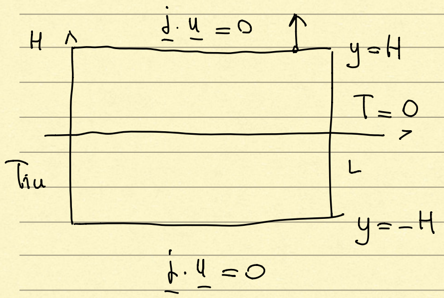
Caso con raffreddamento



$\underline{j} = -k\nabla T$

NEWTON'S COOLING LAW

$k\nabla T\cdot\underline{u} = \alpha(T_{ext} - T_{|H})$

α coeff. scambio termico

$T_{ext} = 0$

Cond. su $y = h$

$\underline{u} = (0,1)$

$k T_y = -\alpha T_{|H}$

$y = \tilde{y} H$

$H/L = \varepsilon \ll 1$

Adimensionalizzo come altra volta

$\tilde{T}_{\tilde{y}} = -\left(\frac{\alpha H}{k}\right)\tilde{T}_{|1}$

Numero di Biot

$B_i = \left(\frac{\alpha H}{k}\right)$

Il problema per $\tilde{T}$

$Bi = O(\varepsilon^2) \qquad Bi = \gamma^2\varepsilon^2$

$$\begin{cases} \varepsilon^2 \tilde{T}_{\tilde{x}\tilde{x}} + \tilde{T}_{yy} = 0 \\ \tilde{T}(0,\tilde{y}) = 1 \\ \tilde{T}(1,\tilde{y}) = 0 \\ \tilde{T}_{\tilde{y}}(x,1) = -\gamma^2\varepsilon^2\,\tilde{T}(x,1) \\ \tilde{T}_{\tilde{y}}(x,0) = 0 \end{cases}$$

Cerco una soluzione

$\tilde{T} = \tilde{T}_0 + \varepsilon\tilde{T}_1 + \ldots$

All'ordine 0 come nell'altro caso trovo $\tilde{T}_0(x)$ con $\tilde{T}_0(0) = 1$ $\tilde{T}_0(1) = 0$

" 1 trovo $\tilde{T}_1(x) \equiv 0$

Pb.mo ordine 2

$$\boxed{\begin{cases} \tilde{T}_{0xx} + \tilde{T}_{2yy} = 0 \\ \tilde{T}_{2|x=0} = \tilde{T}_{2|x=1} = 0 \\ \tilde{T}_{2y}\big|_{y=1} = -\gamma^2 T_0\big|_{y=1} \end{cases}}$$

$\tilde{T}_{0xx}(x)$

$\tilde{T}_{2y} = 0$ su $\tilde{y} = 0$

Integro e trovo

$$\tilde{T}_{2yy} = -\tilde{T}_{0xx} \qquad \tilde{T}_{2y} = -\tilde{T}_{0xx}\, y$$

$$\tilde{T}_{2y}\Big|_{\tilde{y}=1} = -\tilde{T}_{0xx} = -\gamma^2 \tilde{T}_0 \quad \Rightarrow \quad \tilde{T}_{0xx} - \gamma^2 \tilde{T}_0 = 0$$

$$\begin{cases} \tilde{T}_0'' - \gamma^2 \tilde{T}_0 = 0 \\ \tilde{T}_0(0) = 1 \\ \tilde{T}_0(1) = 0 \end{cases} \Rightarrow \qquad \tilde{T}_0 = C_1 e^{-\gamma x} + C_2 e^{\gamma x} \qquad \begin{cases} C_1 + C_2 = 1 \\ C_1 e^{-\gamma} + C_2 e^{\gamma} = 0 \end{cases}$$

$$C_1 = 1 - C_2$$

$$e^{-\gamma} + C_2 (e^{\gamma} - e^{-\gamma}) = 0 \qquad C_2 = -\frac{e^{-\gamma}}{(e^{\gamma} - e^{-\gamma})} = -\frac{e^{-\gamma}}{2\sinh\gamma}$$

