

**Momento zero:**

$$\int_{-\infty}^{+\infty} d\omega S(Q, \omega) = S(Q) \quad \int_{-\infty}^{+\infty} d\omega S_{self}(Q, \omega) = 1$$

**Momento primo:**

$$\int_{-\infty}^{+\infty} d\omega \omega S(Q, \omega) = \frac{\hbar Q^2}{2M} = \omega_r \quad \int_{-\infty}^{+\infty} d\omega \omega S_{self}(Q, \omega) = \frac{\hbar Q^2}{2M} = \omega_r$$

**Momento secondo:**

$$\int_{-\infty}^{+\infty} d\omega \omega^2 S(Q, \omega) = \frac{k_B T Q^2}{M} \quad \int_{-\infty}^{+\infty} d\omega \omega^2 S_{self}(Q, \omega) = \frac{k_B T Q^2}{M}$$

**Bilancio dettagliato**

(asimmetria degli spettri che riflette la maggior probabilità di processi Stokes)

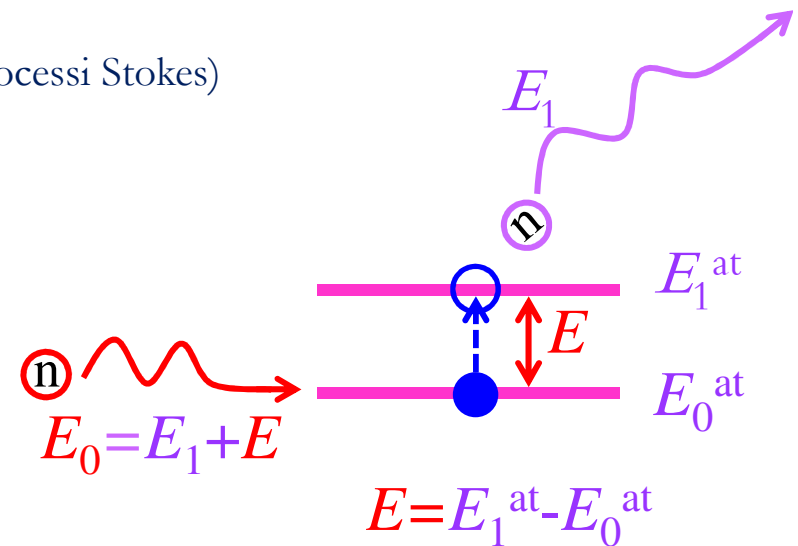
$$S(Q, -\omega) = S(Q, \omega) \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

$$P_{0 \rightarrow 1} \propto e^{-\beta E_0^{at}}$$

$$P_{1 \rightarrow 0} \propto e^{-\beta E_1^{at}}$$

$$P_{0 \rightarrow 1} \propto e^{-\beta E_0^{at}} = e^{-\beta E_0^{at}} e^{-\beta E_1^{at}} e^{\beta E_1^{at}} = e^{-\beta E_1^{at}} e^{\beta(E_1^{at} - E_0^{at})} = P_{1 \rightarrow 0} e^{\beta E}$$

$$P_{0 \rightarrow 1} > P_{1 \rightarrow 0} \Rightarrow P_{1 \rightarrow 0} = P_{0 \rightarrow 1} e^{-\beta \hbar \omega}$$



Il fattore di struttura dinamico nel limite del continuo ( $Q \rightarrow 0$ )  
(tripletto Rayleigh-Brillouin simmetrico)

$$S_{sym}(Q, \omega) = \frac{S(Q)}{\pi} \left[ A_0 \frac{z_H}{\omega^2 + z_H^2} + A_s \frac{z_s + (\omega + \omega_s)b}{(\omega + \omega_s)^2 + z_s^2} + A_s \frac{z_s - (\omega - \omega_s)b}{(\omega - \omega_s)^2 + z_s^2} \right]$$

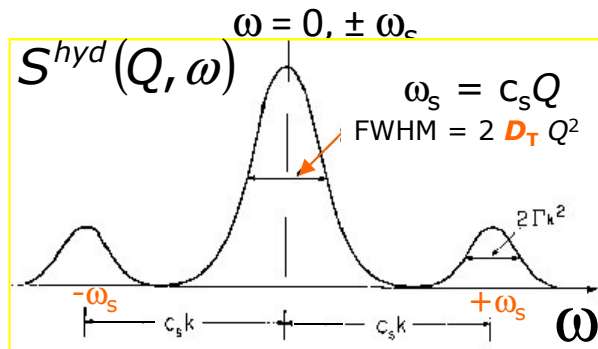
$$\gamma = \frac{c_p}{c_v} \quad A_0 = \frac{\gamma - 1}{\gamma} \quad A_s = \frac{1}{2\gamma} \quad b = \frac{((\gamma - 1)z_H + z_s)}{\omega_s} \quad z_H = D_T Q^2 \quad z_s = \Gamma Q^2 \quad \omega_s = c_s Q$$

