Esercizi funzioni

ES ① $f(x)=3x+2a\ln(1+x)+2$ $\quad a\in\mathbb{R}$ parametro

$$D_a=\begin{cases} x>-1 & \text{se } a\neq 0 \\ \mathbb{R} & \text{se } a=0 \end{cases}$$

$a=0$ $\quad f(x)=3x+2$ funzione affine (il grafico è una retta)

$a\neq 0$ $\quad \lim_{x\to -1^+} f(x) = -\text{sign}(a)\cdot +\infty \Rightarrow$ la retta $x=-1$ è AS. VERTICALE dx

$\lim_{x\to+\infty} f(x)=+\infty$ $\forall a\neq 0$ essendo $\ln(1+x)$ infinito di ordine inferiore a x.

considero $\frac{f(x)}{x}=3+2a\frac{\ln(1+x)}{x}+\frac{2}{x}\xrightarrow[x\to+\infty]{} 3$

$f(x)-3x=\cancel{3x}+2a\ln(1+x)+2-\cancel{3x}=2a\ln(1+x)+2\to \begin{matrix}+\infty & a>0\\ -\infty & a<0\end{matrix}$

$\Rightarrow \nexists$ asintoti obliqui né AS. ORIZZ.

$$f'(x)=3+\frac{2a}{1+x}$$

zeri di f': $f'=0$ sse $3+\frac{2a}{1+x}=0$ sse $\frac{2a}{1+x}=-3$ sse $x+1=-\frac{2}{3}a$

sse $x=-\frac{2}{3}a-1=-\left(\frac{2}{3}a+1\right)$

OSS: $-\left(\frac{2}{3}a+1\right)>-1$ sse $\frac{2}{3}a+\cancel{1}<\cancel{1}$ $\quad a<0$

cioè se $a>0$ $\quad -\left(\frac{2}{3}a+1\right)\notin D$.

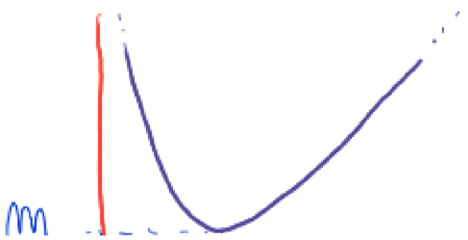
segno f': $f'>0$ sse $\frac{2a}{1+x}>-3$ sse ($x\in D\Rightarrow x>-1$) $\quad -3(x+1)<2a$

$x+1>-\frac{2}{3}a \qquad x>-\left(\frac{2}{3}a+1\right)$

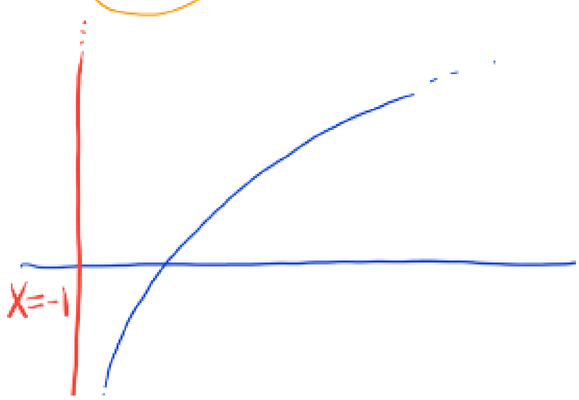
se $a<0$:

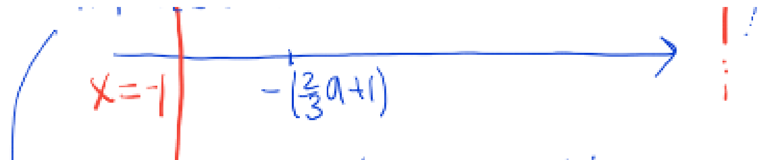
f': $\quad - \quad \bullet_{-\left(\frac{2}{3}a+1\right)} \quad +$

f: $\quad \searrow \quad \nearrow$



se $a>0$ $\quad f'>0$ $\forall x\in D$





? la concavità rappresentata è corretta ?

