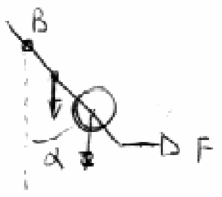
27/04/2017 - Problema 1

α positivo antiorario (verso positivo per le rotazioni quello uscente dal foglio)

a) 

Condizione di equilibrio statico

$$M_\alpha^{(e)} = 0 = -m_1 g \cdot \frac{L}{4}\sin\alpha - m_2 g \cdot \frac{L}{2}\sin\alpha + F \cdot \frac{3}{4}L\cos\alpha$$

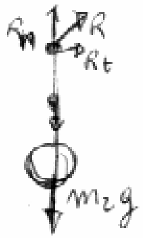
$$\Rightarrow F = g\,\frac{\sin\alpha\,(L/4\, m_1 + L/2\, m_2)}{\cos\alpha} \cdot \frac{4}{3L} = g \cdot \tan\alpha \cdot \frac{(m_1 + 2m_2)}{3}$$

b) Soppressa F - Conservazione energia meccanica

$$\frac{1}{2}I\dot\alpha^2 = m_1 g \frac{L}{4}(1-\cos\alpha) + m_2 g \frac{L}{2}(1-\cos\alpha)$$ con $I = \frac{1}{12}m_1L^2 + m_1\left(\frac{L}{4}\right)^2 + \frac{2}{5}m_2R^2 + m_2\left(\frac{L}{2}\right)^2 = m_1L^2\left(\frac{1}{12}+\frac{1}{16}\right) + m_2\left(\frac{2}{5}R^2 + \frac{L^2}{4}\right)$

$$\dot\alpha = \sqrt{\frac{2}{I}\left[m_1 g \frac{L}{4}(1-\cos\alpha) + m_2 g \frac{L}{2}(1-\cos\alpha)\right]}$$

c) Reazione vincolare in B nella condizione del punto b)



$$z_{cm} = \frac{L/4\, m_1 + L/2\, m_2}{m_1 + m_2} = L \cdot \frac{(m_1 + 2m_2)}{4(m_1+m_2)}$$

II^a eq. d. m. $I\ddot\alpha = M_\alpha^{(e)}$ ma $M_\alpha^{(e)} = 0 \Rightarrow \ddot\alpha = 0$

I^a eq. d. m. $\vec{F}^{(e)} = (m_1+m_2)\vec{a}_{cm}$ con $\vec{a}_{cm} = z_{cm}\cdot\ddot\alpha\,\underline{t} + \frac{v_{cm}^2}{z_{cm}}\,\underline{n}$

componente tangenziale $R_t = -z_{cm}\ddot\alpha = 0 \Rightarrow R_t = 0$

componente normale $R_n - (m_1+m_2)g = (m_1+m_2)\frac{v_{cm}^2}{z_{cm}} = (m_1+m_2)\frac{z_{cm}^2\dot\alpha^2}{z_{cm}}$

Infine $R_n = (m_1+m_2)\left[g + z_{cm}\dot\alpha^2\right]$

d) Come b) ma con momento frenante costante di modulo $\mathcal{M}$

Conservazione energia

$$\frac{1}{2}I\dot\alpha^2 = m_1 g \frac{L}{4}(1-\cos\alpha) + m_2 g \frac{L}{2}(1-\cos\alpha) - \mathcal{M}\alpha$$ da cui

$$\dot\alpha = \sqrt{\frac{2}{I}\left[m_1 g \frac{L}{4}(1-\cos\alpha) + m_2 g \frac{L}{2}(1-\cos\alpha) - \mathcal{M}\alpha\right]}$$

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e) a) $F = g \cdot \tan\alpha \cdot \frac{(m_1+2m_2)}{3} = 9.8 * \sqrt{3} * \frac{(5+2)}{3} = 39.6$ N

b) $I = m_1\frac{L^2}{4}\left(\frac{1}{3}+\frac{1}{4}\right) + m_2\left(\frac{2}{5}R^2 + \frac{L^2}{4}\right) = 5 * \frac{1}{4} * \frac{7}{12} + 1 * \left(\frac{2}{5} * 0.01 + \frac{1}{4}\right) = 0.983$ kg m²

$$\dot\alpha = \sqrt{\frac{2}{0.983}[9.8 * 1] * \left[\frac{5}{4} * \left(1-\frac{1}{2}\right) + \frac{1}{2}\left(1-\frac{1}{2}\right)\right]} = \sqrt{\frac{19.6}{0.983}\left[\frac{5}{8}+\frac{1}{4}\right]} = \sqrt{\frac{19.6}{0.983} * \frac{7}{8}} = 4.18 \text{ rad/s}$$

