Equazione strato limite (ometto i $\sim$) $\qquad \varepsilon \ll 1$

$$\begin{cases} V_1 V_{1x} + V_2 V_{1y} = -p_x + \alpha(\varepsilon^2 V_{1xx} + V_{1yy}) \\ V_1 V_{2x} + V_2 V_{2y} = -\dfrac{p_y}{\varepsilon^2} + \alpha(\varepsilon^2 V_{2xx} + V_{2yy}) \\ V_{1x} + V_{2y} = 0 \end{cases} \qquad \begin{aligned} & V_1 \big|_{y=+\infty} = 1 \\ & p\big|_{y=+\infty} = p_0 \\ & V_1 = V_2 = 0 \text{ su } y = 0 \end{aligned}$$

Ordine 0 dello appross.

$$\begin{cases} V_1 V_{1x} + V_2 V_{1y} = \cancel{-p_x} + \alpha V_{1yy} \\ p_y = 0 \implies p = p(x) \implies p = p_0 \text{ ovunque} \implies p_x = 0 \\ V_{1x} + V_{2y} = 0 \end{cases}$$

$$\begin{cases} V_1 V_{1x} + V_2 V_{1y} = \alpha V_{1yy} \\ V_{1x} + V_{2y} = 0 \end{cases} + \text{C.C.}$$

Cerchiamo soluzioni autosimilari (Blasius)

$$V_1(x,y) = f'(s) \qquad s = \frac{y}{\sqrt{\alpha x}}$$

$$V_{1x} = f''(s)\frac{\partial s}{\partial x} \qquad \frac{\partial s}{\partial x} = \frac{y}{\sqrt{\alpha}}\left(x^{-1/2}\right)' = -\frac{y}{2\sqrt{\alpha}} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{x} = -\frac{s}{2x}$$

$$V_{1x} = -f''(s)\frac{s}{2x} \qquad V_{1y} = f''(s)\frac{1}{\sqrt{\alpha x}} \qquad V_{1yy} = f'''(s)\frac{1}{\alpha x}$$

$$\frac{\partial V_2}{\partial s} = \frac{\partial V_2}{\partial y}\cdot\frac{\partial y}{\partial s} = -\frac{\partial V_1}{\partial x}\cdot\frac{\partial y}{\partial s} = -\frac{\partial V_1}{\partial x}\sqrt{\alpha x} = f''(s)\, s\,\frac{\sqrt{\alpha x}}{2x}$$

$$V_2 = \int \frac{\partial V_2}{\partial s}\,ds = \frac{\sqrt{\alpha x}}{2x}\int f''(s)\,s\,ds = \frac{\sqrt{\alpha x}}{2x}\left[f's - f\right]$$

$$V_1 V_{1x} + V_2 V_{1y} = \alpha V_{1yy}$$

$$-f'f''\frac{s}{2\cancel{x}} + \frac{\cancel{\sqrt{\alpha x}}}{2\cancel{x}}\left[f's - f\right]f''\frac{1}{\cancel{\sqrt{\alpha x}}} = \cancel{\alpha}\,\frac{f'''}{\cancel{\alpha}\cancel{x}}$$

$$-\cancel{f'f''s} + (\cancel{f's} - f)f'' = 2f'''$$

$$s = \frac{y}{\sqrt{\alpha x}}$$

$$\Rightarrow \boxed{2f'''(s) + f''(s)f(s) = 0 \quad (*)}$$

Eq.ne di Blasius

Eq.ne non lineare del 3° ordine

C.C. $\quad V_1\big|_{y=+\infty} = 1 \quad \rightsquigarrow \quad V_1 = f'(s) \quad \boxed{f'(0)=0 \quad f'(\infty)=1}$

$\quad V_1 = V_2\big|_{y=0} = 0 \qquad V_2 = \frac{\sqrt{\alpha x}}{2x}\left[f's - f\right] \quad \boxed{f(0)=0}$