$$m = f\left(-\left(\tfrac{2}{3}a+1\right)\right) = -3\left(\tfrac{2}{3}a+1\right)+2a\ln\left(\cancel{1}-\left(\tfrac{2}{3}a+\cancel{1}\right)\right)+2 =$$
$$= -2a-3+2a\ln\left(\tfrac{2}{3}|a|\right)+2 =$$
$$= -2a-1+2a\ln\left(\tfrac{2}{3}|a|\right) = 2a\left(\ln\left(\tfrac{2}{3}|a|\right)-1\right)-1$$

$\Rightarrow$ il segno del minimo di f dipende dai valori di a

<u>es</u> $a=-\frac{3}{2}$ si ha $\min f = -3(-1)-1 = 2>0$

$a\to 0^-$ si ha $\min f = -1 < 0$

concavità di f: $f''(x) = -\dfrac{2a}{(1+x)^2} \Rightarrow$ f concava in D per $a>0$
f convessa in D per $a<0$

$\nexists$ punti di flesso.

- $\sup\limits_{x\in D} f = +\infty \quad \forall a\in\mathbb{R}$
- $\inf\limits_{x\in D} f = \begin{cases} -\infty & \text{se } a\geq 0 \\ f\left(-\left(\frac{2}{3}a+1\right)\right) & \text{se } a<0 \end{cases}$ $\Leftarrow$ in questo caso $\inf f = \min f$.

<u>Es</u> 6 $f(x) = \operatorname{arctg}\left((a+2)x\right)+x \qquad D=\mathbb{R}$

- <u>OSS</u> se $a=-2$ $f(x)=x$ (retta)
- considero $a\neq -2$

$\lim\limits_{x\to+\infty} f(x) = +\infty \qquad$ essendo $-\frac{\pi}{2}\leq \operatorname{arctg}\xi \leq \frac{\pi}{2} \quad (\cdot)$

$\lim\limits_{x\to-\infty} f(x) = -\infty$

considero $\dfrac{f(x)}{x} = \dfrac{\operatorname{arctg}\left((a+2)x\right)}{x}+1 \longrightarrow 1$ per la $(\cdot)$

$x \qquad x \qquad x\to\pm\infty$

$$f(x)-x = \operatorname{arctg}((a+2)x) \xrightarrow[x\to+\infty]{} \begin{cases} \frac{\pi}{2} & \text{se } a+2>0 \\ -\frac{\pi}{2} & \text{se } a+2<0 \end{cases}$$

$$f(x)-x = \operatorname{arctg}((a+2)x) \xrightarrow[x\to-\infty]{} \begin{cases} \frac{\pi}{2} & \text{se } a+2<0 \\ -\frac{\pi}{2} & \text{se } a+2>0 \end{cases}$$

$\Rightarrow$ per $a>-2$ la retta $y = x+\frac{\pi}{2}$ è AS. OBL. per $x\to+\infty$
$y = x-\frac{\pi}{2}$ è AS. OBL per $x\to-\infty$

per $a<-2$ la retta $y = x-\frac{\pi}{2}$ è AS. OBL. per $x\to+\infty$
$y = x+\frac{\pi}{2}$ è AS. OBL. per $x\to-\infty$

$$f'(x) = \frac{(a+2)}{1+(a+2)^2x^2}+1$$

. segno f': . $a+2>0$ allora $f'>0$ $\forall x\in\mathbb{R} \Rightarrow f \nearrow$ in $\mathbb{R}$

