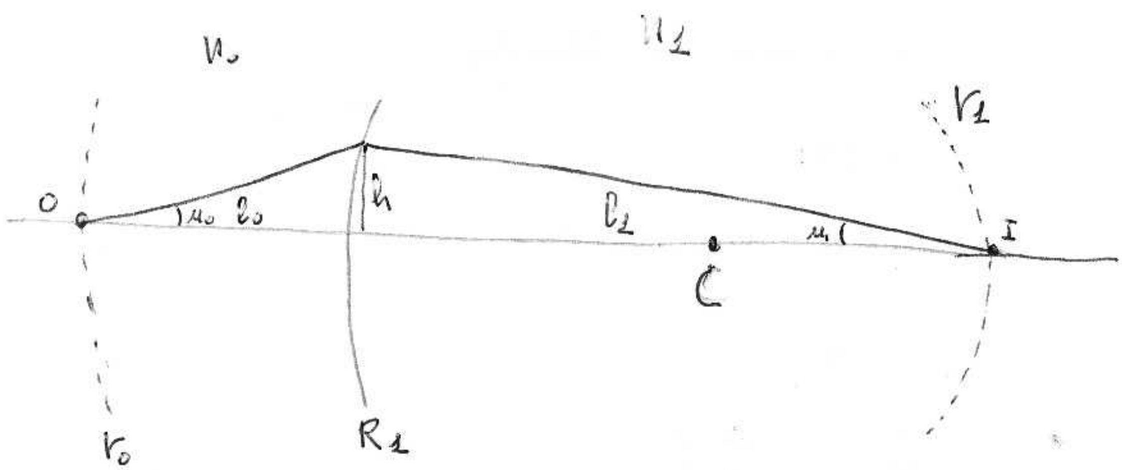
PETZVAL Theorem



$$n_1 u_1 = n_0 u_0 - h\,\frac{n_1 - n_0}{R_1}$$

$$\begin{cases} u_0 = -\dfrac{h}{l_0} \\[2mm] u_1 = -\dfrac{h}{l_1} \end{cases}$$

$$-n_1 \frac{h}{l_1} = -n_0 \frac{h}{l_0} - h\,\frac{n_1 - n_0}{R_1}$$

$$\frac{n_1}{l_1} = \frac{n_0}{l_0} + \frac{n_1 - n_0}{R_1}$$

Vogliamo eliminare le quantità l_i, n_i e mettere tutto in funzione di r_i, R_i
cioè esprimere $r_1 = r_1(R_1)$

$-l_0 = r_0 - R_1$ $\quad$ $l_1 = R_1 - r_1$ con segni opportuni!

$$\frac{n_1}{R_1 - r_1} = \frac{n_0}{R_1 - r_0} + \frac{n_1 - n_0}{R_1} \Rightarrow R_1 (R_1 - r_0)\cancel{(R_1 - r_1)}\frac{n_1}{\cancel{R_1 - r_1}} = n_0 (R_1 - r_1) R_1 + (n_1 - n_0)(R_1 - r_1)(R_1 - r_0)$$

$$n_1 R_1 (R_1 - r_0) - n_0 R_1 (R_1 - r_1) = (n_1 - n_0)(R_1 - r_0)(R_1 - r_1)$$

$$\cancel{n_1 R_1^2} - \cancel{n_1 r_0 R_1} - \cancel{n_0 R_1^2} + \cancel{n_0 r_1 R_1} = \cancel{n_1 R_1^2} - \cancel{n_0 R_1^2} - \cancel{n_1 r_0 R_1} + n_0 r_0 R_1 - n_1 r_1 R_1 +$$
$$+ \cancel{n_0 R_1 r_1} + n_1 r_0 r_1 - n_0 r_0 r_1$$