Diffusività termica  $D_T = \frac{\lambda}{\rho c_p}$   
( $\lambda$  conducibilità termica)

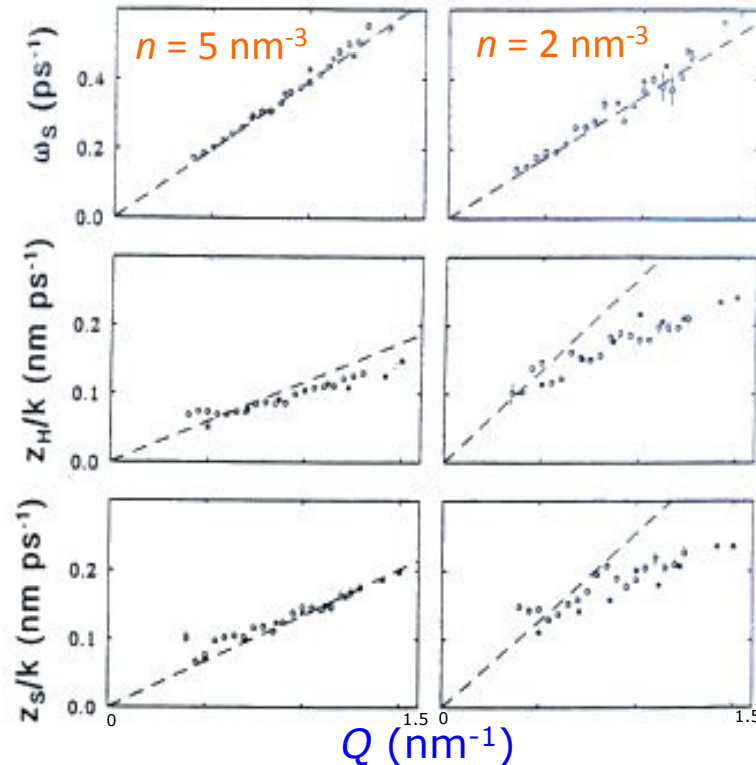
Viscosità cinematica  $\nu = \left( \eta_b + \frac{4}{3} \eta_s \right) \frac{1}{M\rho}$   
( $\eta_b$  viscosità di bulk,  $\eta_s$  viscosità di shear)

Coefficiente di attenuazione del suono  $\Gamma = \frac{(\gamma - 1)D_T + \nu}{2}$

3 lorentziane centrate a



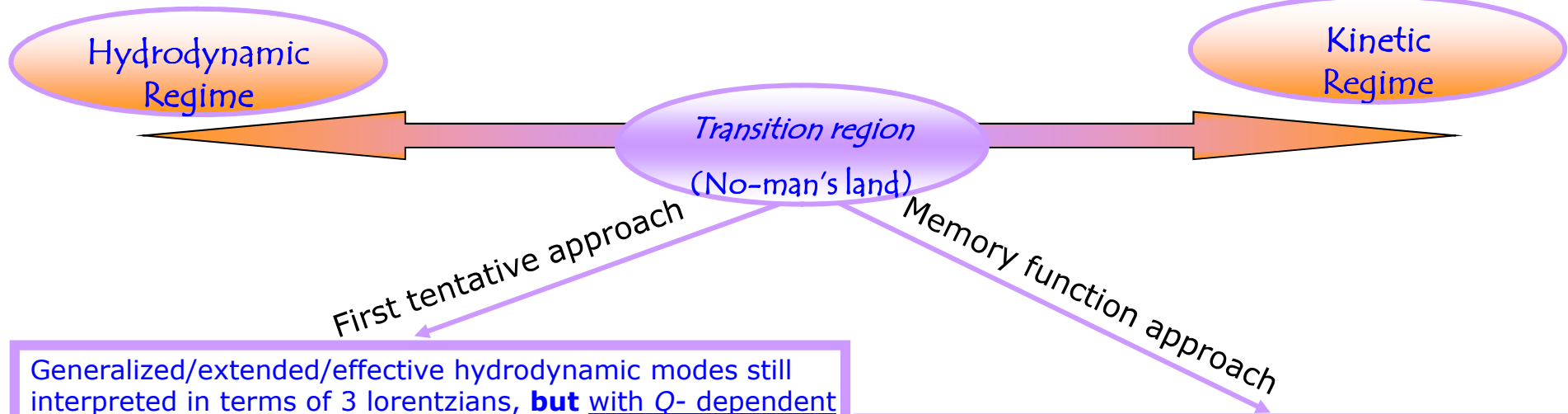
Velocità adiabatica del suono  $c_s = \frac{1}{\sqrt{\rho M \chi_s}}$   
( $\chi_s$  compressibilità adiabatica)



