ESERCIZI

1 Determinazione del periodo

- $\cos(7x) \Rightarrow T = \frac{2\pi}{7}$

- $f(x) = \cos\left(\frac{x}{3} + \frac{\pi}{8}\right)$

$$f(x) = f(x+T)$$

NO!!! $\rightarrow \cos\left(\frac{x}{3} + \frac{\pi}{8}\right) = \cos\left(\frac{x}{3} + \frac{\pi}{8} + 2\pi\right) \leftarrow$ NO!!!

$$f(x+T) = \cos\left(\frac{x+T}{3} + \frac{\pi}{8}\right) = f(x) = \cos\left(\frac{x}{3} + \frac{\pi}{8}\right)$$

$$\frac{x+T}{3} + \cancel{\frac{\pi}{8}} = \frac{x}{3} + \cancel{\frac{\pi}{8}} + 2\pi$$

$$\frac{T}{3} = 2\pi \Rightarrow T = 6\pi$$

- $f(x) = 2\sin x + 5\cos(2x)$

$$f(x+T) = 2\,\text{sen}(x+T) + 5\cos(2x+2T)$$

$$T = 2\pi$$

- $|\text{sen}\,x| \Rightarrow T = \pi$
- $\text{tg}(\pi x) = \text{tg}(\pi x + \pi T) \Rightarrow T = 1$
- $(\text{sen}\,x)(\cos x) = \frac{1}{2}\,\text{sen}(2x) \Rightarrow T = \pi$
- $(\text{sen}\,x + 1)\cos x = \frac{1}{2}\,\text{sen}(2x) + \cos x \Rightarrow T = 2\pi$

Equazioni

2
- $\sin x = 3$ non ammette soluzione perché $|\text{sen}\,x| \leq 1 \;\; \forall x \in \mathbb{R}$

- $2\sin^2 x + \sin x = 0$

$$\sin x(2\sin x + 1) = 0$$

$\sin x = 0$ o $2\sin x = -1$

$x = k\pi,\; k \in \mathbb{Z}$ $\qquad \sin x = -\frac{1}{2} \Rightarrow x = -\frac{\pi}{6} + 2k\pi$

$x = +\frac{7}{6}\pi + 2k\pi \qquad k \in \mathbb{Z}$

- $\sin x = \text{sen}\,1$

$x = 1 + 2k\pi$

o $x = \pi - 1 + 2k\pi \qquad k \in \mathbb{Z}$

E • $5\sin(\pi - x) + 4 - 2\cos^2 x = 0$

$5\sin x + 4 - 2 + 2\sin^2 x = 0$

(1) $2\sin^2 x + 5\sin x + 2 = 0$

$\sin x = t$

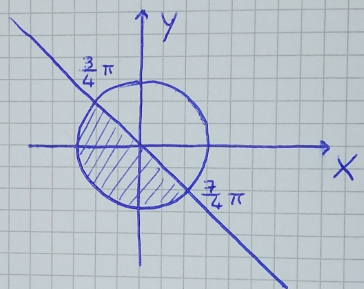
$2t^2 + 5t + 2 = 0 \qquad t = \dfrac{-5 \pm \sqrt{25 - 16}}{4} = \dfrac{-5 \pm 3}{4} = \begin{cases} -\frac{1}{2} \\ -2 \leftarrow \text{No} \end{cases}$

$\sin x = -\dfrac{1}{2}$

$x = -\dfrac{\pi}{6} + 2k\pi \quad \text{o} \quad x = \dfrac{7}{6}\pi + 2k\pi \quad \forall k \in \mathbb{Z}$

(3) Disequazioni e domini

• $\sin x < -\cos x$



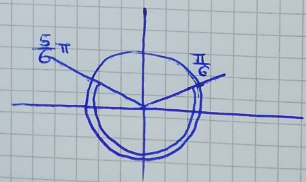
$\sin x = Y$

$\cos x = X \qquad X^2 + Y^2 = 1$

$Y + X < 0$

$x \in \bigcup_{k \in \mathbb{Z}} \left(\dfrac{3}{4}\pi + 2k\pi, \dfrac{7}{4}\pi + 2k\pi\right)$