. $a+2<0$ allora $f'=0$ sse $\frac{-(a+2)}{1+(a+2)^2x^2}=1$

sse $x^2(a+2)^2+(2+a)+1=0$

sse $x^2(a+2)^2 = -(2+a)-1$

$\exists$ soluz sse $-(2+a)-1\geq 0$ sse $2+a\leq -1$

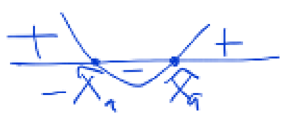
$x^2 = -\frac{(a+3)}{(2+a)^2}$ sse $x=\sqrt{-\frac{a+3}{(2+a)^2}}$ ✓

$x = -\sqrt{\frac{a+3}{(2+a)^2}}$

segno di f': $\frac{a+2}{1+(a+2)^2x^2}+1>0$ sse $\frac{a+2}{1+(a+2)^2x^2}>-1$ sse

$(a+2)>-1-(a+2)^2x^2$ sse $(a+2)^2x^2+a+3>0$

$x^2>-\frac{(a+3)}{(a+2)^2}$



sia $\hat{x}_a = \sqrt{\frac{-(a+3)}{(a+2)^2}}$

	$-\hat{x}_a$		$\hat{x}_a$	
f':	$+$	$-$		$+$
f:	$\nearrow$	$\searrow$		$\nearrow$

in $x=-\hat{x}_a$ f ha MAX Rel
$x=\hat{x}_a$ f ha MIN Rel

$$f''(x) = \frac{-(a+2)^3\cdot 2x}{(1+(a+2)^2x^2)^2}$$ $\Rightarrow$ segno di f'':

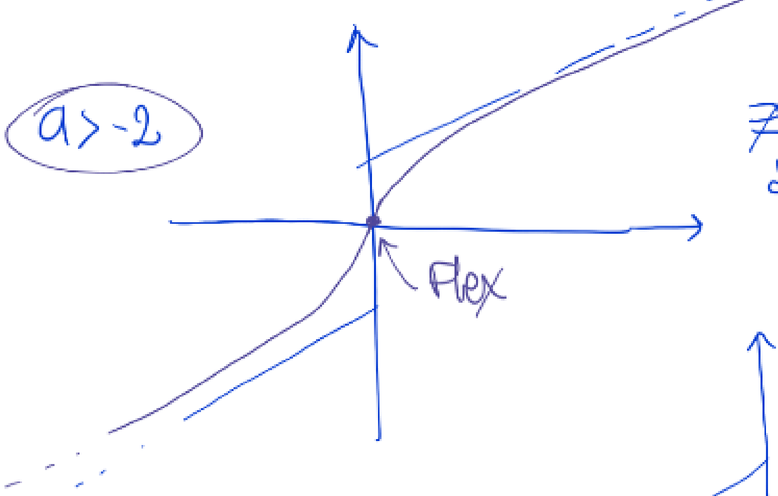
$a>-2$ f'': $+ \overset{x=0}{\bullet} -$ flex

