

# ESERCIZI sui LIMITI di SUCCESSIONE

verificare tramite la definizione, i seguenti limiti

- $\lim_n \frac{1}{n} = 0$
  - $\lim_n \frac{(-1)^n}{n} = 0$
  - $\lim_n \frac{n^2-n}{n+1} = +\infty$
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| } | <ul style="list-style-type: none"> <li>• <math>\lim_n 2^n = +\infty</math></li> <li>• <math>\lim_n \left(\frac{2}{3}\right)^n = 0</math></li> <li>• <math>\lim_n \frac{3n-1}{n+3} = 3</math></li> </ul> |
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$$\lim_n \frac{a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0}{b_m n^m + b_{m-1} n^{m-1} + \dots + b_1 n + b_0} = \begin{cases} \pm \infty & \text{se } k > m \\ a_k/b_k & \text{se } k = m \\ 0 & \text{se } k < m \end{cases}$$

Calcolare i seguenti limiti:

1.  $\lim_n \frac{(n+1)(n+2)(n+3)}{n^3}$
2.  $\lim_n \sqrt{n+1} - \sqrt{n}$
3.  $\lim_n \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$
4.  $\lim_n \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n-1}{n^2}\right)$
5.  $\lim_n n^2 (1 - \cos(1/n))$
6.  $\lim_n \left(\cos\left(\frac{1}{n}\right)\right)^{n^2}$
7.  $a_n = \begin{cases} A > 0 & n \text{ pari} \\ B > 0 & n \text{ dispari} \end{cases}, B > A. \quad \lim_n \sqrt[n]{a_n}$
8.  $\lim_n \frac{n}{\sqrt[n]{n!}}$
9.  $\lim_n \sqrt[n]{2^n + 3^n}$
10.  $\lim_n \sqrt[n]{A^n + B^n} \quad (= \max\{A, B\}, A, B > 0)$
11. sia  $a_n \rightarrow +\infty$ , allora calcolare  $\lim_n \left(1 + \frac{1}{a_n}\right)^{a_n} (= e)$
12.  $\lim_n \left(1 + \sin\left(\frac{1}{n}\right)\right)^{1/\sin(1/n)}$
13.  $\lim_n \left(1 + \frac{3}{n}\right)^{n^2}$
14.  $\lim_n \sin\left(\frac{n\pi}{4}\right)$  [dimostrare che  $\neq$ ]
15.  $\lim_n \left(\sqrt[3]{n+4} - \sqrt[3]{n-1}\right) \sqrt{n}$

16.  $\lim_n \frac{n^2 2^n}{3^n}$
17.  $\lim_n \frac{n^2 - \sin n}{3^n + n^3}$
18.  $\lim_n \frac{\sqrt{n+1}}{n} \sin(n!)$
19.  $\lim_n \frac{n^2 + n^2(1 - \cos(1/n))}{n^2 + n}$
20.  $\lim_n \frac{\sin(1/n)}{n(\cos(1/n) - 1)}$
21.  $\lim_n \sum_{k=1}^n \frac{k^2}{n^3}$
22.  $\lim_n (\sqrt[n]{a} - 1)^n \quad a \geq 0$
23.  $\lim_n \left(1 + \frac{1}{n^2}\right)^n$
24.  $\lim_n \arctan\left(\frac{n\pi}{4}\right)$
25.  $\lim_n \frac{n^2}{n^4 + 3n^2 + 2} \sin\left(\frac{n^2}{n^2 + (-1)^n}\right)$
26.  $\lim_n \frac{(n^2 + 2n + 3)^{2n^2 - 6n + 1}}{(n+1)^2}$
27.  $\lim_n \frac{n^{2n} (2n)!}{\alpha^n (n!)^4} \quad \text{per } \alpha \in \mathbb{R}$
28.  $\lim_n \alpha^n \left[\binom{3n}{n}\right]^{1/n} \quad \text{per } \alpha \in \mathbb{R}$
29.  $\lim_n \left(\cos\left(\frac{1}{n}\right)\right)^{n^2} - \frac{1}{\sqrt{e}}$