ESERCIZI SU SUP & INF: Determinare sup e inf dei seguenti insiemi.

Specificare in quali casi si tratta di max e min.

(1) $A_1 = \left\{ \frac{1+n}{n} \mid n \in \mathbb{N}^* \right\}$

(2) $A_2 = \left\{ \frac{1-n}{n} \mid n \in \mathbb{N}^* \right\}$

(3) $A_3 = \left\{ (-1)^n \mid n \in \mathbb{N}^* \right\}$

(4) $A_4 = \left\{ (-1)^n \frac{n-1}{n} \mid n \in \mathbb{N}^* \right\}$

(5) $A_5 = \left\{ \frac{xy}{x^2+y^2} \mid (x,y) \in \mathbb{R}^2_*, x < y \right\}$

(6) $A_6 = \left\{ \frac{xy}{x+y} \mid (x,y) \in (0,1) \times (0,1) \right\}$

(7) $A_7 = \{ |\pi - n| \mid n \in \mathbb{N} \}$

(8) $S_\lambda \doteq \left\{ \frac{n^\lambda + k^{1/\lambda}}{n+k} \mid k,n \in \mathbb{N},\ (k,n) \neq (0,0) \right\}$
al variare di $\lambda \in \mathbb{R}$

(9) $A_9 = \left\{ x \in \mathbb{R} \mid \frac{x(x^2-8x+7)}{\sqrt{x^2-2x-2}} \geq 0 \right\}$

(10) $A_{10} = \{ x \in \mathbb{R} \mid \sin x < 0,\ x^2 - 13x + 22 \leq 0 \}$

(11) Se a_n, b_n sono successioni limitate, allora
$\sup(a_n + b_n) \leq \sup a_n + \sup b_n$
$\inf(a_n + b_n) \geq \inf a_n + \inf b_n$