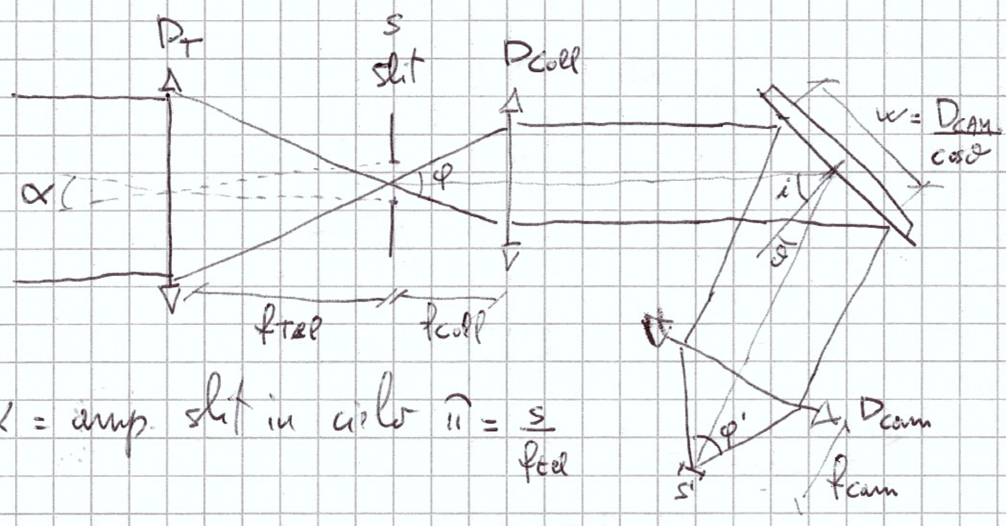
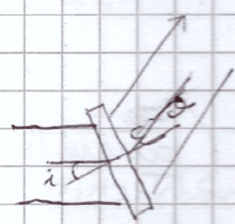
GRATING



ρ = densità di righe

m = ordine

$F \equiv f/\#$



α = amp. slit in cielo $= \frac{s}{f_{tel}}$

$$\boxed{d \,\text{sen}\, i = m\lambda} \overset{\text{estesa}}{\Longrightarrow} \text{sen}\, i + \text{sen}\, \vartheta = \rho m \lambda \qquad \langle 1 \rangle$$

$$\lambda = \frac{1}{\rho m}(\text{sen}\, i + \text{sen}\, \vartheta) \Rightarrow \frac{\Delta\lambda}{\Delta\vartheta} = \frac{\cos\vartheta}{\rho m} \qquad \langle 2 \rangle$$

$$x = f_c \cdot \vartheta_x \Rightarrow \Delta x = f_c \cdot \Delta\vartheta_x \Rightarrow \frac{\Delta\lambda}{\Delta x} f_c = \frac{\cos\vartheta}{\rho m} \Rightarrow \frac{\Delta\lambda}{\Delta x} = \frac{\cos\vartheta}{f_c \cdot \rho m} \qquad \langle 3 \rangle$$

$$\text{Ris. Teorica} = R^* \equiv \boxed{\frac{\lambda}{\Delta\lambda} = mN} \Rightarrow m \cdot \rho W = R^* \qquad \langle 4 \rangle$$

Lagrange $n \Omega A = \text{cost.}$

$A K \Omega$
n

$$s\varphi = s'\varphi'$$

$$s \frac{D_{coll}}{f_{coll}} = s' \frac{D_{cam}}{f_{cam}}$$

$$s' = s \frac{D_{coll}}{f_{coll}} \frac{f_{cam}}{D_{cam}} = s \frac{F_{cam}}{F_{coll}}$$

$$\Delta\lambda \overset{(3)}{=} \Delta x \cdot \frac{\cos\vartheta}{f_c \cdot \rho m} \overset{\text{slit}}{=} s' \frac{\cos\vartheta}{f_c \cdot \rho m} = s \frac{\overset{= f_c/D_c}{F_{cam}}}{F_{coll}} \frac{\cos\vartheta}{f_c \cdot \rho m} = \frac{s \cdot \cos\vartheta}{\rho m \cdot D_{cam} \cdot F_{coll}}$$