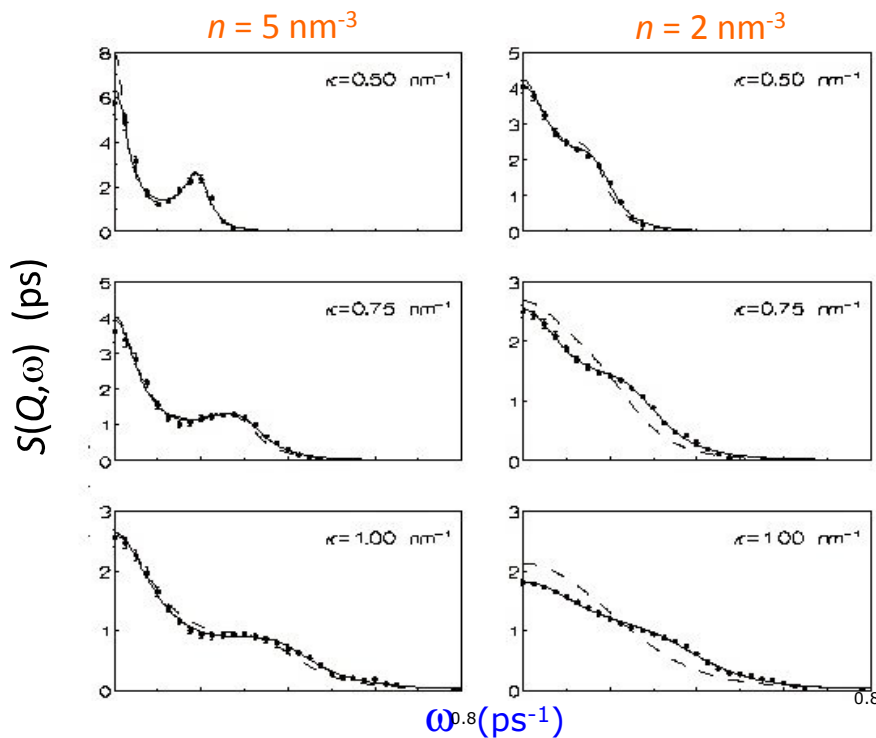
$^{36}\text{Ar}$

A densità più alta le deviazioni cominciano a  $Q$  maggiori per via del cammino libero medio minore

# Probing departures from hydrodynamic behaviour



Generalized/extended/effective hydrodynamic modes still interpreted in terms of 3 lorentzians, **but** with  $Q$ - dependent parameters



$$\ddot{F}(Q, t) + \int_0^t dt' M(Q, t-t') \dot{F}(Q, t') + \langle \omega_Q^2 \rangle F(Q, t) = 0,$$

Se:

$$M(Q, t) = 2\nu Q^2 \delta(t) + (\gamma - 1) \omega_Q^2 e^{-\gamma D_0 Q^2 t} \rightarrow \text{R-B}$$

Per  $\gamma=1$  DHO  $\rightarrow I(Q, \omega) = \frac{1}{\pi} \frac{2\Gamma\Omega^2}{(\omega^2 - \Omega^2)^2 + 4\Gamma^2\omega^2}$ ,  
 doppietto a  $\omega \neq 0$  in underdamp. e ad  $\omega=0$  in overdamp.

$$M(Q, t) = (\omega_L^2 - \gamma\omega_Q^2) e^{-t/\tau} + (\gamma - 1) \omega_Q^2 e^{-\Gamma_0 t}$$

Modello viscoelastico

# Confronto idrodinamica generalizzata e modello visco elastico

$$\ddot{F}(Q, t) + \int_0^t dt' M^{(2)}(Q, t-t') \dot{F}(Q, t') + \langle \omega^2 \rangle F(Q, t) = 0,$$

*GH model*       $M^{(2)}(Q, t) = 2B(Q)\delta(t) + (\gamma(Q) - 1) \langle \omega_Q^2 \rangle e^{-\Gamma_T(Q)t}$       *three-line spectrum*

*VE model*       $M^{(2)}(Q, t) = \left[ \left( \frac{\langle \omega^4 \rangle}{\langle \omega^2 \rangle} - \langle \omega^2 \rangle \right) - \gamma(Q) \langle \omega^2 \rangle \right] e^{-t/\tau(Q)} + [\gamma(Q) - 1] \langle \omega^2 \rangle e^{-\Gamma_T(Q)t}$

*four-line spectrum*

$$I(Q, \omega) = \frac{1}{\pi} \left( I_h \frac{z_h}{\omega^2 + z_h^2} + I_2 \frac{z_2}{\omega^2 + z_2^2} + I_s \frac{z_s + b_s(\omega + \omega_s)}{(\omega + \omega_s)^2 + z_s^2} + I_s \frac{z_s - b_s(\omega - \omega_s)}{(\omega - \omega_s)^2 + z_s^2} \right)$$