$$(17)\quad x\left(\operatorname{atn}(\log x)-\operatorname{atn}x\right)\xrightarrow{x\to+\infty}-\infty$$

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$$f(x)=x\left(-\operatorname{atn}\left(\frac{1}{\log x}\right)+\frac{\pi}{2}+\operatorname{atn}\frac{1}{x}-\frac{\pi}{2}\right)=$$

$$=-\underbrace{\frac{x}{\log x}}_{\to+\infty}\cancel{\log x\operatorname{atn}\frac{1}{\log x}}^{\to 1}+\cancel{x\operatorname{atn}\frac{1}{x}}^{\to 1}$$

$$(18)\quad x\cos(\operatorname{atn}x)\xrightarrow{x\to+\infty}1$$

$$x\cos(\operatorname{atn}x)=x\cos\left(\frac{\pi}{2}-\operatorname{atn}\frac{1}{x}\right)=$$

$$=x\,\frac{\cancel{\operatorname{sen}\left(\operatorname{atn}\frac{1}{x}\right)}}{\cancel{\operatorname{atn}\frac{1}{x}}}^{\to 1}\cdot\operatorname{atn}\frac{1}{x}\longrightarrow 1$$

(14) $\dfrac{(\sqrt{e})^{\operatorname{sen} x} - \cos\sqrt{x}}{(\ln(1+\sqrt{x}))^2} \xrightarrow{x\to 0} 1$

$$f(x) = \left[\frac{e^{\frac{\operatorname{sen} x}{2}} - 1}{\frac{\operatorname{sen} x}{2}} \cdot \frac{\operatorname{sen} x}{2x} + \frac{1-\cos\sqrt{x}}{x} \right] \cdot \frac{x}{\ln(1+\sqrt{x})}$$

(15) $(\ln(\ln x))^{\ln x} - x\cdot(\ln x)^{\ln(\ln x)} \xrightarrow{x\to+\infty} +\infty$

$$f(x) = \exp\left(\ln x \cdot \ln(\ln(\ln x))\right) +$$
$$- \exp\left(\ln x + (\ln(\ln x))^2\right)$$

$$\lim_{x\to+\infty} f(x) \overset{y=\ln x}{=} \lim_{y\to+\infty} \left(e^{y\ln(\ln y)} - e^{y+\ln^2 y} \right) =$$

$$= \lim_{y\to+\infty} e^{y\ln(\ln y)} \left(1 - e^{y+\ln^2 y - y\ln(\ln y)} \right)$$

$$= +\infty$$

(16) $\left(1-\cos\frac{1}{x}\right)\ln\left(\frac{x^2\operatorname{sen} x}{2} + e^x\right) \xrightarrow{x\to+\infty} 0$

$$\ln\left(\frac{x^2\operatorname{sen} x}{2} + e^x\right) = x + \ln\left(1 + \frac{x^2\operatorname{sen} x}{2e^x}\right) \Rightarrow$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{1}{2x^2}\left(x + \ln\left(1+\frac{x^2\operatorname{sen} x}{2e^x}\right)\right) =$$

$$= \lim_{x\to+\infty} \frac{\operatorname{sen} x}{4e^x} = 0$$

(13)* $\lim\limits_{x\to 0} x \operatorname{tg}\left(ax + \operatorname{atn}\frac{b}{x}\right) =$

$$= \begin{cases} \dfrac{b}{1-ab} & ab \neq 1 \\ \infty(\operatorname{sgn} b) & ab = 1 \end{cases}$$