• $\sin x \cos x < \dfrac{1}{4}$

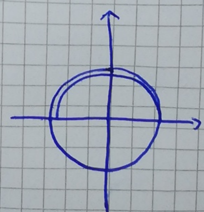


$\dfrac{1}{2}\sin 2x < \dfrac{1}{4}$

$\sin 2x < \dfrac{1}{2}$

$\dfrac{5}{6}\pi + 2k\pi < x < \dfrac{13}{6}\pi + 2k\pi \qquad k \in \mathbb{Z}$

• Dominio di $f(x) = \ln(\sin x)$



$\sin x > 0$

$D_f = \bigcup_{k \in \mathbb{Z}} \left]2k\pi, \pi + 2k\pi\right[$

• Dominio di $f(x) = \dfrac{1}{\sqrt{3} + tg(3x)}$

$$tg(3x) \neq -\sqrt{3}$$

$$3x \neq -\frac{\pi}{3} + k\pi \qquad k \in \mathbb{Z}$$

$$x \neq -\frac{\pi}{9} + k\frac{\pi}{3} \qquad k \in \mathbb{Z}$$

$$\mathcal{D}_f = \mathbb{R} \setminus \left\{ x = -\frac{\pi}{9} + k\frac{\pi}{3} \;\middle|\; k \in \mathbb{Z} \right\}$$

• $f(x) = \dfrac{1}{\sqrt{arctg\left(\frac{x+2}{x}\right)}}$

$$\begin{cases} arctg\left(\frac{x+2}{x}\right) > 0 \\ x \neq 0 \end{cases} \qquad \begin{cases} \frac{x+2}{x} > 0 \\ x \neq 0 \end{cases}$$

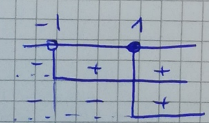
$$\mathcal{D}_f = \{ x \in \mathbb{R} \mid x < -2 \ \vee \ x > 0 \} = (-\infty, -\ \ ) \cup (0, +\infty)$$

• $f(x) = \ln\left(\arccos\left(\frac{x-3}{x+1}\right)\right)$

$$\arccos \frac{x-3}{x+1} > 0$$

$$-1 \leq \frac{x-3}{x+1} < 1$$

$$\begin{cases} \frac{x-3}{x+1} < 1 \\ \frac{x-3}{x+1} \geq -1 \end{cases} \longrightarrow \begin{cases} \frac{-4}{x+1} < 0 \longrightarrow x > -1 \\ \frac{2x-2}{x+1} \geq 0 \longrightarrow x < -1 \vee x \geq 1 \end{cases}$$



$$\mathcal{D}_f = [1, +\infty)$$

- Dimostrare che $f(x) = e^{2x} + 4e^{x}$ è invertibile su $\mathbb{R}$. Determinare f^{-1}.

- $f(x)$ è iniettiva, infatti $\forall x_1, x_2 \in \mathbb{R}$ t.c. $f(x_1) = f(x_2)$

risulta
$$e^{2x_1} + 4e^{x_1} = e^{2x_2} + 4e^{x_2}$$

$$e^{2x_1} - e^{2x_2} = 4(e^{x_2} - e^{x_1})$$

$$(e^{x_1} - e^{x_2})(e^{x_1} + e^{x_2}) = -4\,(e^{x_1} - e^{x_2})$$

→ Se $e^{x_1} - e^{x_2} = 0$,

allora $\begin{cases} e^{x_1} = e^{x_2} \\ 0 = 0 \end{cases} \Rightarrow x_1 = x_2$

→ Se $e^{x_1} - e^{x_2} \neq 0$

allora $\cancel{(e^{x_1} - e^{x_2})}(e^{x_1} + e^{x_2}) = -4\,\cancel{(e^{x_1} - e^{x_2})}$

$e^{x_1} + e^{x_2} = -4$: impossibile.

$\Downarrow$

$x_1 = x_2 \Rightarrow f$ è iniettiva.

- f è suriettiva. Sia $y \in Im(f)$, cioè $y > 0$.

$$y = e^{2x} + 4e^{x}$$

$$e^{x} = t \Rightarrow y = t^2 + 4t$$

$$t^2 + 4t - y = 0$$

$$t = -2 \pm \sqrt{4 + y}$$

$$\Downarrow$$

$$e^{x} = -2 + \sqrt{4+y}$$

$$x = \log\left(\sqrt{4+y} - 2\right) \in \mathbb{R} \text{ se } y > 0.$$

- f è iniettiva e suriettiva $\Rightarrow$ biettiva e $f^{-1}(y) = \log\left(\sqrt{4+y} - 2\right)$

$f^{-1} : (0, +\infty) \longrightarrow \mathbb{R}$

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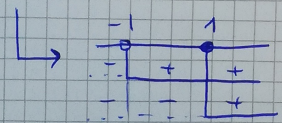

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