$$W = \frac{D_{cam}}{\cos\vartheta} \Rightarrow \Delta\lambda = \frac{s \cos\vartheta}{\rho m \cdot (W \cdot \cos\vartheta) \cdot F_{coll}} = \frac{s}{\rho m \cdot W \cdot F_{coll}}$$

$$R^{reale} = \frac{\lambda}{\Delta\lambda} \overset{\langle 6\rangle}{=} \frac{\rho m \cdot W \cdot F_{coll}}{s} \cdot \lambda \overset{\langle 4\rangle}{=} R^* \frac{F_{coll}}{s} \cdot \lambda$$

R NON dipende dalla camera!!

Ma
$$\begin{cases} \alpha f_{TEL} = s & \text{Fenditura espressa in angolo cielo } (\alpha \text{ in rad}) \\ F_{COLL} = F_{TEL} & \text{@ condizioni IDEALI (direct feed) NO FIBRE} \end{cases}$$

$$\Downarrow$$

$$\frac{f_{coll}}{D_{coll}} = \frac{f_{tel}}{D_{tel}}$$

$$R = R^* \frac{F_{coll}}{s} \cdot \lambda = R^* \cdot \frac{F_{COLL}}{\alpha f_{TEL}} \lambda = R^* \cdot \frac{F_{COLL} \, D_{COLL}}{\alpha \cdot f_{coll} \cdot D_{TEL}} \cdot \lambda \Rightarrow$$

($f_{tel} = F_{coll}$)

$$R = R^* \frac{F_{COLL}}{\alpha \cdot F_{COLL}} \cdot \frac{\lambda}{D_{TEL}} \Rightarrow \boxed{R = R^* \frac{\lambda}{\alpha \cdot D_{TEL}}}$$

$$\overset{\langle 4\rangle}{=} \rho m W \cdot \frac{\lambda}{\alpha \cdot D_{TEL}} \overset{\langle 5\rangle}{=} \rho m \frac{D_{CAM}}{\cos\vartheta} \cdot \frac{\lambda}{\alpha \cdot D_{TEL}}$$

<5>

$$\Rightarrow R \leq R^* \quad \lambda \leq \alpha \cdot D_{TEL} = \lambda_{DIFF.LIM.}$$

ES. $\rho = 100 \, l/mm$ $D_{COLL} = 300 \, mm$ $i = 60°$ $m = 1$

$\alpha_{slit} = 1''$ $D_{TEL} = 8 \, m$ $\lambda = 500 \, nm$

$$\sin i + \sin\vartheta = \rho m \lambda \Rightarrow \vartheta = \arcsin(\rho m \lambda - \sin i)$$

$$\vartheta = \arcsin\left(100 \frac{l}{mm} \cdot 1 \cdot 500 \cdot 10^{-9} m - \sin 60\right) =$$

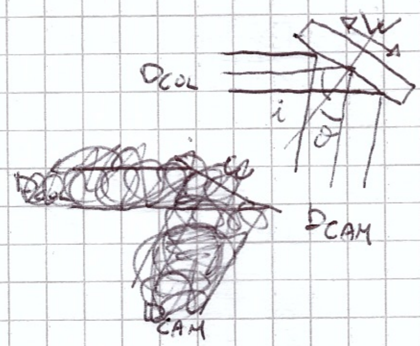
$$= \arcsin(0{,}05 - 0{,}866) = -54°$$

$$W = \frac{D_{COLL}}{\cos i} = \frac{300 \, mm}{0{,}5} = 150 \, mm$$

$$R^* = m \rho W = 1 \cdot 100 \cdot 150 = 15000$$

$$R = R^* \cdot \frac{\lambda}{\alpha \cdot D_{TEL}} = 15000 \cdot \frac{500 \cdot 10^{-9} m}{1'' \cdot 8} = 193$$

INGRANDIMENTO:

