functions.

That resolution is only part of the information for judging image quality is evident from the shapes of curves 1 and 4. Both show the same cutoff frequency, i.e., the same limiting resolution, but vary strongly with regard to contrast. This confirms the remarks made in Fig. 9.8. Another observation substantiates the visual renditions of samples 1 and 2. The limiting resolution of sample 2 is about half the one of sample 1; however, its contrast is much higher at lower frequencies. All this indicates that much can be predicted and confirmed with modulation transfer functions.

A typical MTF analyzer is shown in Fig. 9.10. Such a system can be equipped with a number of convenient attachments for evaluating a lens at different conjugates and off-axis positions. Polychromatic tests can be conducted and defocusing effects can be analyzed.