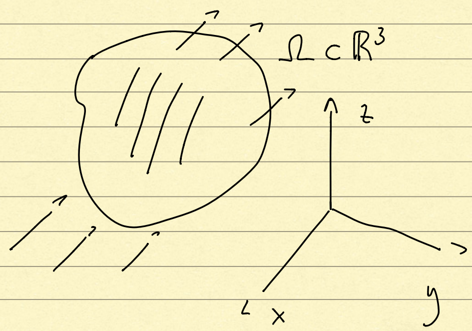
Lun.　　Lun matt. 3 ore　　9.00 - 12.30

Mer　　Merc.　　13.00 - 14.30

Ven.

Termodinamica



$\Omega \subset \mathbb{R}^3$

$T(\underline{x}, t) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$

temperatura

$$H = \int_\Omega \rho e \, d\underline{x} \qquad \text{(entalpia)}$$

e energia interna　　$e(\underline{x}, t)$

$$\frac{dH}{dt} = \int_\Omega \frac{d}{dt}(\rho e) \, d\underline{x}$$

$e = cT$　　　c = calore specifico

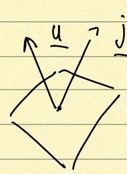
c costante　　　ρ costante

$\underline{j}$ = flusso di calore

$$\frac{dH}{dt} = \int_\Omega \rho c \frac{\partial T}{\partial t} \, d\underline{x} = ??$$

$\underline{j} = -K \nabla T$　　(Fourier)

$K(T)$ conducibilità termica



$d\sigma \subset \partial\Omega$

$\underline{j} \cdot \underline{u} \, d\sigma$ = quantità di calore che fluisce attraverso $d\sigma$ nell'unità di tempo.

$$\frac{dH}{dt} = \int_\Omega \rho c \frac{\partial T}{\partial t} \, d\underline{x} = -\int_{\partial\Omega} \underline{j} \cdot \underline{u} \, d\sigma = \underbrace{\int_{\partial\Omega} K \nabla T \cdot \underline{u} \, d\sigma}_{\text{teo div.}}$$

$$\int_\Omega \rho c \frac{\partial T}{\partial t} \, d\underline{x} \underset{\text{div}}{=} \int_\Omega \operatorname{div}(K \nabla T) \, d\underline{x} \implies \int_\Omega \left(\rho c \frac{\partial T}{\partial t} - K \Delta T\right) d\underline{x} = 0$$

Data l'arbitrarietà di Ω

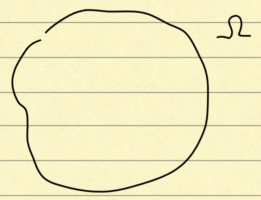
$$\boxed{\rho c \frac{\partial T}{\partial t} - K \Delta T = 0}$$

Eq. ne parabolica in T

$$\Delta T = T_{xx} + T_{yy} + T_{zz}$$

aggiungere C.C. e C.I.

(cond. cont.) (Cond. iniziali)

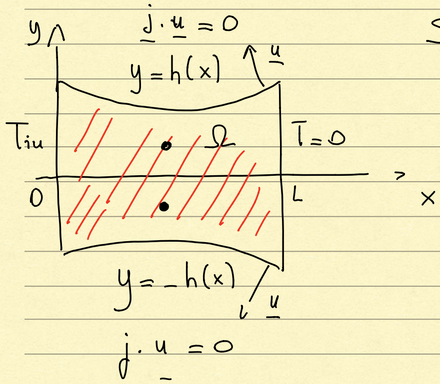


$$T\big|_{\partial\Omega} \qquad T(\underline{x},0) = T_0(\underline{x})$$

$$\frac{\partial T}{\partial n}\Big|_{\partial\Omega} = \nabla T \cdot \underline{u}\Big|_{\partial\Omega}$$

ESEMPIO 2D

Studiamo la conduz. del calore nella sbarretta $\Omega \subset \mathbb{R}^2$



$$\boxed{\begin{cases} \rho c \frac{\partial T}{\partial t} - k(T_{xx} + T_{yy}) = 0 \\ T(x,y,0) = T_0(x,y) \\ T(0,y,t) = T_{in} > 0 \\ T(L,y,t) = 0 \\ -T_x h_x + T_y = 0 \quad \text{su } y = h \\ -T_x h_x - T_y = 0 \quad \text{su } y = -h \end{cases}}$$

$$\underline{j} = -K(T_x, T_y)$$

$$\underline{u} = \frac{(-h_x, 1)}{\sqrt{1 + h_x^2}} \qquad y = h$$

$$T(x,y,t) = T(x,-y,t) \quad \text{(pari)}$$

$$\underline{u} = \left(\frac{-h_x, -1}{\sqrt{1+h_x^2}}\right) \qquad y = -h$$

$$\Rightarrow \quad T_y(x,0,t) = 0$$

posso sostituire a questa

Hyp (di strato sottile) $\quad H = \max_{[0,L]} h(x) \qquad \frac{H}{L} = \varepsilon \ll 1$

RISCALATURA

$$x = L\tilde{x} \qquad y = H\tilde{y} \qquad y = \varepsilon L \tilde{y} \qquad T = T_{in}\tilde{T}$$

$$h = H\tilde{h} \qquad h = \varepsilon L \tilde{h} \qquad t = \theta \tilde{t}$$

$$\rho c \bar{T}_t - k(T_{xx}+T_{yy}) = 0$$

$$\left(\frac{\rho c \cancel{T_{su}}}{\theta}\right)\tilde{T}_{\tilde t} - k\left(\frac{1}{L^2}\tilde T_{\tilde x\tilde x} + \frac{1}{H^2}\tilde T_{\tilde y\tilde y}\right)\cancel{T_{su}} = 0$$

