PROPOSIZIONE: $tg\,x$ NON è INTEGRABILE tra $-\frac{\pi}{2}$ e $\frac{\pi}{2}$

dim: ① $tg\,x$ NON è integrabile in $[0,\frac{\pi}{2})$ $\Longrightarrow$ NON può ESSERLO IN un intervallo che contiene $[0;\pi/2)$.

''' Poichè questo NON vi ha sicuramente convinto, consideriamo $\int_{-\pi/2}^{\pi/2} tg\,x\,dx$.

② $\int_{-\frac{\pi}{2}}^{\pi/2} tg\,x\,dx \underset{\text{per def}}{=} \lim_{t,s\to\frac{\pi}{2}} \int_{-s}^{t} tg\,x\,dx$ vogliamo dim che questo limite $\nexists$

(a) • considero $s=t$ cioè $\int_{-t}^{t} tg\,x\,dx = 0 \quad \forall t \Rightarrow \lim_{t\to\frac{\pi}{2}} \int_{-t}^{t} tg\,x\,dx = 0$

(b) • considero $t = s-\sqrt{\frac{\pi}{2}-s}$ allora $\int_{-s}^{t} f(x)\,dx = \int_{-s}^{s-\sqrt{\frac{\pi}{2}-s}} tg\,x\,dx \underset{\text{essendo } \int_{-s}^{s} tg\,x = 0 \ \forall s.}{=} \int_{s}^{s-\sqrt{\frac{\pi}{2}-s}} tg\,x\,dx = \int_{s-\sqrt{\frac{\pi}{2}-s}}^{s} -tg\,x\,dx$

calcolo $\int_{s-\sqrt{\frac{\pi}{2}-s}}^{s} -tg\,x\,dx = \ln(\cos x)\Big|_{s-\sqrt{\frac{\pi}{2}-s}}^{s} = -\ln\left(\frac{\cos(s-\sqrt{\frac{\pi}{2}-s})}{\cos s}\right) =$

$= -\ln\left(\cos\sqrt{\frac{\pi}{2}-s} + \sin s\,\frac{\sin\sqrt{\frac{\pi}{2}-s}}{\cos s}\right) \underset{\cos s = \sin\left(\frac{\pi}{2}-s\right)}{=} -\ln\left(\cos\sqrt{\frac{\pi}{2}-s} + \sin s\,\frac{\sin\sqrt{\frac{\pi}{2}-s}}{\sin\left(\frac{\pi}{2}-s\right)}\right) =$

$= -\ln\left(\underset{\to 1}{\cos\sqrt{\frac{\pi}{2}-s}} + \underset{\to 1}{\sin s}\;\underset{\to 1}{\frac{\sin\sqrt{\frac{\pi}{2}-s}}{\sqrt{\frac{\pi}{2}-s}}}\cdot\underset{\to +\infty}{\frac{\sqrt{\frac{\pi}{2}-s}}{\frac{\pi}{2}-s}}\;\underset{\to 1}{\frac{\frac{\pi}{2}-s}{\sin\left(\frac{\pi}{2}-s\right)}}\right)$

Passando al $\lim_{s\to\frac{\pi}{2}}$ si ottiene $\lim_{s\to\frac{\pi}{2}} \int_{s}^{s-\sqrt{\frac{\pi}{2}-s}} tg\,x\,dx = -\infty$

da (a) e (b) si deduce che

$\nexists \lim_{s,t\to\frac{\pi}{2}} \int_{-s}^{t} tg\,x\,dx$. Cioè $tg\,x$ non è integrabile in senso improprio in $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$.