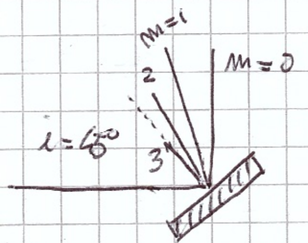


$$\begin{cases} M_\lambda = \dfrac{s'}{s} = \dfrac{F_{CAM}}{F_{COL}} = \dfrac{f_{CAM}}{f_{COL}} \cdot \dfrac{D_{COL}}{D_{CAM}} \\[2ex] M_x = \dfrac{f_{CAM}}{f_{COL}} \quad \text{essendo } D^s_{COL} = D^s_{CAM} \end{cases}$$

$$A = \frac{M_x}{M_\lambda} = \frac{f_{CAM}}{f_{COL}} \left(\frac{f_{COL}}{f_{CAM}} \cdot \frac{D_{CAM}}{D_{COL}} \right) = \frac{D_{CAM}}{D_{COL}} = \frac{W \cos\vartheta}{W \cos i} = \frac{\cos\vartheta}{\cos i}$$

— o —

ESEMPIO



$\rho = 600\ \ell/mm$

$i = 45°$

- $m=0$

  $\operatorname{sen}\vartheta = -\operatorname{sen} i \Rightarrow \vartheta = -45°$ (specchio) NO DISP.

- $m=1$

  $\vartheta \overset{(1)}{=} \arcsin(\rho m \lambda - \operatorname{sen} i) \overset{\arcsin}{=} \left(\dfrac{600}{10^{-3} m} \cdot 10^{+3} \cdot 1 \cdot 400 \cdot 10^{-9} m - 0{,}71 \right) =$

  $$\begin{cases} \vartheta_{\lambda=400} = -28° \\ \vartheta_{\lambda=700} = -17° \end{cases} \quad \overset{vis}{\simeq} 10°$$

- $m=2$

  $\vartheta = \arcsin\left(600 \dfrac{10^3}{m} \cdot 2 \cdot \lambda - 0{,}71\right) =$

  $$\begin{cases} \vartheta_{\lambda=400} = -13° \\ \vartheta_{\lambda=700} = 7{,}5° \end{cases} \quad \overset{vis}{\simeq} 20°$$

- $m = 3$

$$\vartheta = \arcsin\left(600 \cdot \frac{10^3}{[mm]} \cdot 3 \cdot \lambda - 0{,}71\right) =$$

$$\begin{cases} \vartheta_{\lambda=400} = 0{,}75^\circ \\ \vartheta_{\lambda=700} = 33{,}5^\circ \end{cases}$$

vis $\approx 33^\circ$ sovrapposto a $m = 2$

- @ $\vartheta_1 - \vartheta_2 \rightarrow 500\text{-}800$ nm   Quale angolo per $m = 1$?

$$i = \arcsin(\rho m \lambda - \sin\vartheta) = \arcsin\left(600 \cdot \frac{10^3}{[mm]} \cdot 1 \cdot 500 \cdot 10^{-9}[m] - \sin 28^\circ\right)$$

$= 50^\circ$ grating più inclinato

- Dispersione? $f_{CAM} = 100$ mm

ANG.

$$\begin{cases} \left.\frac{\Delta\lambda}{\Delta\vartheta}\right|_{28^\circ} \overset{\langle 2\rangle}{=} \frac{\cos\vartheta}{\rho m} = \frac{\cos(-28^\circ)}{600 \cdot \frac{10^3}{[m]} \cdot 1} = 1{,}47 \cdot 10^{-6}\ \text{rad}\left[\frac{m}{rad}\right] = 2570\ \text{nm}/^\circ \\ \left.\frac{\Delta\lambda}{\Delta\vartheta}\right|_{17^\circ} = \frac{\cos(-17^\circ)}{600 \cdot 10^3} = 2780\ \text{nm}/^\circ \end{cases}$$

LIN.