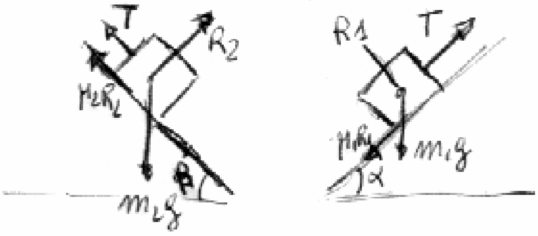
c) $z_{cm} = \frac{5/4 + 1/2}{4(5+1)} = \frac{7}{24} = 0.292$ m

$R_n = (m_1+m_2)\left[g + z_{cm}\cdot\dot\alpha^2\right] = [6]\left[9.8 + 0.292 * 4.18^2\right] = 89.4$ N

d) $\dot\alpha = \sqrt{\frac{19.6}{0.983}\cdot\frac{7}{8} - \frac{2}{0.983}\cdot 5 \cdot \frac{\pi}{3}} = \sqrt{17.45 - 10.65} = 2.61$ rad/s

27/04/2017 - Problema 2

a) m_{2max} per equilibrio statico $\rightarrow \vec{F}^{(e)} = 0$ $\qquad \alpha = 30°, \beta = 60°$



Per il corpo 2 $\rightarrow R_2 = m_{2max}\, g \cos\beta$

$T + \mu_2 R_2 = m_{2max}\, g \,\text{sen}\,\beta$

$T = m_{2max}\, g\, \text{sen}\beta - \mu_2 m_{2max}\, g \cos\beta$

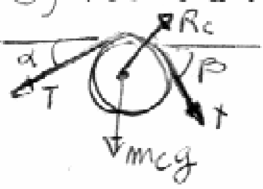
Per il corpo **1** $\rightarrow R_1 = m_1 g \cos\alpha$

$T = m_1 g\, \text{sen}\alpha + \mu_1 m_1 g \cos\alpha$

Uguagliando le due espressioni di T

$$m_{2max} \cdot g(\text{sen}\beta - \mu_2 g\cos\beta) = m_1 g(\text{sen}\alpha + \mu_1\cos\alpha) \rightarrow m_{2max} = \frac{m_1 (\text{sen}\alpha + \mu_1\cos\alpha)}{(\text{sen}\beta - \mu_2\cos\beta)}$$

b) Reazione vincolare su carrucola



Per equilibrio carrucola: $R_{cy} = m_c g + T\,\text{sen}\alpha + T\,\text{sen}\beta$

$R_{cx} = T\cos\alpha - T\cos\beta = T(\cos\alpha - \cos\beta)$

c) $m_2 = 2 \cdot m_{2max} \rightarrow$ accelerazione $\ddot{s}$

Per la rigidità del sistema $\ddot{s} = \ddot{s}_1 = \ddot{s}_2$

Applicando la I^a eq. card. dinamica si ha

corpo 1 $\rightarrow m_1 \ddot{s} = T - m_1 g\,\text{sen}\alpha - \mu_1 m_1 g\cos\alpha$

corpo 2 $\rightarrow m_2 \ddot{s} = m_2 g\,\text{sen}\beta - T - \mu_2 m_2 g \cos\beta$

Ricavando T dalla prima e sostituendolo nella seconda:

$m_2\ddot{s} = m_2 g(\text{sen}\beta - \mu_2 \cos\beta) - m_1\ddot{s} - m_1 g(\text{sen}\alpha + \mu_1\cos\alpha)$ da cui si ottiene

$$\ddot{s} = \frac{m_2 g(\text{sen}\beta - \mu_2 \cos\beta) - m_1 g(\text{sen}\alpha + \mu_1\cos\alpha)}{m_1 + m_2}$$

d) come c ma rotazione carrucola (senza strisciamento corda)

Avremo che $\ddot{s} = \ddot{s}_1 = \ddot{s}_2$ e inoltre $\ddot{s} = R\ddot{\varphi}$

Le eq. cardinali diventano

corpo 1 $\rightarrow m_1\ddot{s} = T_1 - m_1 g\,\text{sen}\alpha - \mu_1 m_1 g\cos\alpha$

corpo 2 $\rightarrow m_2\ddot{s} = -T_2 + m_2 g\,\text{sen}\beta - \mu_2 m_2 g\cos\beta$

carrucola $\rightarrow I_O\ddot{\varphi} = R(T_2 - T_1) = \frac{1}{2} m_c R^2 \ddot{\varphi} = \frac{1}{2} m_c R\ddot{s}$

Sommando e sostituendo il valore di $T_2 - T_1$ si ottiene

$(m_1 + m_2)\ddot{s} = -\frac{1}{2} m_c \ddot{s} + m_2 g(\text{sen}\beta - \mu_2 \cos\beta) - m_1 g(\text{sen}\alpha + \mu_1\cos\alpha)$ da cui

$$\ddot{s} = \frac{m_2 g(\text{sen}\beta - \mu_2 \cos\beta) - m_1 g(\text{sen}\alpha + \mu_1\cos\alpha)}{(m_1 + m_2 + m_c/2)}$$