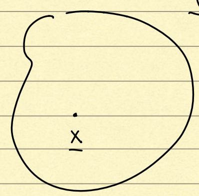
$$C_1 = \frac{e^{\gamma}}{2\sinh\gamma} \qquad \tilde{T}_0(x) = \frac{e^{\gamma(1-x)} - e^{-\gamma(1-x)}}{2\sinh\gamma}$$

$$\tilde{T}_0(x) = \frac{\cancel{2}\sinh(\gamma(1-x))}{\cancel{2}\sinh\gamma} \qquad \gamma^2 \varepsilon^2 = Bi = \frac{\alpha H}{k}$$

$$\alpha = 0 \iff \gamma = 0$$

Se $\gamma = 0$ $\qquad \tilde{T}_0(x) = 1 - x$

Problemi di convezione e diffusione

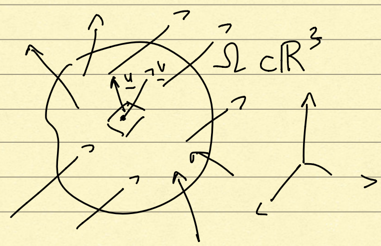


$\Omega \subset \mathbb{R}^3$

$$\underline{j}_d = -k \nabla T \quad \text{(Fourier)}$$

$$\underline{j}_{conv} = \rho c T \underline{v} \quad \text{(Trasporto convettivo)}$$

$\underline{v}$ è la velocità della particella di fluido che all'ist. t occupa la posizione $\underline{x}$

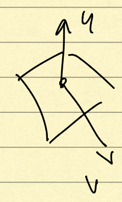


$$m(\Omega) = \int_\Omega \rho(\underline{x}, t)\, d\underline{x}$$

Per vedere come varia la massa

$$\frac{dm}{dt} = \int_\Omega \frac{\partial \rho}{\partial t}\, d\underline{x} = -\int_{\partial\Omega} \rho \underline{v} \cdot \underline{u}\, d\sigma$$

$\rho \underline{v} \cdot \underline{u}\, d\sigma$ è la massa che fluisce attraverso $d\sigma$ nell'unità di tempo



dal teo della divergenza

$$\int_\Omega \left[\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \underline{v})\right] d\underline{x} = 0$$

Data l'arbitrarietà di Ω trovo

$$\boxed{\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \underline{v}) = 0}$$

Se ρ è costante $\Longrightarrow$ $\boxed{\operatorname{div} \underline{v} = 0}$ (fluido incomprimibile)

Eq.ne del calore con convezione e diffusione

$\underline{j}_d = -k\nabla T \qquad \underline{j}_c = \rho c T \underline{v}$

Bilancio di energia

$$\frac{dH}{dt} = \int_\Omega \rho c \frac{\partial T}{\partial t}\, d\underline{x} = -\int_{\partial\Omega} \underline{j}_d \cdot \underline{u}\, d\sigma - \int_{\partial\Omega} \underline{j}_{con} \cdot \underline{u}\, d\sigma$$

$$\int_\Omega \rho c \frac{\partial T}{\partial t}\, d\underline{x} = \int_\Omega k \Delta T\, d\underline{x} - \int_\Omega \mathrm{div}(\rho c T \underline{v})\, d\underline{x}$$

Per l'arbitrarietà di Ω $\quad \rho c \frac{\partial T}{\partial t} = k\Delta T - \mathrm{div}(\rho c T \underline{v})$

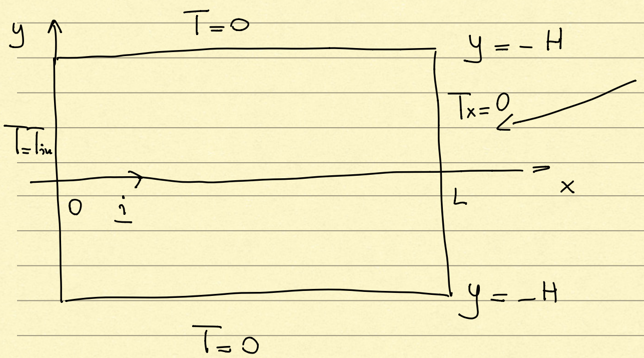
$$\rho c \left(\frac{\partial T}{\partial t} + \mathrm{div}(T\underline{v}) \right) = k \Delta T$$

ρ cost. fluido omogeneo

$$\mathrm{div}(T\underline{v}) = T \cdot \underbrace{\mathrm{div}\,\underline{v}}_{=0} + \nabla T \cdot \underline{v}$$

poiché ρ è costante

Eq.ne cond. e conv. è $\boxed{\rho c \left(\cancel{\frac{\partial T}{\partial t}} + \nabla T \cdot \underline{v} \right) = k \Delta T}$ (*)