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$a = 0$: $f(x) \equiv b$

$a \neq 0$: $f(x) = x \dfrac{\operatorname{tg}(ax) + \frac{b}{x}}{1 - \operatorname{tg}(ax)\frac{b}{x}}$

se $ab \neq 1$:
$$\lim_{x\to 0} f(x) = \frac{b}{1-ab}$$

se $ab = 1$: (*) $\operatorname{atn} y + \operatorname{atn}\frac{1}{y} = \frac{\pi}{2}\operatorname{sgn} y \qquad y \neq 0$

$$\lim_{x\to 0^\pm} f(x) \underset{y = \frac{x}{b}}{=} \lim_{y\to 0^\pm(\operatorname{sgn} b)} b\, y \cdot \operatorname{tg}\left(y + \operatorname{atn}\frac{1}{y}\right) =$$

$$\lim_{y\to(\operatorname{sgn} b)0^\pm} b\, y \operatorname{tg}\left(y - \operatorname{atn} y + \frac{\pi}{2}\operatorname{sgn} y\right) =$$

(*) Più semplice: $y = x/b$

$$\lim_{y\to(\operatorname{sgn} b)0^\pm} (-by)\operatorname{ctg}\left(y - \operatorname{atn} y\right) =$$

$$f(x) = by\,\frac{\operatorname{tg} y + 1/y}{1 - \frac{\operatorname{tg} y}{y}} = b\,\frac{y \operatorname{tg} y + 1}{1 - \frac{\operatorname{tg} y}{y}}$$

$$= \lim_{y\to(\operatorname{sgn} b)0^\pm} (-by)\,\frac{1 + y\operatorname{tg} y}{y - \operatorname{tg} y} =$$

$$= -b \lim_{y\to(\operatorname{sgn} b)0^\pm} \frac{1 + y\operatorname{tg} y}{1 - \frac{\operatorname{tg} y}{y}} = +\infty \cdot \operatorname{sgn} b$$

(EX) $$\lim_{x\to 0^+}\left(1 - \operatorname{tg}(x^{1/\alpha})\right)^{\frac{1}{x^\alpha}} = \begin{cases} 1 & 0 < \alpha < 1 \\ e & \alpha = 1 \\ +\infty & \alpha > 1 \; (\nexists \; \alpha < 0 \end{cases}$$

difficile

la f.ne NON è definita in alcun intv.llo $(0,a)$ con $a >$

$$g_\alpha(x) = \frac{1 + x^{\alpha-3}}{1 + \frac{\operatorname{sen} x + x}{x^3}} \xrightarrow{x \to +\infty} \begin{cases} +\infty & \alpha > 3 \\ 2 & \alpha = 3 \\ 1 & \alpha < 3 \end{cases}$$

da cui se $\alpha \geq 3$:

$$\lim_{x \to +\infty} x \ln(g_\alpha(x)) = +\infty$$

segue:

$$\lim_{x \to +\infty} (g_\alpha(x))^x = +\infty$$

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se $\alpha < 3$:

$$\lim_{x \to +\infty} x \ln(g_\alpha(x)) = \lim_{x \to +\infty} x \cdot (g_\alpha(x) - 1)$$

$$x \cdot (g_\alpha(x) - 1) = x \cdot \frac{x^{\alpha-3} - \frac{\operatorname{sen} x + x}{x^3}}{1 + \frac{\operatorname{sen} x + x}{x^3}} =$$

$$= \frac{x^{\alpha-2} + \frac{\operatorname{sen} x + x}{x^2}}{1 + \frac{\operatorname{sen} x + x}{x^3}}$$

da cui:

$$\lim_{x \to +\infty} x \ln(g_\alpha(x)) = \begin{cases} +\infty & 2 < \alpha < 3 \\ 1 & \alpha = 2 \\ 0 & \alpha < 2 \end{cases}$$

da cui:

$$\lim_{x \to +\infty} (g_\alpha(x))^x = \begin{cases} +\infty & \alpha > 2 \\ e & \alpha = 2 \\ 1 & \alpha < 2 \end{cases}$$

$$= \lim_{x\to+\infty} \frac{\ln x + \ln\left(1+\frac{1}{x}\right)}{\ln(2x)+\ln\left(1-\frac{1}{2x}\right)} = \lim_{x\to+\infty} \frac{\ln x}{\ln(2x)} = 1$$