Trasformo (*) in un sistema del 1° ordine

$$u = f'$$
$$u' = f'' = \eta$$
$$\eta' = f''' = -\left(\frac{f''f}{2}\right)$$

$$\frac{d}{ds}\begin{pmatrix} f \\ u \\ \eta \end{pmatrix} = \begin{pmatrix} f' \\ u' \\ \eta' \end{pmatrix} = \begin{pmatrix} u \\ \eta \\ -\frac{\eta f}{2} \end{pmatrix} = \underline{F}(f,u,\eta) \quad (**)$$

C.C. $\quad f(0)=0 \quad u(0)=0 \quad u(\infty)=1$

Trasformo il pb.mo (**)(*) in un pb.mo di Cauchy

$$\frac{d}{ds}\begin{pmatrix} f\\ u\\ \eta\end{pmatrix} = \underline{F}(f,u,\eta) \qquad \begin{cases} f(0)=0\\ u(0)=0\\ \eta(0)=\gamma\end{cases}$$

γ è incognito

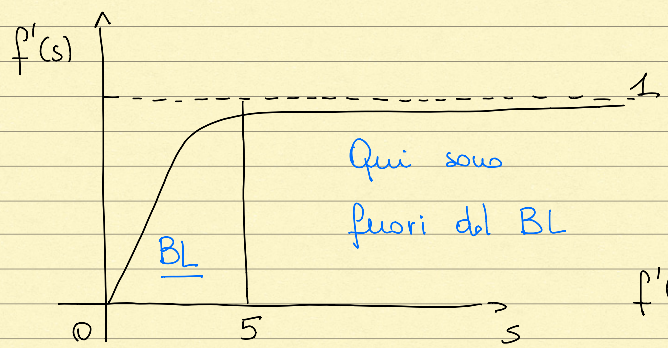
$\Rightarrow$ ottengo $f(s;\gamma)$; $u(s;\gamma)$; $\eta(s;\gamma)$

$$U'(s) = \frac{\partial U}{\partial s} = \eta \qquad \int_0^{+\infty} U'(s)\,ds = U(\infty) - \cancel{U(0)} = 1$$

Scelgo γ in modo che $\int_0^{+\infty} \eta(s;\gamma)\,ds = 1$

Una volta che il pb.mo è risolto numericamente (col γ "giusto")

faccio il plot di $f'(s) = U(s) = V_1$

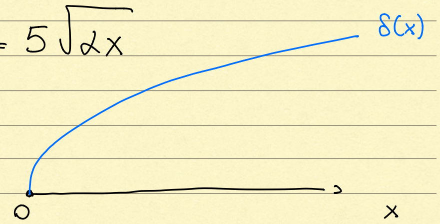


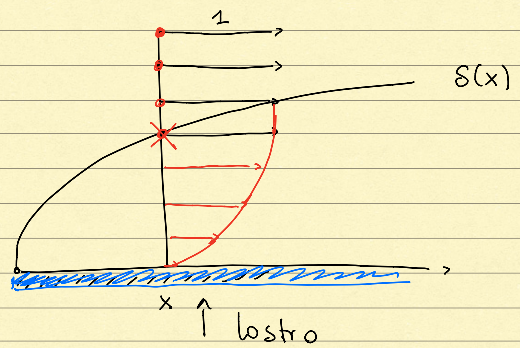
$$\lim_{s \to +\infty} f'(s) = 1$$

$f'(5) \simeq 0.99$

$$s = \frac{y}{\sqrt{\alpha x}} \qquad 5 = \frac{y}{\sqrt{\alpha x}} \qquad y = \delta(x) = 5\sqrt{\alpha x}$$

Se $y > 5\sqrt{\alpha x}$ (sono fuori dal BL)





