EXERCISES FOR THE COURSE
FUNCTIONS OF COMPLEX AND HYPERCOMPLEX VARIABLE

As usual, $\mathbb{H}$ denotes the real algebra of quaternions. Let $\Omega$ be a symmetric slice domain in $\mathbb{H}$ and let $(\text{Reg}(\Omega), +, *, ^c)$ denote the $*$-algebra of regular functions $f : \Omega \to \mathbb{H}$. The zero set of such an $f$ is denoted as $Z(f)$. Recall that we have set $f^* := f^* f = f^c f$.

Exercise 1. Define $f : \mathbb{H} \to \mathbb{H}$ by setting $f(q) := (q - 1 + k) * (q + i)$.
- Prove that $f^*(q) = (q^2 - 2q + 2) * (q^2 + 1) = (q^2 - 2q + 2)(q^2 + 1)$.
- Find 2-spheres $S_1, S_2$ such that $Z(f^*) = S_1 \cup S_2$.
- Determine $Z(f)$.

Exercise 2. Define $f : \mathbb{H} \to \mathbb{H}$ by setting $f(q) := (q + k) * (q + i)$.
- Compute $f^*(q)$.
- Determine $Z(f^*)$.
- Determine $Z(f)$.

Exercise 3. Prove that there are no zero divisors in $\text{Reg}(\Omega)$. In other words, prove that: $f^* g \equiv 0$ implies $f \equiv 0$ or $g \equiv 0$.

Exercise 4. Provide an example of regular function $f : \mathbb{H} \setminus \mathbb{R} \to \mathbb{H}$ for which the Maximum Modulus Principle does not hold. Same question for the Minimum Modulus Principle and the Open Mapping Theorem.

Exercise 5. Prove that the regular function $f : \mathbb{H} \to \mathbb{H}$ defined by $f(q) = q^2 + qi$ is an open mapping.