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e) a) $m_{2max} = \dfrac{10 \cdot (0.5 + 0.1\cdot\sqrt{3}/2)}{(\sqrt{3}/2 - 0.2\cdot 0.5)} = 10 \cdot 0.587/0.766 = 7.66\ kg$

b) $T = m_1 g(\text{sen}\alpha + \mu_1\cos\alpha) = 10 \cdot 9.8 \cdot (0.587) = 57.5\,N$

$R_{cy} = m_c g + T(\text{sen}\alpha + \text{sen}\beta) = 1\cdot 9.8 + 57.5\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 88.3\,N$

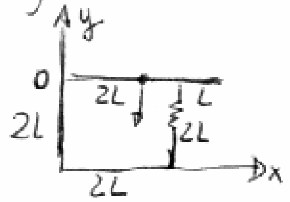
$R_{cx} = T(\cos\alpha - \cos\beta) = 57.5\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 21.0\,N$

c) $\ddot{s} = 9.8 \cdot [2\cdot 7.66(\sqrt{3}/2 - 0.2\cdot 0.5) - 10\cdot(0.5 + 0.1\cdot\sqrt{3}/2)]/(2\cdot 7.66 + 10) = 2.27\,m\,s^{-2}$

d) $\ddot{s} = 9.8 \cdot [\qquad]/(2\cdot 7.66 + 10 + 0.5) = 2.23\,m\,s^{-2}$

27/04/2017 - Problema 3

a) Posizione orizzontale di equilibrio statico



$M_O^{(e)} = 1.5L \cdot m_a g - 2L \cdot k(4L - 2L) = 0$

$\Rightarrow 1.5L \cdot m_a g = 4L^2 k \Rightarrow k = \frac{1.5 \cancel{L} m_a g}{4L^{\cancel{2}}} = \frac{3}{8}\frac{m_a g}{L}$

b) Reazione vincolare in O $\rightarrow R_y, R_x$

Per equilibrio statico asta $\rightarrow \vec{F}^{(e)} = 0 \rightarrow \vec{F}_x^{(e)} = R_x = 0$

$\rightarrow F_y^{(e)} = R_y - m_a g + k \cdot 2L = 0$

$R_y = m_a g - 2kL$

c) Caduta libera corpo di massa m_A

$y = y_0 - \frac{1}{2} g t^2 \rightarrow y_f - y_0 = -L = -\frac{1}{2} g T^2 \rightarrow T = \sqrt{\frac{2L}{g}}$

d) Urto anelastico $\rightarrow$ conservazione del momento angolare L

$L_{\text{prima urto}} = m_A \cdot v \cdot 3L$

$L_{\text{dopo urto}} = I_{asta} \cdot \dot{\varphi}_0 + m_A \cdot 3L \cdot 3L \dot{\varphi}_0 = \dot{\varphi}_0 (I_{asta} + 9 m_A L^2)$

Per la caduta libera $v = \sqrt{2g \cdot L}$ e quindi uguagliando i due L si ha

$m_A \cdot \sqrt{2gL} \cdot 3L = \dot{\varphi}_0 (I_{asta} + 9 m_A L^2) = \dot{\varphi}_0 (\frac{1}{12} m_a 9L^2 + 9 m_A L^2 + m_a (1.5L)^2)$

ed infine $\dot{\varphi}_0 = \frac{m_A \cdot \sqrt{2gL} \cdot 3L}{(\frac{1}{12} m_a 9L^2 + 9 m_A L^2 + m_a \cdot 2.25L^2)} = \frac{m_A \cdot \sqrt{2gL} \cdot 3L}{(\frac{1}{3} m_a 9L^2 + 9 m_A L^2)} = \frac{m_A \sqrt{2gL}}{L(m_a + 3m_A)}$

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e) a) $k = \frac{3}{8}\frac{m_a g}{L} = \frac{3}{8} \cdot \frac{8 \cdot 9.8}{0.5} = 58.8\ N/m$

b) $R_x = 0$, $R_y = m_a g - 2kL = 8 \cdot 9.8 - 2 \cdot 58.8 \cdot 0.5 = 19.6 N$

c) $T = \sqrt{\frac{2L}{g}} = \sqrt{\frac{1}{9.8}} = 0.319\ s$ $\quad v = \sqrt{2gL} = 3.13\ m/s$

d) $\dot{\varphi}_0 = \frac{m_A \cdot \sqrt{2gL} \cdot 3L}{(\frac{1}{12} m_a L^2 + 9 m_A L^2 + m_a 2.25L^2)} = \frac{1 \cdot \sqrt{2 \cdot 9.8 \cdot 0.5} \cdot 3 \cdot 0.5}{(\frac{1}{12} \cdot 8 \cdot 0.5^2 + 3 \cdot 1 \cdot 0.5^2)} = \frac{4.70}{0.917} = 5.13\ rad/s$