$$\begin{cases} \left.\frac{\Delta\lambda}{\Delta x}\right|_{28^\circ} = \frac{\cos\vartheta}{f_{cam} \cdot \rho m} = 14{,}7 \cdot 10^{-6} = \frac{10^{-9}}{10^{-3}}\ \frac{nm}{mm} \\ \left.\frac{\Delta\lambda}{\Delta x}\right|_{17^\circ} = 16\ \text{nm}/\text{mm} \end{cases}$$

— o — DISPERSIONE & RISOLUZIONE

Da $\langle 5\rangle$

$$\begin{cases} R = R^* \frac{\lambda}{\alpha D_{TEL}} = \rho m \frac{D_{CAM}}{\cos\vartheta} \cdot \frac{\lambda}{\alpha D_{TEL}} \\ \text{ma } \frac{\Delta\lambda}{\Delta\vartheta} \overset{\langle 2\rangle}{=} \frac{\cos\vartheta}{\rho m} \end{cases}$$

ALTA RIS
⇕
ALTA DISP.

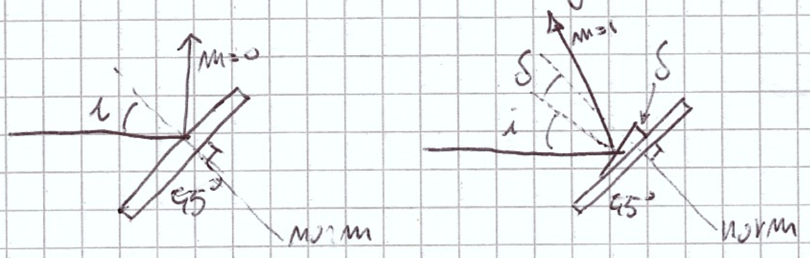
se $\frac{\Delta\lambda}{\Delta\alpha} \rightarrow 0$ (H_{RES})
$\Rightarrow R >> 0$ @ $\Delta\lambda \rightarrow 0$

BASSA DISP
⇓
BASSA RIS.

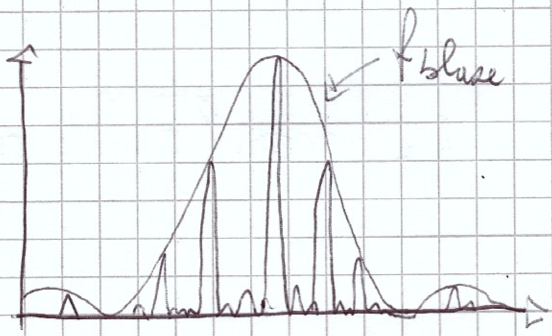
$$R = \frac{\lambda}{\Delta\lambda} = \frac{D_{CAM}}{D_{TEL}} \cdot \frac{\lambda}{\alpha \cdot \frac{\Delta\lambda}{\Delta\vartheta}} \Rightarrow \Delta\lambda = \frac{D_{TEL}}{D_{CAM}} \cdot \alpha \cdot \frac{\Delta\lambda}{\Delta\vartheta}$$

BLAZE ANGLE [δ]

$\vartheta = 0 \quad m = 0$ MAX energia: inutile!!



spazio fra i solchi $= d \cos\delta$



{ Per grating a bassi ordini $\delta \lesssim 20°$
{ Per grating "echelle" $\delta \sim 60°-70°$

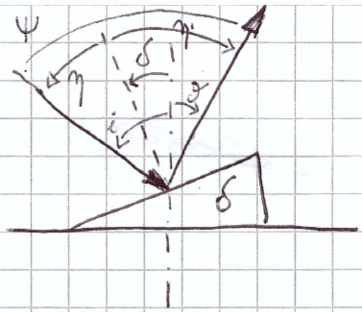
$$I = A_0^2 \frac{\operatorname{sen}^2\beta}{\beta^2} \frac{\operatorname{sen}^2 N\gamma}{\operatorname{sen}^2\gamma}$$

funz. di BLAZE

intensità diffratta dalla singola fenditura larga b

$$\begin{cases} 2\gamma = \dfrac{2\pi}{\lambda} d \operatorname{sen}\vartheta = k\, d \operatorname{sen}\vartheta & \Delta_{\text{Fase}} \text{ tratti adiac.} \\ \beta = \dfrac{\pi}{\lambda} b \operatorname{sen}\vartheta = k \dfrac{b}{2} \operatorname{sen}\vartheta & \Delta_{\text{FASE}} \text{ centro-bordo } \forall \text{ tratto} \end{cases}$$