(9) $$\lim_{x\to 0} \frac{|\operatorname{sen}(\alpha x)|}{|x|^\alpha} = \begin{cases} 0 & \alpha<1 \\ 1 & \alpha=1 \\ +\infty & \alpha>1 \end{cases}$$

4

$$\frac{|\operatorname{sen}(\alpha x)|}{|x|^\alpha} = \frac{|\operatorname{sen}(\alpha x)|}{|\alpha x|} |\alpha| |x|^{1-\alpha}$$

(10) $$\lim_{x\to 0} \frac{1-\cos x}{x^2+|x|^\alpha} = \begin{cases} 0 & \alpha<2 \\ 1/4 & \alpha=2 \\ 1/2 & \alpha>2 \end{cases}$$

$$\frac{1-\cos x}{x^2+|x|^\alpha} = \frac{1-\cos x}{x^2} \frac{x^2}{x^2+|x|^\alpha} =$$

$$= \frac{1-\cos x}{x^2} \frac{1}{1+|x|^{\alpha-2}}$$

(EX) $$\lim_{x\to 0} \frac{1-\cos(\alpha x)}{|x|^2+|x|^{4-\alpha}} = \begin{cases} \alpha^2/2 & \alpha<2 \\ 1 & \alpha=2 \\ 0 & \alpha>2 \end{cases}$$

(11) $$\lim_{x\to+\infty} \frac{x\cdot\left(x^{\frac{1}{x}}-1\right)}{\ln x} =$$

$$\frac{\ln x}{x} = \ln\left(x^{\frac{1}{x}}\right)$$

$$\lim_{x\to+\infty} f(x) \overset{t=x^{\frac{1}{x}}}{=} \lim_{t\to 1} \frac{t-1}{\ln t} = 1$$

(12)* $$\lim_{x\to+\infty} \left(\frac{x^3+x^\alpha}{x^3+\operatorname{sen} x+x}\right)^x = \begin{cases} +\infty & \alpha>2 \\ e & \alpha=2 \\ 1 & \alpha<2 \end{cases}$$

$$= g_\alpha(x) \xrightarrow{x\to+\infty} \frac{\infty}{\infty}$$

Calcolo:

$$\lim_{x\to+\infty} x\cdot\ln(g_\alpha(x))$$

(6) Siano $a,b \in \mathbb{R}$ con $a \neq b$:

3

$$f(x) = \frac{\ln\left(1+\frac{a}{x}\right)}{e^{\frac{a}{x}} - e^{\frac{b}{x}}} \xrightarrow{x \to +\infty} 0$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\frac{a}{x}}{e^{\frac{a}{x}} - e^{\frac{b}{x}}} =$$

$$= \lim_{x\to+\infty} \frac{\frac{a}{a-b}}{e^{\frac{b}{x}}} = \begin{cases} \cancel{\frac{a}{a-b}}\ 1 & b=0 \\ \frac{a}{a-b} & b\neq 0 \end{cases}$$

(Ex) $\lim_{x\to 1} \frac{2\,\text{sen}(x-1)}{x^2-1} \cdot \frac{e^{x^2-1}-1}{x-1} = 2$

(7) $\lim_{x\to 0} \underbrace{\frac{x(2^x-3^x)}{1-\cos(3x)}}_{f(x)} = \frac{1}{9}\ln\left(\frac{2}{3}\right)$

$3^x\left(\left(\frac{2}{3}\right)^x - 1\right)$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2\cancel{x}}{9x^{\cancel{2}}}(2^x-3^x) =$$

$$= \lim_{x\to 0} \frac{3^x}{9} \frac{\left(\frac{2}{3}\right)^x - 1}{x}$$

(Ex) $\lim_{x\to 0} \frac{7^x - 2^x}{\text{sen}(\text{sen}^2(\text{sen}\sqrt{x}))} = \ln\frac{7}{2}$

(8) $\lim_{x\to+\infty} \ln(1+x)\,\text{sen}\left(\frac{1}{\ln(2x-1)}\right) = 1$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\ln(1+x)}{\ln(2x-1)} =$$