Canale in cui scorre il mio fluido omogeneo

Suppongo che $\underline{v} = V\underline{i}$

Caso stazionario

SCALING di (*) $\qquad T = T_{in}\tilde{T} \qquad x = L\tilde{x} \qquad y = H\tilde{y}$

$$\varepsilon = \frac{H}{L} \ll 1$$

$$\rho c \,(T_x U) = K\,(T_{xx} + T_{yy})$$

$$\left(\frac{\rho c U}{L}\right)\tilde{T}_x = \frac{K}{H^2}\left(\varepsilon^2 \tilde{T}_{xx} + \tilde{T}_{yy}\right)$$

$$\underbrace{\left(\frac{\rho c U H^2}{LK}\right)}_{= Pe} \tilde{T}_x = \varepsilon^2 \tilde{T}_{xx} + \tilde{T}_{yy}$$

$$Pe = \frac{\left(\frac{\rho c H^2}{K}\right)}{(L/U)} = \frac{t_d}{t_c}$$

$= Pe$ numero di Peclet

t_d tempo caratt. diffusivo

t_c " " convettivo

$Pe = O(1) \implies Pe = 1$

$$\boxed{\begin{cases} \tilde{T}_x = \varepsilon^2 \tilde{T}_{xx} + \tilde{T}_{yy} \\ \tilde{T}(0,\tilde{y}) = 1 \\ \tilde{T}(x,1) = 0 \\ \tilde{T}_x(1,y) = 0 \\ \tilde{T}_y(x,0) = 0 \end{cases}}$$

Ordine 0 $\implies$

$\tilde{T} = \tilde{T}_0 + \varepsilon \tilde{T}_1 \dots$

$$\boxed{\begin{cases} \tilde{T}_x = \tilde{T}_{yy} \\ \tilde{T}(0,\tilde{y}) = 1 \\ \tilde{T}(x,1) = 0 \\ \cancel{\tilde{T}_x(1,y) = 0} \\ \tilde{T}_{\tilde{y}}(x,0) = 0 \end{cases}}$$

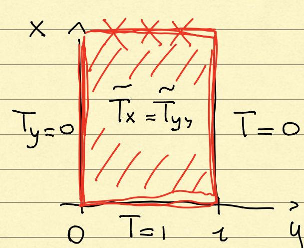
(O metto il pedice 0)

??

(*)

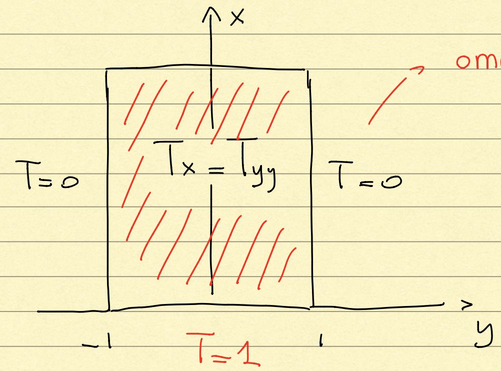
Questo pb.mo è mal posto

Il problema (*) è una PDE parabolica lineare in cui x gioca il ruolo del tempo



$x = L\tilde{x}$

Mi dimentico della condizione $\cancel{T_x\big|_{x=1}=0}$ e risolvo il pb.ma



ometto i ~

Lo risolvo separando le variabili

Cerco

$$T_0(x,y) = X(x)Y(y)$$

$$X'Y = Y''X$$

$$\frac{X'}{X} = \frac{Y''}{Y} = \text{Costante} \begin{cases} \lambda^2 > 0 \\ -\lambda^2 < 0 \\ 0 \end{cases}$$

Se Cost $= \lambda^2 > 0$ $\Downarrow$

$$Y'' - \lambda^2 Y = 0 \qquad Y(y) = Ae^{-\lambda y} + Be^{\lambda y}$$

$$T_0\big|_{y=\pm 1} = 0 \implies \begin{cases} Ae^{-\lambda} + Be^{\lambda} = 0 \\ Ae^{\lambda} + Be^{-\lambda} = 0 \end{cases} \implies Y \equiv 0$$

$\Downarrow$

$$T_0 \equiv 0$$

Se $\lambda^2 = 0$

Questo nega $T_0(0,y) = 1$

$$Y'' = 0 \implies Y(y) = Ay + B$$

Imponendo $Y(\pm 1) = 0 \implies \begin{cases} A + B = 0 \\ A - B = 0 \end{cases} \implies Y \equiv 0$ $\Downarrow$