Per quanto riguarda l'Int. di volume

$$\int_{B(x,r)} u(y)\,dV(y) = \int_0^r \left(\int_{\partial B(x,\rho)} u(y)\,dS(y) \right) d\rho$$

$$= \int_0^r |\partial B(x,\rho)| \underbrace{\left(\fint_{\partial B(x,\rho)} u(y)\,dS(y) \right)}_{\text{=} u(x) \text{ per la formula prec.}} d\rho$$

$$= u(x) \int_0^r \underbrace{|\partial B(x,\rho)|}_{\int_{\partial B(x,\rho)} dS(y)} d\rho = u(x) \int_{B(x,r)} dV(y)$$

$$= u(x)\ |B(x,r)|$$