(3) $\lim_{x\to 0^+} (\ln(e+x))^{\frac{1}{\sqrt{x}}\sin\left(\frac{1}{x}+\frac{2}{x^3}\right)} = 1$

2

Basta calcolare:

$$\lim_{x\to 0^+} \frac{1}{\sqrt{x}} \sin\left(\frac{1}{x}+\frac{2}{x^3}\right) \ln(\ln(e+x)) = (+\infty \cdot 0$$

$$\ln(e+x) = \cancel{\ln e}^{1} + \ln(1+e^{-1}x)$$

$$\frac{\ln(\ln(e+x))}{\ln(1+e^{-1}x)} \longrightarrow 1$$

da cui se $\exists$ $\lim_{x\to 0^+}(\ldots) = \lim_{x\to 0^+} \frac{\sin\left(\frac{1}{x}+\frac{2}{x^3}\right)\ln(1+e^{-1}x)}{\sqrt{x}}$

e:

$$\sin\left(\frac{1}{x}+\frac{2}{x^3}\right)\frac{\ln(1+e^{-1}x)}{\sqrt{x}} \xrightarrow{x\to+\infty} 0$$

(4)* $\lim_{x\to+\infty} \frac{a_p x^p + a_{p-1}x^{p-1} + \ldots + a_1 x + a_0}{b_q x^q + b_{q-1}x^{q-1} + \ldots + b_1 x + b_0} =$

$$= \begin{cases} \operatorname{sgn}\left(\frac{a_p}{b_q}\right)\cdot(+\infty) & p>q \\ \frac{a_p}{b_q} & p=q \\ 0 & p<q \end{cases} \qquad \text{con } a_p \cdot b_q \neq 0$$

(5) $\lim_{x\to 0} \frac{\log(\cos x)}{\operatorname{tg}^2 x} = -\frac{1}{2}$

$$\frac{\log(\cos x)}{\operatorname{tg}^2 x} = \frac{\log(1+(\cos x - 1))}{(\cos x - 1)} \quad \frac{\cos x - 1}{\operatorname{tg}^2 x}$$

Uso dei limiti notevoli nei calcoli di alcuni limiti:

1

$$(1)\quad \lim_{x\to 0} (1-x)^{\frac{1}{\cos(x-\frac{\pi}{2})}} = e^{-1}$$

$$\cos\left(x-\frac{\pi}{2}\right) = \operatorname{sen} x$$

$$(1-x)^{\frac{1}{\operatorname{sen} x}} = e^{\frac{\ln(1-x)}{\operatorname{sen} x}} = e^{-\frac{\ln(1-x)}{(-x)}\cdot\frac{x}{\operatorname{sen} x}} \longrightarrow e^{-1}$$

$$(2)^{*} \lim_{x\to\frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg}(2x)} = e^{-1}$$

$$(\operatorname{tg} x)^{\operatorname{tg}(2x)} = e^{\operatorname{tg}(2x)\ln(\operatorname{tg} x)}$$

mi riduco al calcolo del limite:

$$\lim_{x\to\frac{\pi}{4}} \operatorname{tg}(2x)\ln(\operatorname{tg} x) = (\pm\infty\cdot 0)$$

$$\operatorname{tg}(2x) = \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x}$$

~~calcolo i limiti dx e sin x che~~ ~~cosx~~ posso usare la sostituzione $t = \operatorname{tg} x$:

$$\lim_{x\to\frac{\pi}{4}} \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x}\ln(\operatorname{tg} x) = \lim_{t\to 1} \frac{2t\ln t}{1-t^2} =$$

Idea: $\ln t = \ln(1+(t-1))$

se $t\to 1 \Rightarrow t-1 \longrightarrow 0$, da cui:

$$-\frac{2t}{t+1}\,\frac{\ln(1+(t-1))}{t-1} \longrightarrow -1$$