f: ⌣ ⌒

$a < -2$ $\quad f''$: $-$ $\underset{x=0}{\bullet}$ $+$ flex

f: ⌒ ⌣

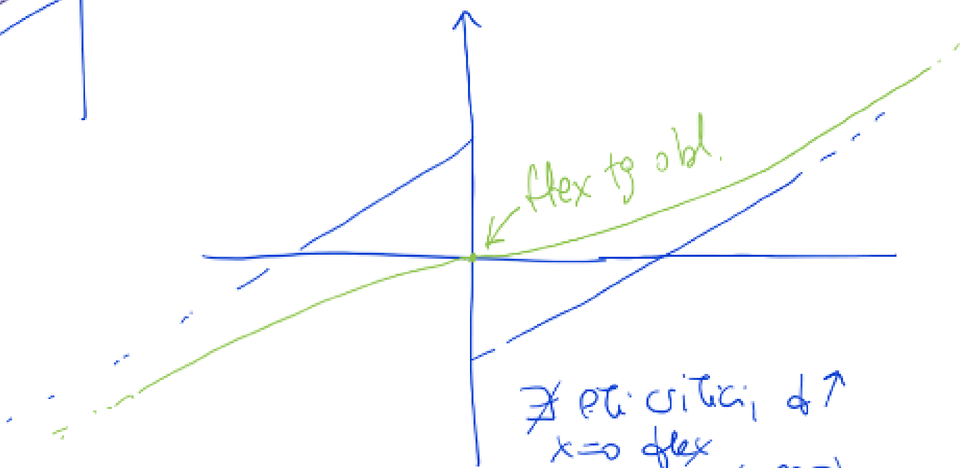
$a > -2$



∄ p.ti critici, $f \nearrow$

f conv. in $(-\infty, 0)$,
conc in $(0, +\infty)$

$-3 < a < -2$



∄ p.ti critici, $f \nearrow$

$x=0$ flex

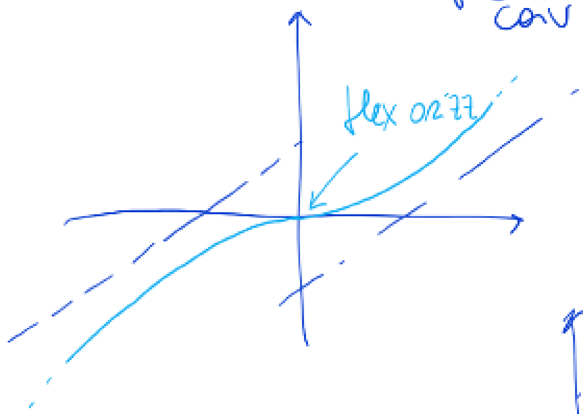
f conc in $(-\infty, 0)$
conv in $(0, +\infty)$.

$a = -3$

∃! punto critico $x=0$
flex a tg orizz

$f \nearrow$

f conc in $(-\infty, 0)$
conv in $(0, +\infty)$



$a < -3$

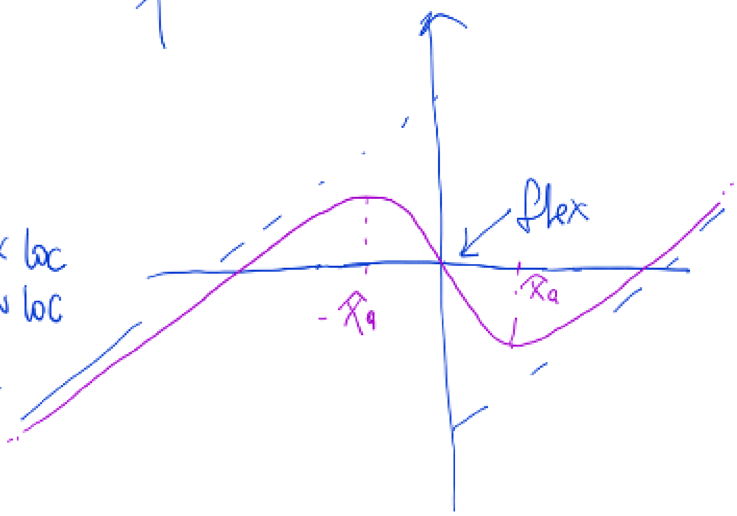
∃ 2 p.ti critici

$x = -\bar{x}_a$ p.to di max loc

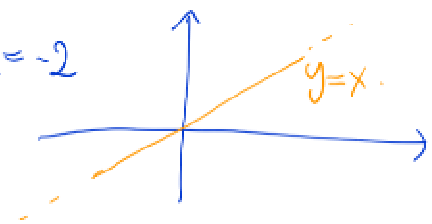
$x = \bar{x}_a$ p.to di min loc

∃! p.to di flex $x=0$.

f conc in $(-\infty, 0)$
conv in $(0, +\infty)$.