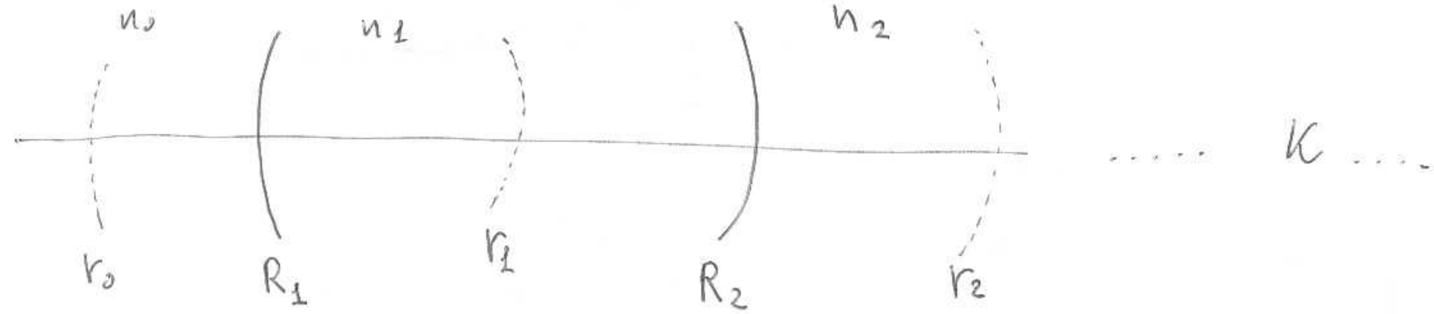
Riarr. $$\frac{n_0 r_0 R_1 - n_1 r_1 R_1 + (n_1 - n_0) r_0 r_1 = 0}{n_0 n_1 \cdot r_0 r_1 \cdot R_1} \Rightarrow$$

$$\Rightarrow \frac{1}{n_1 r_1} - \frac{1}{n_0 r_0} + \frac{n_1 - n_0}{n_0 n_1} \cdot \frac{1}{R_1} = 0 \qquad \text{ma} \quad \frac{n_1 - n_0}{R_1} = \varphi_1 \ (\text{potere})$$

$$\frac{1}{n_0 r_0} - \frac{1}{n_1 r_1} = \frac{\varphi_1}{n_0 \cdot n_1} \qquad \langle 1 \rangle$$

cioè $r_1 = r_1(\varphi)$ che è la relazione cercata

Se considero ulteriori superfici



$\frac{1}{n_1 r_1} - \frac{1}{n_2 r_2} = \frac{\varphi_2}{n_1 n_2}$ se applicata alle superfici $r_1\, R_2\, r_2\, n_1\, n_2$ $\Rightarrow$

Allora la $\langle 1 \rangle$ diviene

$$\frac{1}{n_0 r_0} - \frac{1}{n_2 r_2} - \frac{\varphi_2}{n_1 n_2} = \frac{\varphi_1}{n_0 n_1} \Rightarrow \frac{1}{n_0 r_0} - \frac{1}{n_2 r_2} = \frac{\varphi_1}{n_0 n_1} + \frac{\varphi_2}{n_1 n_2} \Rightarrow$$

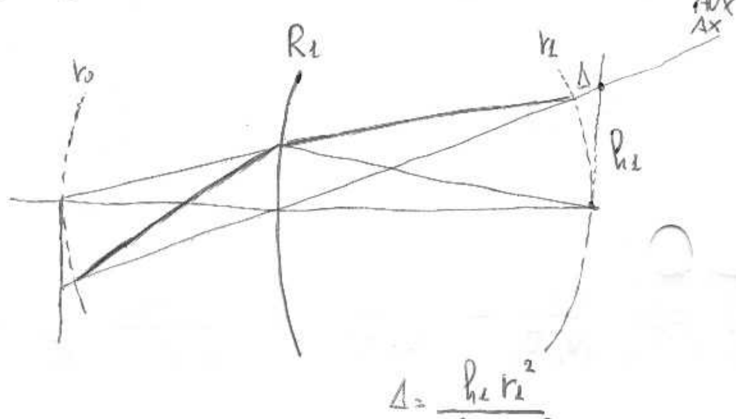
$$\frac{1}{n_0 r_0} - \frac{1}{n_K r_K} = \sum_{i=1}^{K} \frac{\varphi_i}{n_{i-1} \cdot n_i}$$

Se $r_0 = \infty \Rightarrow$ $\frac{1}{n_K r_K} = -\sum_{i=1}^{K} \frac{\varphi_i}{n_{i-1} \cdot n_i} \Rightarrow$ r_K $\boxed{\frac{1}{r_K} = -n_K \sum_{i=1}^{K} \frac{\varphi_i}{n_{i-1} \cdot n_i}}$

Se ho un singoletto di indice di rifrazione n_1:

$$\frac{1}{r_2} = -n_2 \left(\frac{\varphi_1}{n_0 n_1} + \frac{\varphi_2}{n_1 n_2} \right) \quad \text{ma} \quad n_2 = n_0 = 1 .$$

$$-\frac{1}{r_2} = \frac{\varphi_1 + \varphi_2}{n}$$



$$\Delta = \frac{h_1 r_1^2}{R_1^2 - 2 r_1^2}$$