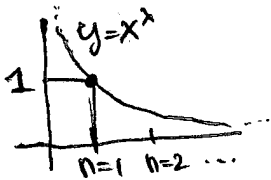


ES. (8) SUP & INF SOLUZIONE

$$S_\lambda = \left\{ \frac{n^\lambda + k^{1/\lambda}}{n+k} \mid k, n \in \mathbb{N}, (k, n) \neq (0, 0) \right\} \quad \lambda > 0$$

- $\lambda > 1$  considero  $(k, n) = (0, n) \Rightarrow \frac{n^\lambda + k^{1/\lambda}}{n+k} = n^{\lambda-1}$  illimitato per  $n \in \mathbb{N}$   
 $\Rightarrow \sup S_\lambda = +\infty$   
 considero  $(k, n) = (k, 0) \Rightarrow \frac{n^\lambda + k^{1/\lambda}}{n+k} = k^{\lambda-1} \rightarrow 0$  per  $k \rightarrow +\infty$   
 essendo  $0 < \frac{n^\lambda + k^{1/\lambda}}{n+k} \forall (n, k) \in \mathbb{N} \times \mathbb{N}$  segue  
 $\inf S_\lambda = 0$
- $\lambda < 1$  analogamente al caso precedente ma considerando  
 $(k, n) = (k, 0) \Rightarrow \frac{n^\lambda + k^{1/\lambda}}{n+k} = k^{\lambda-1} \rightarrow 0$  illimitato  $\Rightarrow \sup S_\lambda = +\infty$   
 $(k, n) = (0, n) \Rightarrow \frac{n^\lambda + k^{1/\lambda}}{n+k} = n^{\lambda-1} \rightarrow 0$  per  $n \rightarrow +\infty$   $\Rightarrow \inf S_\lambda = 0$
- $\lambda = 1$   $\frac{n^\lambda + k^{1/\lambda}}{n+k} \equiv 1 \Rightarrow \sup S_\lambda = \inf S_\lambda = 1 = \max = \min$

Per  $\lambda < 0$  considero  $S_\lambda = \left\{ \frac{n^\lambda + k^{1/\lambda}}{n+k} \mid k, n \in \mathbb{N}^* \right\}$



0 < n  $\forall n \in \mathbb{N}^* \quad n^\lambda \leq 1 \leq n$   
 $\forall k \in \mathbb{N}^* \quad k^{1/\lambda} \leq 1 \leq k \Rightarrow 0 < \frac{n^\lambda + k^{1/\lambda}}{n+k} \leq 1$

quindi 1 è maggiorante. Inoltre per  $n=k=1$  si ha  
 $\frac{n^\lambda + k^{1/\lambda}}{n+k} = \frac{1+1}{1+1} = 1 \Rightarrow 1 \text{ è } \max S_\lambda$

considero  $k=1$

$$0 < \frac{n^\lambda + k^{1/\lambda}}{n+k} = \frac{n^\lambda + 1}{n+1} \xrightarrow{\lim_{n \rightarrow +\infty}} 0 \Rightarrow \inf S_\lambda = 0$$

RICAPITOLANDO

$$\sup S_\lambda = \begin{cases} +\infty & \text{se } \lambda \in (0, 1) \cup (1, +\infty) \\ 1 & \lambda = 1 \text{ (è max)} \\ 1 & \lambda < 0 \text{ (è max)} \end{cases} \quad \inf S_\lambda = \begin{cases} 0 & \text{se } \lambda \in (0, 1) \cup (1, +\infty) \\ 1 & \lambda = 1 \text{ (è min)} \\ 0 & \lambda < 0 \end{cases}$$