$a = -2$



Es 13 $f(x)=x^2e^{(a+1)x}$ $\qquad D=\mathbb{R}\ \ \forall a\in\mathbb{R}$

$$\lim_{x\to+\infty} f(x)=\begin{cases}+\infty & \text{se } a+1\geq 0\\ 0 & \text{se } a+1<0\end{cases} \Rightarrow \text{la retta } y=0 \text{ è AS. ORIZZ per } x\to+\infty$$

$$\lim_{x\to-\infty} f(x)=\begin{cases}0 & \text{se } a+1>0\\ +\infty & \text{se } a+1<0\end{cases} \Rightarrow \text{la retta } y=0 \text{ è AS. ORIZZ per } x\to-\infty$$

mentre se $a=-1$ si ha $f(x)=x^2$ Parabola

OSS. NON ci sono AS. OBLIQUI

$$f'(x)\underset{a\neq-1}{=} e^{(a+1)x}\left(2x+(a+1)x^2\right)=e^{(a+1)x}\cdot x\left((a+1)x+2\right)$$

Punti critici: $f'=0 \iff x=0 \ \vee\ x=-2/(a+1)\doteq x_a$

Segno f':

$a>-1$

		x_a		0	
$e^{(a+1)x}$	$+$		$+$		$+$
x	$-$		$-$		$+$
$(a+1)x+2$	$-$		$+$		$+$
	$+$	$\cdot$	$-$	$\cdot$	$+$
f:	$\nearrow$		$\searrow$		$\nearrow$

$a<-1$

		0		x_a	
$e^{(a+1)x}$	$+$		$+$		$+$
x	$-$		$+$		$+$
$(a+1)x+2$	$+$		$+$		$-$
	$-$	$\cdot$	$+$	$\cdot$	$-$
f:	$\searrow$		$\nearrow$		$\searrow$

$\forall a\neq-1$: in $x=x_a$ f ha MAX loc

$x=0$ f ha min loc $\equiv$ min ASS. $=0$

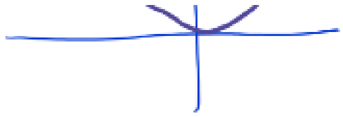
$$f''(x)=e^{(a+1)x}\left((a+1)x\left((a+1)x+2\right)+(a+1)x+2+(a+1)x\right)=$$

$$=e^{(a+1)x}\left(\underbrace{(a+1)^2x^2+4(a+1)x+2}_{=\left((a+1)x+2\right)^2-2}\right)$$

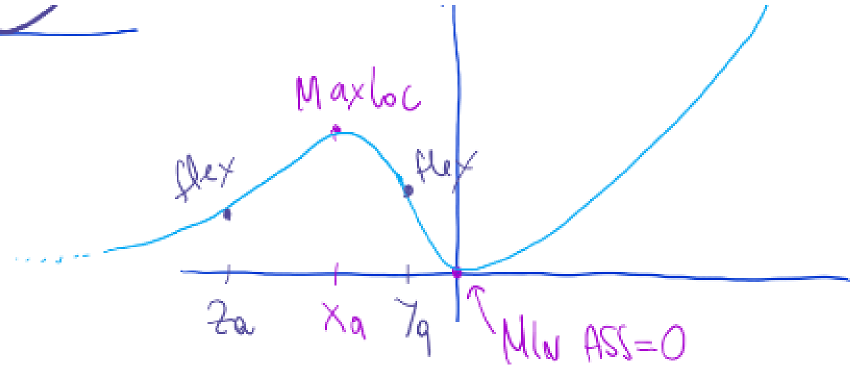
$$f''=0 \iff \left((a+1)x+2\right)^2=2 \iff \begin{matrix}(a+1)x+2=\sqrt2 \ \vee\\ (a+1)x+2=-\sqrt2\end{matrix} \iff$$

$$\iff x=y_a \ \vee\ x=z_a \text{ dove } y_a=\frac{\sqrt2-2}{a+1} \ \vee\ z_a=\frac{-\sqrt2-2}{a+1}$$

$\boxed{a=-1}$



$\boxed{a>-1}$



oss $z_a < x_a < y_a < 0$

$\boxed{a<-1}$

oss $0 < y_a < x_a < z_a$