Grating x ordini bassi: $\delta < 20°$
Grating x ordini alti: $\delta = 60°-70°$ (ECHELLE)



$$I = \frac{\operatorname{sen}^2 \beta}{\beta^2}$$

$$\beta = \frac{\pi}{\lambda} b \operatorname{sen}\vartheta = \frac{\pi}{\lambda} b \left(\operatorname{sen}\eta + \operatorname{sen}\eta'\right) =$$

$$= \frac{\pi}{\lambda} b \left[\operatorname{sen}(i-\delta) + \operatorname{sen}(\vartheta - \delta)\right] =$$

$$\left\{ p m \lambda = \operatorname{sen} i + \operatorname{sen}\vartheta \Rightarrow \frac{1}{\lambda} = \frac{pm}{\operatorname{sen} i + \operatorname{sen}\vartheta} \right.$$

$$= \pi b p m \frac{\operatorname{sen}(i-\delta) + \operatorname{sen}(\vartheta-\delta)}{\operatorname{sen} i + \operatorname{sen}\vartheta} =$$

$$\left\{ \operatorname{sen}x + \operatorname{sen}y = 2 \operatorname{sen}\frac{x+y}{2} \cos\frac{x-y}{2} \right.$$

$$= \pi b p m \frac{2 \operatorname{sen}\frac{i+\vartheta-2\delta}{2} \cos\frac{i-\vartheta}{2}}{2 \operatorname{sen}\frac{i+\vartheta}{2} \cos\frac{i-\vartheta}{2}} =$$

$$= \pi b p m \frac{\operatorname{sen}\frac{i+\vartheta-2\delta}{2}}{\operatorname{sen}\frac{i+\vartheta}{2}} = \pi b p m \frac{\operatorname{sen}\left(\frac{i+\vartheta}{2} - \delta\right)}{\operatorname{sen}\frac{i+\vartheta}{2}} =$$

$$= \pi b p m \frac{\operatorname{sen}\frac{i+\vartheta}{2}\cos\delta - \operatorname{sen}\delta \cos\frac{i+\vartheta}{2}}{\operatorname{sen}\frac{i+\vartheta}{2}} \Rightarrow$$

$$\beta = \pi b p m \left[\cos\delta - \frac{\operatorname{sen}\delta}{\operatorname{tg}\frac{i+\vartheta}{2}}\right]$$

- $\beta = 0 \overset{Max}{\Rightarrow} \cos\delta - \frac{\operatorname{sen}\delta}{\operatorname{tg}\frac{i+\vartheta}{2}} = 0 \Rightarrow \left(\operatorname{tg}\frac{i+\vartheta}{2}\right)^{-1} \operatorname{sen}\delta = \cos\delta \Rightarrow$

$$\operatorname{tg}\delta = \operatorname{tg}\frac{i+\vartheta}{2} \Rightarrow 2\delta = i + \vartheta_b \text{ max} \Rightarrow @\, i = \vartheta \quad \left| \delta = i = \vartheta \right|$$

<10>

- $\beta = \pm\pi$ MIN $\quad \pi b p m \left[\cos\delta - \frac{\operatorname{sen}\delta}{\operatorname{tg}\frac{i+\vartheta}{2}}\right] = \pm\pi = \pm\frac{1}{bpm} \Rightarrow$

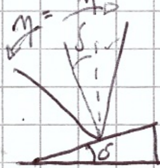
$$\Rightarrow \operatorname{tg}\frac{i+\vartheta}{2} = \frac{\operatorname{sen}\delta}{\cos\delta \pm \frac{1}{pmb}}$$

= o =

MAX BLAZE $\Rightarrow \beta = 0 \Rightarrow \eta = -\eta' \Rightarrow i + \vartheta_{blaze} = 2\delta$