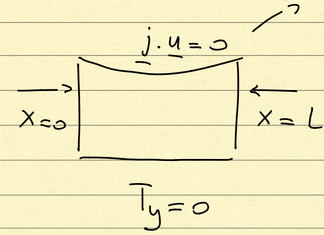
$$\underbrace{\cancel{\left(\frac{\rho c H^2}{k\theta}\right)}}_{=\text{ Fourier number}} \tilde T_t - \left(\varepsilon^2 \tilde T_{\tilde x\tilde x} + \tilde T_{\tilde y\tilde y}\right) = 0$$

$$Fo = \frac{\left(\frac{\rho c H^2}{k}\right)}{\theta}$$ tempo caratt. diffusivo nella direz. y

Supponiamo $\theta \gg \dfrac{\rho c H^2}{k} \Rightarrow Fo \ll 1$

$$\boxed{\begin{array}{l} \varepsilon^2 \tilde T_{\tilde x\tilde x} + \tilde T_{\tilde y\tilde y} = 0 \quad \text{(Ellittica)} \\ \tilde T(0,\tilde y) = 1 \\ \tilde T(1,\tilde y) = 0 \qquad (+) \\ -\varepsilon^2 \tilde h_{\tilde x} \tilde T_{\tilde x} + \tilde T_{\tilde y} = 0 \quad \text{su } \tilde y = \tilde h \\ \tilde T_{\tilde y}(\tilde x, 0) = 0 \end{array}}$$

$\rightarrow -T_x h_x + T_y = 0$



$$-\frac{H}{L^2}\tilde h_{\tilde x}\tilde T_{\tilde x} + \frac{1}{H}\tilde T_{\tilde y} = 0$$

$U_x + U = 0$

$U_x U + U_t = 0$

Strategia: cerco $T(x,y)$ come sviluppo in ε

$$\tilde T = \tilde T_0 + \tilde T_1 \varepsilon + \tilde T_2 \varepsilon^2 + \tilde T_3 \varepsilon^3 + \dots\dots \qquad (*)$$

Sostituisco $(*)$ in $(+)$ e butto via tutti i termini che contengono ε

ORDINE 0 dello sviluppo $(*)$ (ometto i $\sim$ per semplicità)

$$\boxed{\begin{cases} T_{0yy} = 0 \\ T_0|_{(x=0)} = 1 \qquad T_0|_{(x=1)} = 0 \\ T_{0y}|_{y=h} = 0 \qquad T_{0y}|_{y=0} = 0 \end{cases}} \Rightarrow T_0(x) \quad \text{t.c.} \quad T_0(0) = 1 \quad T_0(1) = 0$$

∞ soluzioni

ORDINE 1

$$\boxed{\begin{array}{ll} T_{1yy} = 0 & \\ T_1\big|_{(x=0)} = 0 & T_1\big|_{(x=1)} = 0 \\ T_{1y}\big|_{y=h} = 0 & T_{1y}\big|_{y=0} = 0 \end{array}}$$

$T_1 \equiv 0$ (Anche l'ordine 1 non mi da informazioni)

ORDINE 2

$$\varepsilon^2 T_{xx} + T_{yy} = \varepsilon^2 (T_0 + \varepsilon T_1 + \varepsilon^2 T_2 \dots)_{xx} + (T_0 + \varepsilon T_1 + \varepsilon^2 T_2 \dots)_{yy} = 0$$

$$\boxed{\begin{array}{ll} T_{0xx} + T_{2yy} = 0 & \\ T_2\big|_{x=0} = 0 \quad T_2\big|_{x=1} = 0 & \\ -h_x T_{0x} + T_{2y} = 0 & y = h \\ T_{2y}\big|_{y=0} = 0 & \end{array}}$$

$$\underbrace{\cancel{T_{0yy}}}_{=0} + \varepsilon \underbrace{\cancel{T_{1yy}}}_{=0} + \varepsilon^2 \underbrace{(T_{0xx} + T_{2yy})}_{=0} + \varepsilon^3 \underbrace{(\dots)}_{=0} = 0$$

$$T_{2yy} = -T_{0xx}(x) \qquad T_{2y} = -T_{0xx}\, y$$

$$-h_x T_{0x} - T_{0xx} h = 0 \quad \Rightarrow \quad (h T_{0x})_x = 0 \qquad h(x) T_{0x}(x) = C$$

$$T_{0x} = \frac{C}{h(x)} \qquad \int_0^x T_{0x}\, dx = T_0(x) - \underbrace{T_0(0)}_{1} = C \int_0^x \frac{1}{h(\xi)} d\xi$$

$$T_0(x) = 1 + C \int_0^x \frac{d\xi}{h(\xi)} \qquad 0 = 1 + C \int_0^1 \frac{d\xi}{h(\xi)} \qquad C = -\frac{1}{\int_0^1 h^{-1}(\xi)\, d\xi}$$

$$T_0(x) = \left(1 - \frac{\int_0^x h^{-1} d\xi}{\int_0^1 h^{-1} d\xi}\right)$$

<u>Rk</u>: Se $h \equiv 1$

$$T_0(x) = 1 - x$$

$$\frac{T_0}{T_{in}} = 1 - \frac{x}{L}$$

<u>dim</u>

$$T_0 = T_{in}\left(1 - \frac{x}{L}\right)$$