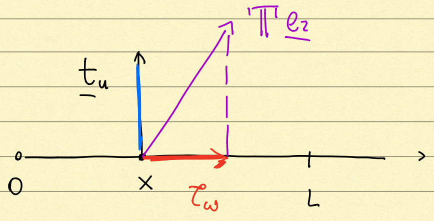
Ritorniamo a variabili dimensionali $\tilde{\delta}(\tilde{x}) = 5\sqrt{\alpha \tilde{x}}$

$$\alpha = \frac{1}{Re\,\varepsilon^2} = \frac{\mu}{\rho V_\infty L} \cdot \frac{L^2}{H^2} \qquad \tilde{\delta} = \frac{\delta}{H}$$

$$\frac{\delta}{\cancel{H}} = 5\sqrt{\frac{\mu}{\rho V_\infty \cancel{L}} \cdot \frac{\cancel{L^2}}{\cancel{H^2}} \cdot \frac{x}{\cancel{L}}} \qquad \nu = \left(\frac{\mu}{\rho}\right) \text{ viscosità cinematica}$$

$$\delta = 5\sqrt{\frac{\nu x}{V_\infty}}$$

Calcoliamo infine lo "sforzo" (forza di trascinamento) sulla lastra



$$\tau_w = \mathbb{T}\,\underline{e}_2 \cdot \underline{e}_1$$

$$\tau_w = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{12} & T_{22} & T_{23} \\ T_{13} & T_{23} & T_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} T_{12} \\ T_{22} \\ T_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = T_{12}$$

$\tau_w = T_{12}$

$$\mathbb{T} = -p\mathbb{I} + \mu(\nabla \underline{v} + \nabla \underline{v}^T) \quad \text{(fluidi newtoniani)} \qquad \nabla \underline{v} = \begin{pmatrix} v_{1x} & v_{1y} & v_{1z} \\ v_{2x} & v_{2y} & v_{2z} \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$T_{12} = \mu \left(\frac{\partial v_1}{\partial y} + \cancel{\frac{\partial v_2}{\partial x}} \right)$$ lo voglio T_{12} su $y=0$

$\rightarrow$ su $y = 0$

$$v_2(x,0) = 0 \quad \Rightarrow \quad \frac{\partial v_2}{\partial x}(x,0) = 0$$

la forza di trascinamento dimensionale su $y=0$ è

$$\boxed{\tau_w = \mu \frac{\partial v_1}{\partial y}\Big|_{y=0}}$$

Se ritorno a variabili adimensionali (con i $\sim$)

$$\tau_w = \left(\frac{\mu V_\infty}{H}\right) \frac{\partial \tilde{v}_1}{\partial \tilde{y}}\Big|_{\tilde{y}=0} = \left(\frac{\mu V_\infty}{H}\right) \frac{f''(s=0)}{\sqrt{\alpha \tilde{x}}} = \frac{\mu V_\infty}{H\sqrt{\alpha \tilde{x}}} \overset{0.34}{f''(0)}$$

$$\tau_w = \frac{\mu V_\infty \, f''(0)}{\cancel{H}\sqrt{\underbrace{\frac{\mu}{\rho V_\infty \cancel{L}} \frac{\cancel{L}^2}{\cancel{H}^2}}_{=\alpha} \underbrace{\frac{x}{\cancel{L}}}_{\tilde{x}}}} = \frac{\mu V_\infty \, f''(0)}{\sqrt{\frac{\nu x}{V_\infty}}} = \frac{\mu V_\infty \sqrt{V_\infty}}{\sqrt{\nu x}} f''(0)$$

Se voglio la forza di trascinamento totale su $y=0$ (la lastra)

$$\tau_w = \frac{\mu V_\infty^{3/2}}{\sqrt{\nu}} f''(0) \int_0^L \frac{dx}{\sqrt{x}} = \frac{\mu V_\infty^{3/2}}{\sqrt{\nu}} f''(0)\, 2\sqrt{L}$$

$$\nu = \frac{\mu}{\rho}$$

$$\boxed{\tau_w = \sqrt{\rho\mu}\, V_\infty^{3/2} f''(0)\, 2\sqrt{L}}$$