$\uparrow (\operatorname{sen}\eta + \operatorname{sen}\eta' = 0)$

$$\left\{ \begin{array}{l} \eta = \eta' \\ i + \vartheta_B = 2\delta \end{array} \right.$$



BLAZE ANGLE & RISOLUZIONE

Max

$$\begin{cases} i + \vartheta_b \overset{\text{<10°}}{=} 2\delta \ , \ n = n' & \text{condizioni x il max} \\ \operatorname{sen} i + \operatorname{sen} \vartheta_b = \rho m \lambda_b & \text{eq. del grating} \end{cases}$$

$$2 \operatorname{sen} \frac{i+\vartheta_b}{2} \cos \frac{i-\vartheta_b}{2} = \rho m \lambda_b \Rightarrow 2 \cdot \operatorname{sen}\delta \cdot \cos \frac{i-2\delta+i}{2}$$

$$\Rightarrow \boxed{\rho m \lambda_b = 2 \operatorname{sen}\delta \cos \frac{\psi}{2}}$$

ψ = angolo coll. - camera $= i - \vartheta_b$

$\psi = 0$ Littrow Config. $\Rightarrow \rho m \lambda_b^{LIT} = 2 \operatorname{sen}\delta \Rightarrow$

$$\Rightarrow \rho m \lambda_b = \underbrace{\rho m \lambda_b^{LIT}}_{2\operatorname{sen}\delta} \cdot \cos\frac{\psi}{2} \Rightarrow$$

$$\Rightarrow \lambda_b = \lambda_b^{LIT} \cdot \cos \frac{\psi}{2}$$

$$R^{LIT} = \rho m W \frac{\lambda}{\alpha D_{TEL}} = \rho m W \frac{2 \operatorname{sen}\delta \cos\frac{\psi}{2}}{\rho m} \frac{1}{\alpha D_{TEL}} =$$

$$= \frac{D_{COLL}}{\cos\delta} \cdot 2 \operatorname{sen}\delta \cdot \frac{1}{\alpha D_{TEL}} = \frac{2 D_{COLL}}{\alpha D_{TEL}} \cdot \operatorname{tg}\delta$$

$$\boxed{\lambda_b = \frac{2 \operatorname{sen}\delta \cdot \cos(i-\delta)}{\rho m}}$$

angolo camera collimatore

$\psi = i - \vartheta_b$

$$i - \delta = i - \frac{i+\vartheta_b}{2} = \frac{2i - i - \vartheta_b}{2} = \frac{i-\vartheta_b}{2} = \frac{\psi}{2}$$

$$\boxed{\lambda_b = \frac{2 \operatorname{sen}\delta \cdot \cos\frac{\psi}{2}}{\rho m}}$$

$\beta = 0$ MAX $\Rightarrow \delta = \frac{i+\theta}{2}$ — condizione blaze max

$R = pm \frac{D_{CAM}}{\cos\theta} \cdot \frac{\lambda}{\alpha D_{TEL}}$ — è la <5>, eq. del grating.

$\lambda_b = \frac{2 \operatorname{sen}\delta \cos\psi/2}{pm}$ — condizione blaze max

$$R = pm\lambda \frac{D_C}{\cos\theta} \cdot \frac{1}{\alpha D_T} = pm \frac{2 \operatorname{sen}\delta \cos\psi/2}{pm} \cdot \frac{D_C}{\cos\theta} \cdot \frac{1}{\alpha D_{TEL}} \cdot \lambda$$

$$R = 2 \operatorname{tg}\delta \cdot \cos\frac{\psi}{2} \cdot D_{coll} \cdot \frac{1}{\alpha \cdot D_{TEL}}$$

$$= 2 \cdot \operatorname{tg}\delta \cdot \cos\frac{\psi}{2} \cdot f_C/S \cdot \frac{F_T}{F_C}$$

essendo $\begin{cases} F_C = f_C/D_C \\ F_T = f_T/D_T \end{cases}$

S: diametro fibra

$F_T = \eta_{FRD} F_C = \eta_{FRD} \cdot F_{FIBER}$